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The Quarterly Projection Model of the SARB

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Abstract

The macroeconomic modelling and forecasting process at the South African Reserve Bank makes use of a suite of models. This paper provides an update of the Quarterly Projection Model (QPM) – a so-called *gap* model – which has played an integral role in the suite since 2007. Details of the structure and functioning of the QPM model, with particular focus on the four most important gaps – the output gap, real exchange rate gap, real interest rate gap, and inflation gap (or inflation from target) – are provided. The model is then used to decompose these four gaps in order to tell a coherent story of South Africa's macroeconomic dynamics since the inception of inflation targeting. From the perspective of the policy maker, the QPM provides a tool that quantifies the consequences of its actions on the economy, while adequately highlighting the trade-offs that are faced in the process.

JEL Classification: E50, C53, Z00

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1 Introduction

The macroeconomic modelling and forecasting process at the South African Reserve Bank makes use of a suite of models. All models have their specific strengths and weaknesses, and the use of a suite allows the models to complement each other in order to generate better policy outcomes. Over time, the models have evolved along with advances in both econometric theory and computing power. This paper provides an update of the Quarterly Projection Model (QPM) – a so-called *gap* model – which has played an integral role in the suite since 2007.[1]

The QPM is a general equilibrium model (GEM) that has its foundation in modern macroeconomic theory.[2] The underlying assumption is that agents in the economy are forward looking, *i.e.*, expectations about the future matter for decisions that are made today. For example, a firm would set the price for the good it produces by taking into account its expectation for future inflation. In terms of policy making, an inflation-targeting central bank would also set interest rates today based on its expectation for inflation 12 – 24 months into the future.

An equally important characteristic of the QPM is the fact that the economy evolves around an underlying, but well-defined, equilibrium path. The so-called *gaps* develop when the economy deviates from its equilibrium path. In general, these deviations from equilibrium are caused by a variety of shocks. While shocks are mostly transitory, the impacts on gaps (such as the output gap) they induce are quite persistent due to the presence of nominal and real rigidities. The role of monetary policy is to assist in closing the gaps by guiding the economy back to its equilibrium path and, most importantly, inflation back to it's target level.

The use of QPM-style models was pioneered by the Bank of Canada in the 1990s. Since then, many central banks with inflation-targeting mandates have incorporated these models in their suites. In general, QPMs provide longer term economic projections with relative ease. They also enable central banks to analyse the nature of the shocks that are impacting on the economy, as well as the longer term equilibrating forces that influence economic behaviour over time. However, since QPMs generally have the limitation that they are highly aggregated – focusing on GDP as a whole and not the underlying components of aggregate demand – they need to be complemented with other models that explain the behaviour of specific variables that are analysed at a more disaggregated level.

This latest revision of the SARB's Quarterly Projection Model presents further enhancements to the initial model of De Jager (2007) and its revised version by De Jager, Johnston, and Steinbach (2015). It is important to emphasise that these various publications of the model reflect the state of thinking at the time of release. The model will continue to undergo changes as new empirical evidence is uncovered, or as economic theory evolves. The paper is laid out as follows: first, a bird's-eye view of the model is given in the next section; thereafter follows a more technical discussion of the model's key equations in Section 4. The various empirical properties of the model, ranging from its parameterisation to its dynamic responses to shocks, is discussed in Section 5, before Section 6 concludes.

[1] See De Jager (2007) for a detailed description of this initial version of the model, and De Jager, Johnston, and Steinbach (2015) for an extended version thereof.

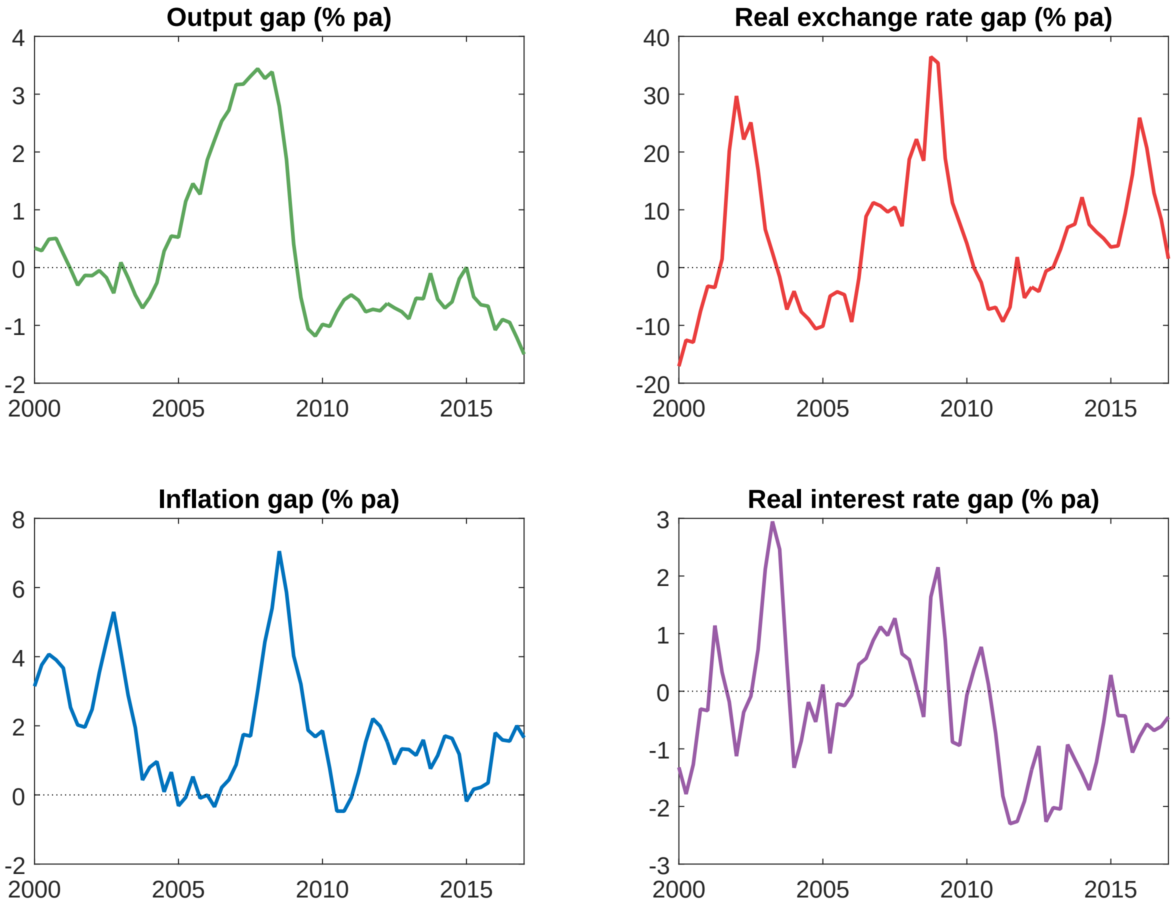
[2] Essentially, the QPM is a simplified, user-friendly version of the more rigorous dynamic stochastic general equilibrium (DSGE) models that are a popular tool in both academic and central banking circles.

2 The four gaps: A bird's-eye view of the QPM

The model is premised on the key assumption that monetary policy (in practice) cannot influence the fundamental long-term equilibrium trends in the economy, and therefore only affects the cyclical features of the economy, or *gaps*, as they are referred to here (*i.e.*, the deviations from long-term equilibrium trends).

Essentially, there are four gaps that matter for policy analysis and forecasting in the QPM: (1) the output gap; (2) the real exchange rate's degree of over/undervaluation; (3) inflation's deviation from the target; and (4) the real interest rate's deviation from it's neutral level. These gaps (or disequilibria) are illustrated in Figure 1, and discussed in more detail below.

Figure 1: The four gaps



2.1 Output gap

The output gap reflects the deviation of the level of real GDP from its potential level. Here, the premise is that if the current level of GDP is the same as its potential level, this gap would be zero and will therefore be at it's equilibrium level, *i.e.* where there is no excess or insufficient demand pressures impacting on inflation.

The key factors in the QPM that influence the output gap are the real interest rate gap, the real exchange rate gap, commodity terms of trade effects and foreign demand, which is expressed as the output gap in the rest of the world.

2.2 Exchange rate gap

The real exchange rate gap – the deviation of the real exchange rate from its equilibrium level – essentially shows to what extent the currency is either over/undervalued, or the pressure that the currency is exerting on inflation. A closed gap (*i.e.* a real exchange rate that is in line with its equilibrium level) is consistent with output at potential, and inflation at its target. Exchange rates in the QPM are defined as the price of foreign currency in terms of the rand. A positive gap therefore reflects a rand that has depreciated beyond its equilibrium level, in real terms.

The real exchange rate in the model is determined by an uncovered interest parity condition (UIP) that relates expected currency movements to the risk-adjusted differential between real interest rates at home and abroad. Similarly, the equilibrium trend of the real exchange rate is defined by an equilibrium UIP condition (i.e. where the interest rates at home and abroad are represented by their neutral levels and the equilibrium country risk premium).

2.3 Inflation gap

The inflation gap is the deviation of the rate of headline CPI inflation from the mid-point of the inflation target band. This is driven, *inter alia*, by the two gaps stated above, as well as wage pressures and the expectations of future inflation. The QPM allows for the headline CPI to be explicitly decomposed into its non-core and core subcomponents, where non-core inflation components such as food, fuel, and electricity are separately defined.

- CPI food inflation has a weight of 17.24 per cent in the overall CPI basket, and is determined by international food prices, the exchange rate, domestic demand, and input costs related to labour and fuel.
- Fuel prices account for 4.58 per cent of CPI, and are determined by the international oil price, the exchange rate, as well as taxes and margins.
- Electricity inflation, with a weight of 3.75 per cent, is generally treated as exogenous, and is assumed to follow a prescribed path over the forecast.
- Core CPI makes up 74.43 per cent of headline CPI, and is split into core services (2/3) and core goods (1/3). Both subcomponents are largely determined by real wage pressures, the real exchange rate gap, imported inflation, the output gap and inflation expectations.

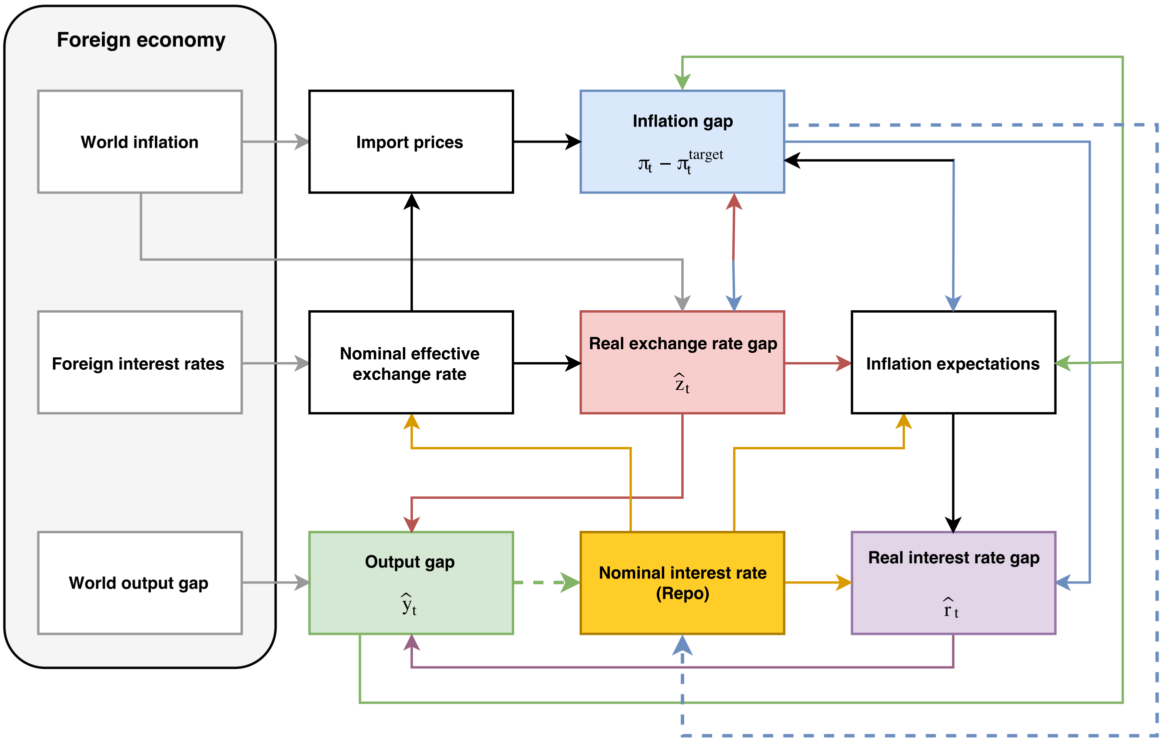
2.4 Real interest rate gap

The real interest rate gap reflects the deviation of the real (short-term) interest rate from its neutral level. The real interest rate is calculated, using the Fischer equation, as the nominal interest rate minus expected inflation (or some forecast of inflation one-period ahead), *i.e.* where the nominal short-term interest rate is determined by the central bank's policy reaction function using a Taylor-type rule. The nominal repo rate in this version of the Taylor rule reacts to the deviation of forecasted inflation from the target midpoint, as well as the output gap.

3 How does monetary policy work in the QPM?

The mechanism by which monetary policy transmits to the real economy, and ultimately inflation, is summarised in Figure 2.[3] To illustrate, assume that the economy starts in a position of equilibrium, *i.e.*, the output gap is closed, the real exchange rate is at its equilibrium (fair) value, the real interest rate is at its neutral level, and inflation is at its target. If the Bank were to raise the repo rate, inflation would be affected via two channels: (1) the real economy, and (2) the exchange rate. A higher repo rate immediately increases the real interest rate as prices are sticky. The real interest rate now exceeds its neutral level. The positive real rate gap induces a negative output gap, as GDP falls below its potential level. Hereafter, inflation starts to decline as the wider output gap creates disinflationary pressure. Within the exchange rate channel, the higher repo rate leads to an appreciation of the nominal and real exchange rate (the latter due to price stickiness). Not only does the stronger exchange rate put further downward pressure on inflation, it also puts the real economy under further strain through a deterioration of the net exports position. The combination of lower inflation and a wider output gap causes the Bank to reduce the repo rate. Reducing the repo rate will: (1) lower the real interest rate; (2) lift the output gap; (3) depreciate the exchange rate; and (4) raise inflation, closing the four gaps.

Figure 2: The monetary policy transmission mechanism in the QPM



This example merely illustrates the workings of the model for one specific shock – a sudden change to monetary policy – that causes the repo rate to move first. However, shocks may emanate from abroad, or in the domestic economy, and cause the exchange rate, inflation, or output to move first. In reality, many shocks occur simultaneously. Nevertheless, the closing of the gaps occurs through the same four basic channels.

[3]For the sake of exposition, certain key channels in the QPM are omitted from this diagram.

4 The model

Before discussing the key equations of the model, it is worth explaining the notation that we have used. Let x_t represent any variable (in terms of log $\times$100). Where a *hat* appears above the variable, $\hat{x}_t$, it indicates that the variable is a gap, or deviation from equilibrium. In turn, when a line appears above the variable, $\bar{x}_t$, reference is being made to the variable's equilibrium value. To indicate the steady state of a variable, a superscript ss is used and the time subscript t is not shown (since steady states remain constant over time): x^{ss}. Finally, the quarter-on-quarter change in a variable is denoted by a Δ that precedes it: Δx_t. The only exception here is inflation – the quarter-on-quarter change in the (log) price level – where we use π's, as it is the custom in most of the literature[4].

4.1 Real economy

The real economy is modelled as an investment-savings (IS) curve. Here, the output gap (i.e., deviations of output from its equilibrium level) is determined by four fundamental variables: (1) the real interest rate gap $\widehat{lr}_t$; (2) the real exchange rate gap $\hat{z}_t$; (3) the foreign output gap $\hat{y}_t^*$; and (4) the deviation of South Africa's export commodity price index (real) from its trend value $\widehat{rcom}_t$. In addition, expectations about the future real economy, $E_t\hat{y}_{t+1}$, as well as recent outcomes $\hat{y}_{t-1}$ matter for the dynamic behaviour of the output gap. Finally, the residual term, $\varepsilon_t^{\hat{y}}$ captures demand shocks, while the a's are the parameters that indicate the direct impact each of the variables above have on the output gap:

$$\begin{aligned} \hat{y}_t &= a_1 E_t\hat{y}_{t+1} + a_2\hat{y}_{t-1} \\ &- a_3\Big[a_4\widehat{lr}_t + (1-a_4)(-\hat{z}_t)\Big] \\ &+ a_5\hat{y}_t^* + a_6\widehat{rcom}_t + \varepsilon_t^{\hat{y}}, \end{aligned} \tag{1}$$

$$a_1 = 0.15, \quad a_2 = 0.60, \quad a_3 = 0.15, \quad a_4 = 0.95, \quad a_5 = 0.15, \quad a_6 = 0.01$$

where the output gap is defined as the difference between the log of real GDP and that of potential GDP, times 100, in order to express the gap in percentage terms:

$$\begin{aligned} \hat{y}_t &= \Big[\log(\text{GDP}_t) - \log(\text{potental GDP}_t)\Big]\times 100 \\ &= y_t - \bar{y}_t. \end{aligned} \tag{2}$$

4.2 Prices and wages

In the QPM, inflation is generally modelled as a New Keynesian Phillips curve. It is the modern version of the original Phillips curve that permeated macroeconomics since the late 1950s, when A.W. Phillips (1958) documented the relationship between wage growth and unemployment in the United Kingdom. Over time, the original specification of this central tenet – that inflation is linked to conditions in the real economy – evolved into its modern version.[5] Accordingly, the New Keynesian Phillips curve expresses current inflation as a function of the inflation rate that price setters expect to hold in the future, past inflation outcomes, as well as price pressures that emanate

[4] See Appendix C for a complete list of the variables and their definitions.

[5] For example, Friedman (1968) correctly argued that the Phillips curve is only valid if it also accounts for inflation expectations.

from changes in the *real* costs of production – commonly referred to as real marginal costs.[6,7] In the model, these real marginal costs are generally driven by real wages, the real exchange rate, and the output gap.

4.2.1 Headline CPI

To ensure that the underlying dynamics in the headline consumer price index (CPI) are adequately captured with sufficient detail, CPI is disaggregated into its core and non-core subcomponents. In turn, the core CPI is split into its services and goods subcomponents, while non-core CPI is made up by separate equations for the CPI indices of food, electricity and fuel prices.

Headline CPI is constructed as follows:

$$\begin{aligned} p_t^{cpi} &= (1 - w_t^{food} - w_t^{elec} - w_t^{petr}) p_t^{core} \\ &+ w_t^{food} p_t^{food} \\ &+ w_t^{elec} p_t^{elec} \\ &+ w_t^{petr} p_t^{petr} \end{aligned} \tag{3}$$

where the p_t's denote the (log) price levels for headline CPI and its subcomponents, and the w_t's denote the weights of the respective subcomponents. Similarly, core CPI is further decomposed into the CPI for services and core goods:

$$\begin{aligned} p_t^{core} &= w_t^{services} p_t^{services} \\ &+ (1 - w_t^{services}) p_t^{coregoods} \end{aligned} \tag{4}$$

4.2.2 Relative prices and their trends

When the price a specific good changes relative to the price of others, such *relative price* movements often alter consumption patterns, as consumers would substitute a less expensive good for the good that has become more expensive. These relative price movements are important drivers of overall inflation dynamics, since, in theory, reduced demand for the good that has become relatively more expensive, should ultimately constrain the price change that caused the initial relative price movement.

In the model, the relative prices of the components of headline CPI are all expressed relative to headline CPI:

$$relp_t^i = p_t^i - p_t^{cpi} \tag{5}$$

where i represents food, electricity, fuel, and core (which is split between services and core goods).

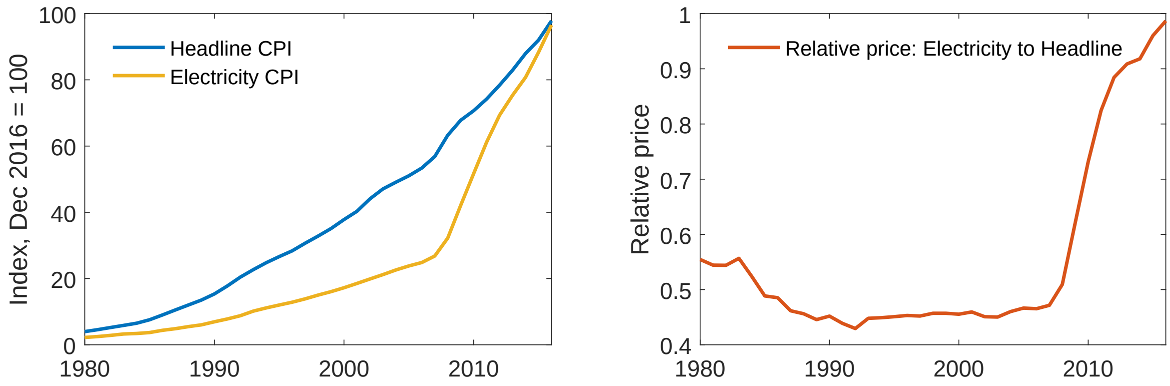
While theory would predict that substitution effects ultimately correct relative price changes, this does not hold in the South African data. On average, CPI electricity inflation is higher than CPI food and fuel inflation. In turn, these non-core inflation rates are higher than core inflation. Within core inflation, services inflation is generally higher than the inflation rate of goods. There are various reasons for these *permanent* differences in relative prices. Some are related to the degree

[6] The pure theoretical New Keynesian Phillips curve abstracts from past inflation outcomes. However, to match the empirical persistence of actual inflation, indexing to past inflation outcomes is added to the specification

[7] Real marginal costs are essentially the increase in production costs (in real terms) that are related to a marginal increase in output at the firm level.

to which the consumption of certain goods is inelastic to the price of that good – food and transport are necessities. Others are structural. A well-known example would be South African electricity tariffs (see Figure 3).

Figure 3: The relative price of electricity



For years, South Africa had excess capacity in terms of electricity generation, which reduced the need for electricity tariffs to increase (while overall CPI was rising). As a result, the *relative* price of electricity was so low, that South Africa enjoyed a considerable comparative advantage in its energy-intensive production sectors (see Mohamed, 1997).[8] Eventually, increasing demand for electricity eroded all the excess capacity. Suddenly, by 2008, substantial increases in electricity tariffs were needed in order to finance the required expenditure on new generation capacity.

Within the model, we account for these permanent differences in the relative prices of the subcomponents of CPI to the headline index through relative price *wedges*. Over the medium- to long-term the wedges of all the inflation components average to zero (when multiplied by the weights of the CPI basket). This for example, allows food inflation to be higher than services inflation for long periods of time, without either being inconsistent with medium-term headline inflation or the long-term inflation target.

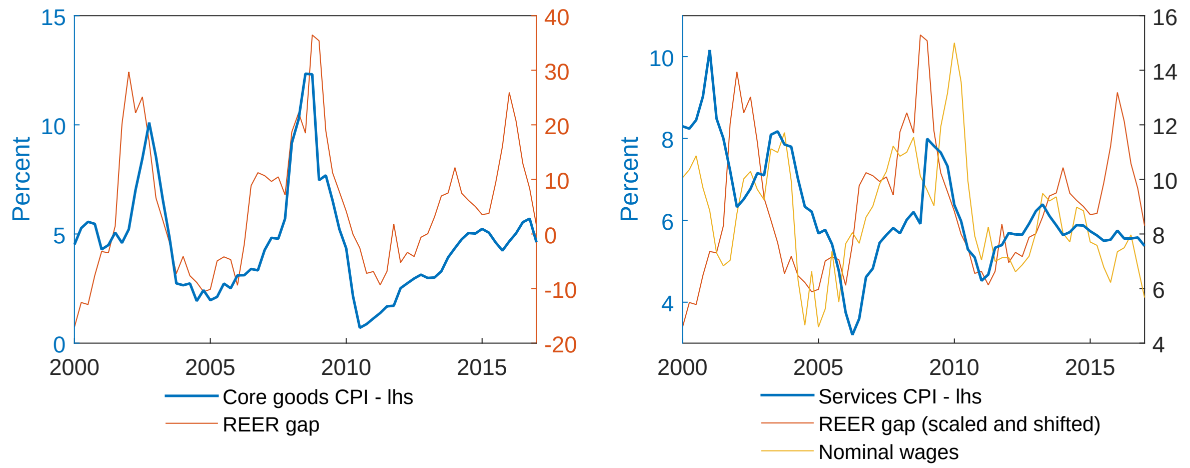
4.2.3 Core inflation

Core inflation is decomposed into its subcomponents of core goods and services.[9] This distinction is made to allow for the differing roles that wages and the exchange rate play in the respective price formation processes of goods relative to that of services. In general, services inflation is more persistent, and wages play an important role in the price setting process. On the contrary, CPI goods are generally more sensitive to exchange rate movements, as a large portion of the goods items in the CPI basket are traded across international borders.

[8]These sectors mostly relate to first-stage beneficiation of raw materials from the mining industry, such as smelters, etc.

[9]All the services in headline CPI are part of core CPI. CPI goods excluding food, electricity and fuel constitute core goods.

Figure 4: Core inflation



Services inflation

The Phillips curve for services CPI is specified as follows:

$$\begin{aligned} \pi_t^{services} &= b_{11}\Big[E_t\pi_{t+1}^{cpi} + \Delta\overline{relp}_t^{services}\Big] + (1-b_{11})\pi_{t-1}^{services} \\ &+ b_{13}rmc_t^{services} + \varepsilon_t^{\pi^{services}} \end{aligned} \tag{6}$$

$$b_{11} = 0.20, \quad b_{13} = 0.20$$

where the quarter-on-quarter (annualised) inflation rate of services CPI, $\pi_t^{services}$ is the change in the (log) price level of services CPI:

$$\pi_t^{services} = (p_t^{services} - p_{t-1}^{services}) \times 4 \tag{7}$$

In Equation (6), services CPI inflation is a function of the expectation for headline CPI, $E_t\pi_{t+1}^{cpi}$, which needs to be adjusted for the difference in equilibrium between the relative price of services to headline CPI, $\Delta\overline{relp}_t^{services}$. In addition, to capture the persistence observed in services inflation, it is driven by its own recent outcomes, $\pi_{t-1}^{services}$. The degree of slack in the economy, as measured by the output gap $\hat{y}_t$, as well as the gaps for real wages and the real exchange rate, combine to create the real marginal costs that price-setters in the services sector face, $rmc_t^{services}$:

$$\begin{aligned} rmc_t^{services} &= b_{14}(b_{15}\hat{y}_t + (1-b_{15})[\widehat{rwage}_t - \widehat{relp}_t^{services}]) \\ &+ (1-b_{14})(\hat{z}_t - \widehat{relp}_t^{services}). \end{aligned} \tag{8}$$

$$b_{14} = 0.85, \quad b_{15} = 0.61$$

Finally, $\varepsilon_t^{\pi^{services}}$ in Equation (6) captures supply shocks to services CPI, *i.e.* idiosyncratic movements in services CPI that are not explained by the model.

Core goods inflation

The Phillips curve for core goods CPI is specified as follows:

$$\begin{aligned} \pi_t^{coregoods} &= b_{21}\Big[E_t\pi_{t+1}^{cpi} + \Delta\overline{relp}_t^{coregoods}\Big] + (1-b_{21})\pi_{t-1}^{coregoods} \\ &+ b_{22}\pi_t^{m} \\ &+ b_{23}rmc_t^{coregoods} + \varepsilon_t^{\pi^{coregoods}} \end{aligned} \tag{9}$$

$$b_{21} = 0.25, \quad b_{22} = 0.04, \quad b_{23} = 0.25$$

where the quarter-on-quarter (annualised) inflation rate of CPI for core goods, $\pi_t^{coregoods}$ is the change in its (log) price level:

$$\pi_t^{coregoods} = (p_t^{coregoods} - p_{t-1}^{coregoods}) \times 4 \tag{10}$$

Similar to services CPI, inflation of core goods prices is a function of the expectation for headline CPI, $E_t\pi_{t+1}^{cpi}$, adjusted for the difference in equilibrium between the relative price of core goods to headline CPI. In addition, core goods inflation is partly driven by its own recent outcomes, $\pi_{t-1}^{coregoods}$. What distinguishes core goods from services, is the fact that certain goods in the CPI basket are imported as a final product from abroad, without any value necessarily being added in the domestic economy.[10] Price changes for these imported final goods will therefore reflect directly in the CPI. Their inflation rate, π_t^m, is defined as follows:

$$\begin{aligned} \pi_t^m &= d_1\pi_{t-1}^m \\ &+ (1-d_1)\left[\Delta s_t + \pi_t^* - \Delta\bar{z}_t + \Delta\overline{relp}_t^{coregoods}\right] \\ &+ \varepsilon_t^{\pi^m} \end{aligned} \tag{11}$$

$$d_1 = 0.10$$

Equation (11) relates quarter-on-quarter import price inflation to its own lag, as well as to changes in the nominal exchange rate, Δs_t, and foreign inflation, π_t^*. The last two terms – the rate of equilibrium real depreciation $\Delta\bar{z}_t$ and growth in the equilibrium relative price of core goods – ensure steady state consistency in the long run. It is worth noting that the presence of the lag term π_{t-1}^m allows for short-run deviations from the law of one price in terms of import price inflation. Real marginal costs for the price setters of core goods, $rmc_t^{coregoods}$, are a function of the output gap $\hat{y}_t$, and both gaps for real wages and the real exchange rate:

$$\begin{aligned} rmc_t^{coregoods} &= b_{24}(b_{25}\hat{y}_t + (1-b_{25})[\widehat{rwage}_t - \widehat{relp}_t^{coregoods}]) \\ &+ (1-b_{24})(\hat{z}_t - \widehat{relp}_t^{coregoods}). \end{aligned} \tag{12}$$

$$b_{24} = 0.90, \quad b_{25} = 0.43$$

As before, $\varepsilon_t^{\pi^{coregoods}}$ in Equation (9) captures supply shocks to CPI inflation of core goods.

4.2.4 Food, electricity, and fuel price inflation

Food price inflation

In the same vein as services and core goods inflation, the first two terms in the food price equation are persistence and expectations of future inflation. Food price inflation also reflects changes in international food prices $\Delta food_t$ (converted to rand terms by the nominal exchange rate Δs_t), the real marginal cost of food, petrol prices, and a supply shock. Similar to the core goods equation, international food prices are intended to capture imports of final goods. The energy (petrol) prices term is unique to this equation and takes into account the importance of fuel prices in the production of food, from the machinery employed in agriculture to the transportation of product throughout the production process. The idiosyncratic supply shock represents deviations in food prices that are

[10] Among these imported final goods would be CPI categories such as appliances, electronic equipment, certain new vehicles, etc.

not covered by the fundamentals of the equation. The impact that a period of drought can have on CPI food prices is an example of such a shock.

$$\begin{aligned}
\pi_t^{food} &= b_{31}\pi_{t-1}^{food} + (1-b_{31}-b_{32}-b_{36})(\pi_{t+1} + \Delta\overline{relp}_t^{food}) \\
&+ b_{32}(\Delta food_t + \Delta s_t) \\
&+ b_{33} rmc_t^{food} \\
&+ b_{36}\pi_t^{4,petrol} \\
&+ \varepsilon_t^{\pi^{food}}
\end{aligned} \tag{13}$$

$b_{31} = 0.65, \quad b_{32} = 0.012, \quad b_{33} = 0.16, \quad b_{36} = 0.033$

Real marginal costs for the price setters of food prices, rmc_t^{food}, are a function of the output gap, and both gaps for real wages and the real domestic price of international food.

$$\begin{aligned}
rmc_t^{food} &= b_{34}[b_{35}\widehat{y}_t + (1-b_{35})(\widehat{rwage}_t - relp_t^{food})] \\
&+ (1-b_{34})[\widehat{rfood}_t + \widehat{z}_t - \widehat{relp}_t^{food})]
\end{aligned} \tag{14}$$

$b_{34} = 0.80, \quad b_{35} = 0.50$

Electricity price inflation

The electricity price equation is very similar to the other price specifications given above, having the simplest form a Philips curve might have in this context. Namely persistence and expectations terms, real marginal costs, and an idiosyncratic supply shock. The use of a simplified Phillips curve is motivated by the fact that electricity prices are regulated and therefore do not necessarily react to market forces.

$$\begin{aligned}
\pi_t^{electricity} &= b_{41}\pi_{t-1}^{electricity} + (1-b_{41})(\pi_{t+1} + \Delta\overline{relp}_t^{electricity}) \\
&+ b_{42} * rmc_t^{electricity} \\
&+ \varepsilon_t^{\pi^{electricity}}
\end{aligned} \tag{15}$$

$b_{41} = 0.65, \quad b_{42} = 0.10$

Real marginal costs for electricity prices are a function of the output gap and the gap for real wages. Since the majority of electricity supplied is generated within South Africa, the real exchange rate gap is omitted.[11] In practice, CPI electricity in the QPM is assumed to follow a prescribed path over the forecast horizon.

$$rmc_t^{electricity} = b_{43}\widehat{y}_t + (1-b_{43})(\widehat{rwage}_t - relp_t^{electricity}) \tag{16}$$

$b_{43} = 0.10$

Fuel price inflation

Fuel price inflation π_t^{petrol} measures the growth rate of the CPI index consisting of petrol and diesel components. It is proxied in the model by the official regulated price of 95 octane unleaded petrol

[11] Less than 4 per cent of the domestic electricity supplied by Eskom is imported. Countries that supply electricity to Eskom are members of the Southern African Power Pool (SAPP), and include Mozambique (Cahora Bassa), Namibia and Lesotho.

in Gauteng, published monthly by the Central Energy Fund (CEF). Due to methodological reasons, and the fact that diesel is not included in the proxy, there is a shock allowing for deviations to what would otherwise be an identity.

$$\pi_t^{petrol} = \Delta petrol_t + \varepsilon_t^{\pi^{petrol}} \tag{17}$$

The petrol price is made up of two major components; the basic fuel price (BFP), bfp_t, and levies and margins, $pettax_t$. Their relative weights are simply the ratio of the respective component to the total price. Over the recent period the weight of BFP in the petrol price has been roughly 45 percent, with levies and margins accounting for the rest.

$$petrol_t = w_t^{bfp} \times bfp_t + (1 - w_t^{bfp}) \times pettax_t \tag{18}$$

$$\Delta petrol = 4 \times (petrol_t - petrol_{t-1}) \tag{19}$$

The BFP is calculated by CEF as the combination of movements in international product prices (sourced crude and refined oils) and the (rand/dollar) exchange rate. These price movements are proxied by the measure of crude oil and exchange rates used in the model.

$$\Delta bfp_t = \Delta oil_t + \Delta s_t + \varepsilon^{\Delta bfp} \tag{20}$$

$$bfp_t = bfp_{t-1} + \Delta bfp_t / 4 \tag{21}$$

Since levies and margins are purely a regulatory component they are modelled agnostically as a mean reverting process (which converges to its calibrated steady-state).

$$\Delta pettax_t = 0.75 \Delta pettax_{t-1} + (1 - 0.75) \Delta pettax^{ss} + \varepsilon_t^{\Delta pettax} \tag{22}$$

$$pettax_t = pettax_{t-1} + \Delta pettax_t / 4 \tag{23}$$

4.2.5 Wages

The wage channel is an important driver of the overall inflation process in the model. Real wage developments – along with the real exchange rate and output relative to its potential – ultimately determine the real marginal costs that feature in the model's price setting equations. The definition of real wages is standard:

$$\Delta rwage_t = \Delta wage_t - \Delta \pi_t, \tag{24}$$

and simply states that real wage growth, $\Delta rwage_t$, is nominal wage growth adjusted for contemporaneous inflation. Importantly, it is not the absolute growth rate of real wages that leads to inflationary pressures, but rather the degree to which real wages deviate from their equilibrium level.

The equilibrium real wage is pinned down by the theoretical condition which states that in the long run, real wage growth should equal productivity growth. In this case, productivity growth is defined as growth in output per worker, or GDP growth adjusted for employment growth. In equilibrium, this definition of productivity growth would therefore become potential GDP growth adjusted for equilibrium employment growth. The equilibrium for real wage growth is therefore specified as follows:

$$\Delta \overline{rwage}_t = (g_t - \Delta \overline{emp}_t) + \varepsilon_t^{\Delta \overline{rwage}}, \tag{25}$$

where $\varepsilon_t^{\Delta\overline{rwage}}$ is a shock that can move the equilibrium real wage rate temporarily.

Having defined both the real wage and its equilibrium, the real wage gap – which features in the real marginal cost component of the model's Phillips curves – is expressed as the difference between the real wage and its equilibrium value:

$$\widehat{rwage}_t = rwage_t - \overline{rwage}_t. \tag{26}$$

Essentially, the real wage gap reflects the real unit labour costs that producers face. It is however a slight modification of the usual unit labour cost definition. Here, the real wage gap reflects real wages that are adjusted for equilibrium productivity, while the more familiar real unit labour cost definition expresses real wages that are adjusted for contemporaneous productivity.

While real wages are the cause of inflation deviating from its target, it is ultimately nominal wages that are settled upon in the labour market. In the model, nominal wages follow a Phillips curve that depends on expectations of future wage inflation as well as recently realised wage inflation. Nominal wages further depend on the real wage gap, $\widehat{rwage}_t$, and productivity. Here the real wage gap plays an equilibrating role: if the real wage gap is positive, not only are nominal wages growing in excess of headline inflation, but real wages are growing in excess of productivity, and nominal wages will therefore be forced to moderate. An empirical feature of the South African labour market is the tendency of wage-setters to index wage increases to past inflation, which is captured by the fifth term. There is also an idiosyncratic supply shock.

$$\begin{aligned}\Delta wage_t = {} & (1 - w_1 - w_3)E\Delta wage_{t+1} \\ & + w_1 \Delta wage_{t-1} \\ & - w_2(\widehat{rwage}_t) \\ & + w_2(\widehat{y}_t - \widehat{emp}_t) \\ & + w_3(\pi^4_{t-1} + \Delta\overline{rwage}_t) \\ & + \varepsilon_t^{wage}\end{aligned} \tag{27}$$

$$w_1 = 0.20, \quad w_2 = 0.08, \quad w_3 = 0.55$$

4.3 Exchange rate

A key structural element of the QPM, as is true for all open-economy DSGE models, is the uncovered interest parity condition (UIP). It links domestic and international financial markets, by relating expected currency movements to the risk-adjusted differential between interest rates at home and abroad. To ensure no opportunity for arbitrage exists, the currency with the higher yield (after adjusting for the country risk premium, $prem_t$) should be expected to depreciate in future in order to equate returns in either currency. More formally, the standard UIP condition is expressed as follows:

$$Es_{t+1} - s_t \quad = \quad i_t - i_t^* - prem_t \tag{28}$$

where s_t is the (log) price of the foreign currency in terms of the domestic currency. As such, an increase in the exchange rate signifies a depreciation.

A drawback of the standard UIP specification in Equation (28), is that it does not mimic the empirical behaviour of the rand exchange rate, which generally shows more persistence and a less than one-to-one reaction to the interest rate.[12] As a result, exchange rate dynamics in the QPM are determined using a modified UIP relationship similar to that found in operational QPM models elsewhere.[13]

$$\Phi_t = e_1 E s_{t+1} + (1-e_1)(s_{t-1} + \frac{2}{4} \cdot [\Delta\bar{z}_t + \pi_t^{target} - \pi^{*target}]) \quad (29)$$

$$\Phi_t - s_t = e_2(i_t - i_t^* - prem_t) + (1-e_2)(\Delta\bar{z}_{t-1} + \pi_{t-1} - \pi_{t-1}^*) - e_t^{ls} \quad (30)$$

$$e_1 = 0.90, \quad e_2 = 0.60$$

The Φ_t term is broken up into two parts in Equation (29). The first, Es_{t+1}, represents purely rational agents with forward-looking expectations – similar to that found in the standard UIP specification. The second part represents myopic, backward-looking agents. These agents forecast future exchange rates based on the equilibrium real exchange rate and the differential between the domestic and foreign inflation targets. Essentially, these three terms represent relative purchasing power parity (PPP) in equilibrium. In Equation (30), the standard UIP condition in which interest rate differentials impact the exchange rate on a one-to-one basis, is reduced by the coefficient e_2. Here, a portion of the exchange rate's movement from one quarter to the next is ascribed to recent developments in the inflation differential. Together, these modifications dampen the pass-through of interest rates onto the exchange rate.

4.4 Interest rates

4.4.1 Taylor rule

The model uses a standard, albeit slightly modified, Taylor-type rule employed in structural models such as those in use at other inflation targeting central banks (Benes et al., 2016, Szilágyi et al., 2013, Kamber et al., 2015).

$$\begin{aligned} i_t &= f_1 i_{t-1} \\ &+ (1-f_1)\Big\{i_t^{neutral} + f_2[\frac{1}{3} \cdot (E\pi_{t+3}^{4,cpi} + E\pi_{t+4}^{4,cpi} + E\pi_{t+5}^{4,cpi}) - E\pi_{t+4}^{target}] + f_3\widehat{y}_t\Big\} \\ &+ \varepsilon_t^i \end{aligned} \quad (31)$$

$$f_1 = 0.79, \quad f_2 = 1.57, \quad f_3 = 0.54$$

An almost universal difference in this style of model from the rule Taylor (1993) proposed is the autoregressive smoothing parameter f_1, which creates the necessary persistence in the policy (repo) rate i_t to fit the observed data. The policy rate is then also a function of the neutral policy (or equilibrium) rate $i_t^{neutral}$:

$$i_t^{neutral} = \bar{r}_t + \pi_t^{target}, \quad (32)$$

which is purely the sum of the neutral real interest rate and the inflation target. In terms of macroeconomic conditions, the rule responds to deviations from the inflation target of (smoothed) inflation expectations over the next year and half, and the output gap. There is also a policy shock, ε_t^i, to account for deviations of actual policy decisions from this rule.[14]

[12] This finding is primarily based on the estimation results of a recursively identified Bayesian vector autoregression (VAR).

[13] See Benes et al. (2016).

[14] Such deviations occur for two reasons, one, the Taylor rule is only a crude approximation of the MPC's reaction function, and two, when the model filters the data through history it uses *ex post* information that the MPC would not

Parameters f_2 and f_3 are the relative weights on deviations of forward-looking inflation from target and output from potential. The expected inflation term emphasises the desire to react over the monetary policy horizon and thus "see through the noise" while concern for the inflation-output trade-off is consistent the SARB's flexible inflation targeting framework. It is necessary that f_2 is greater than 1 for the model to have a solution. That is, the policy maker is required to react more than one-to-one with deviations of expectations from target (inducing real interest rates to rise) in order to effectively anchor expectations and future inflation outcomes.

4.4.2 Term structure of interest rates

While the central bank has control over the short-end of the yield curve via the repo rate, it has less control over longer maturities along the yield curve. Longer-term interest rates are mostly determined by expectations about the future path of short-term interest rates. Ultimately, what matters for aggregate demand is not only the short-term interest rate, but also interest rates of longer maturities, especially for investment and government spending. Since the QPM is forward looking, it allows for the derivation of expected future short-term interest rates. As such, longer-term interest rates can be modelled via the expectations hypothesis of the terms structure, which states that the yield on a long-term bond should equal the average of expected short term interest rates over the life of the bond:

$$i_t^n = \frac{1}{n}\sum_{i=0}^{n-1} i_{t+i} + term_t^n, \tag{33}$$

where i_t^n represents the yield of an n-period bond, the summation term reflects the expected average short-term interest rate, and $term_t^n$ is the term premium required by investors to hold that bond.[15,16]. Using the Fisher equation:

$$r_t = i_t - E_t\pi_{t+1}, \tag{34}$$

which states that a real interest rate is a nominal rate minus inflation, the expected average short-term interest rate of Equation (33) can be further decomposed into expectations for both the average real rate and inflation over the life of the bond:

$$i_t^n = \frac{1}{n}\sum_{i=0}^{n-1} r_{t+i} + \frac{1}{n}\sum_{i=0}^{n-1} \pi^4_{t+1+i} + term_t^n. \tag{35}$$

The real yield for a bond of n-period maturity, r_t^n can therefore be calculated from the first term on the right-hand side of Equation (33):

$$r_t^n = \frac{1}{n}\sum_{i=0}^{n-1} r_{t+i}. \tag{36}$$

have had at its disposal when they made their interest rate decision, *i.e.*, actual inflation outcomes are different from what was forecast at the time, and the then real-time output gap would most likely have been revised.

[15] Since the QPM is a quarterly model, a 10 year bond would consist of 40 quarters, hence $n = 40$.

[16] The inclusion of the term premium is necessitated by the fact that the expectations hypothesis cannot explain the empirical regularity that yield curves slope upwards on average. The liquidity premium theory extends the expectations hypothesis theory, by positing that investors generally prefer to hold shorter-term securities, and therefore need additional compensation – a *term* premium – to induce their holding of longer term securities. The term premium increases with the maturity of the bond, and therefore explains the tendency of the yield curve to slope upwards.

In order to determine the relative importance that yields of different maturities have for aggregate demand in the model, we analyse the debt maturity structure in South Africa. To do this, we consider the lending activities by both banks and non-bank financial institutions, as well as lending via the bond market by government and corporations. Formal bank lending totalled R3.35 trillion in June 2017, with about an equal split going to households and corporations. Of this total, about 40 per cent are mortgage advances, 30 per cent general loans and advances, while the rest includes overdrafts, credit cards, instalment sales, and other lending. The majority of loans occur on a flexible basis, linked directly to the prevailing short-term repurchase rate set by the SARB. As a consequence, the majority of this debt, although possibly of long maturity, is linked directly to the short-term policy rate. An estimate of the effective lending rate from bank lending has a correlation with the policy rate of 0.97. General government, on the other hand, mainly borrows from the bond market, at the longer-end of the yield curve. In 2016/17 fiscal year, government had an estimated term-to-maturity on total debt of 14.7 years. Corporations that lend on the bond market fall somewhere in between the longer-end lending of government and the formal bank lending with estimates of an average maturity of around 6 years. Taking account of total investment and government spending's shares in real GDP, the effective long-run real rate, lr_t, that enters the IS-curve of Equation (2) is specified as follows:

$$\begin{aligned} lr_t &= \alpha_1 r_t + \alpha_2 r_t^8 + \alpha_3 r_t^{20} + \alpha_4 r_t^{40}, \qquad (37) \\ \alpha_1 &= 0.40, \quad \alpha_2 = 0.30, \quad \alpha_3 = 0.20, \quad \alpha_4 = 0.10 \end{aligned}$$

where r_t is the real repo (or short-term) rate, r_t^8 is an 8-quarter (*i.e.*, 2 year) real bond yield, while r_t^{20} and r_t^{40} are the real yields for 5 and 10 year government bonds, respectively. The α's in Equation (37) represent the relative weights of the various yields.

4.4.3 Country risk and term premiums

The country risk premium ($prem_t$) in the model is a key feature of the UIP condition in Equation (28). By re-arranging the UIP condition as follows:

$$prem_t = (i_t - i_t^*) - (E_t s_{t+1} - s_t), \qquad (38)$$

it becomes clear that the country risk premium serves as the *wedge* that equates returns between domestic assets and those abroad. For higher domestic country risk, international investors would require greater return from the domestic economy via a higher i_t. The risk premium itself is modelled in a fairly agnostic (non-structural) way, i.e. as a stochastic process that moves around in the event of risk-premium shocks, ε_t^{prem}, but, in the absence of such shocks, it will ultimately revert to its equilibrium value, $\overline{prem}_t$:

$$\begin{aligned} prem_t &= h_0 prem_{t-1} + (1 - h_0)\overline{prem}_t + \varepsilon_t^{prem} \qquad (39) \\ h_0 &= 0.80 \end{aligned}$$

Similar to the country risk premium, the term premiums of the various bond yields from Equation (33) are modelled as stochastic processes that ultimately converge on their steady state values:

$$\begin{aligned} term_t^n &= h_4 term_{t-1}^n + (1 - h_4) term^{n,ss} + h_n(prem_t - prem^{ss}) + \varepsilon_t^{term^n}. \qquad (40) \\ h_4 &= 0.80 \end{aligned}$$

However, since the same fundamentals that move the country risk premium would also drive changes in term premiums, we enable this co-movement to occur by allowing for spill-overs from the country risk premium onto the various term premiums. Finally, the maturity weighting that applies to the effective long-run real rate lr_t, is also used to determine the effective term premium, $lterm_t$:[17]

$$lterm_t = \alpha_1 \cdot 0 + \alpha_2 term_t^8 + \alpha_3 term_t^{20} + \alpha_4 term_t^{40}. \tag{41}$$

$$\alpha_1 = 0.40, \quad \alpha_2 = 0.30, \quad \alpha_3 = 0.20, \quad \alpha_4 = 0.10$$

4.5 Rest of the world

Due to the small size of the South African economy relative to the rest of the world there is no appreciable feedback from the domestic economy to the rest of the world. Advantage is taken of this fact by treating international price and output variables as exogenously (separately) determined via supplementary models in the SARB's suite of models. These variables include trade-weighted consumer prices, output, and interest rates of the G3 economies (*i.e.* the United States, euro area, and Japan) plus China as well as food, oil, and non-oil commodity prices.

The global variables themselves are modelled in the Global Projection Model (GPM) (Carabenciov et al., 2013) which models the economies of our most important trading partners[18]. The structure of this model is similar in spirit to the QPM, while placing emphasis on the dynamics of cross-country spillovers of global shocks. Some of the salient empirical features of this model are discussed in the next section.

5 Empirical features

This section discusses the empirical properties of the model, including its time-varying equilibrium values, steady states, and the calibration of parameters. In turn, this allows us to generate impulse response functions (*i.e.*, the model's dynamic response to a variety of shocks), decompositions of the key equations into the variables and shocks that drive them, as well as forecasts. In order to generate forecasts from the model, a number of steps are required. First, as was done in the previous section, the appropriate model structure needs to be determined, which is a fine balance between complexity and functionality. Second, the model's parameters need to be calibrated to fit South Africa's empirical properties. Third, actual data on the economy (*i.e.*, CPI, real GDP, interest rates and the exchange rate, among others) is required to filter the model over history, in order to generate the time-varying equilibriums and the accompanying structural shocks that lead to historical deviations from these equilibriums. Fourth, the model is decomposed to determine the accuracy and appropriateness of the model's structure and calibration using shock and equation decompositions, impulse response functions, and forecast accuracy.

5.1 Data

In order to filter and forecast, the QPM requires data of the variables of interest. In the filter step, for a given calibration and dataset, we estimate the unobserved equilibriums (and the gaps they imply). Following the Bank's suite of models approach, not all equilibriums are estimated in the QPM, but rather imposed as observed. The most important equilibrium generated from an

[17] Since there is no term premium for the short rate, it is denoted by a zero in Equation (41)

[18] While the QPM does have a foreign economy block of equations, the equilibriums of most foreign variables are taken from the GPM and not filtered in the QPM. The forecast of these variables are also taken from GPM's *GPM Network* forecast of which the SARB is a contributing member.

auxiliary model is potential growth. We use a modified version of the finance-neutral method used by Anvari et al. (2014). Table A1 shows all the data used as inputs into the QPM.

5.2 Equilibriums

The QPM is a general equilibrium model where deviations from equilibrium – the so-called gaps – drive economic outcomes. In this general equilibrium context, it is assumed that an economy will revert to its long-run equilibrium (steady state) over time, usually in the absence of transitory shocks and other factors affecting the medium term. Take for example an economy that is operating above its potential growth rate for a period of time due to strong demand. This period is characterised by a positive and growing output gap which tends to raise inflation. The best South African example was the mid-2000s, when real GDP growth averaged over 5 per cent, but potential growth was likely just below 4 per cent. This led to a significantly large positive output gap which put upward pressure on inflation, i.e. what's generally referred to as demand-pull inflation. As a consequence, targeted inflation rose from 4.5 per cent in the first quarter of 2004 to just below 8 per cent by the end of 2007, and required monetary policy tightening to help bring inflation back to within the target range. This section provides detail on some of the key equilibriums that are defined in the QPM: the neutral real interest rate; real equilibrium exchange rate; the inflation target; and others. These equilibriums inform the gaps – the deviations from equilibrium – that policymakers often speak about and use to justify their policy choices.

5.2.1 Steady states

It is important to distinguish between the concept of a variable's equilibrium level and that of a steady state. Steady states are a set of fixed values that represent the fundamental long-run properties of an economy. The long-run can be thought of as 8 years and beyond. Equilibriums are slowly evolving medium-term concepts that generally fluctuate around the steady steady over time. The medium-term can be thought of as 3-6 years.

The above example on the pre-crisis boom in South Africa is useful for illustrating this distinction. Assuming that South Africa's (long-run) steady-state potential growth rate is around 2,5 per cent, a number of factors helped to temporarily drive actual potential growth – a medium-term equilibrium concept – above this steady state value during the mid-2000s (strong and rising asset prices, growing credit extension, rising terms of trade linked to significant growth in the price of South Africa's export commodities). Together, these factors explain the rise in South Africa's potential growth from around 2.5 per cent in the late-1990s to to almost 4 per cent during the mid-2000s (see Anvari et al., 2014). However, following the global financial crisis, potential growth once again decelerated. In fact, in recent years, estimates show that potential growth has fallen to below its steady state.

Table 1 provides some of the key steady state values imposed on the QPM. Two main equations pin down the relationship between steady state values at home and abroad: a purchasing power parity (PPP) condition:[19]

$$\Delta \bar{s}^{ss} = \Delta \bar{z}^{ss} + (\pi^{target} - \pi^{*,target}); \tag{42}$$

[19]The notation for steady state values does not have a time subscript t, since these values are constant over time.

and an uncovered interest parity (UIP) condition in real terms:

$$\bar{r}^{ss} = \bar{r}^{*,ss} + \overline{prem}^{ss} + \Delta\bar{z}^{ss}. \tag{43}$$

The steady state PPP condition links the depreciation of the nominal exchange rate ($\Delta\bar{s}^{ss}$) to the real exchange rate's rate of depreciation ($\Delta\bar{z}^{ss}$) and the inflation target differential between South Africa and abroad. The real UIP condition, in steady state, relates the domestic neutral real interest rate ($\bar{r}^{ss}$) to the foreign neutral real interest rate $\bar{r}^{*,ss}$, equilibrium country risk premium ($\overline{prem}^{ss}$), and equilibrium real exchange rate depreciation ($\Delta\bar{z}^{ss}$).

Table 1: Model steady states

Policy Variables	Domestic	Foreign
Inflation target	4.5	2
Neutral real interest rate	2.5	0.5
Neutral nominal interest rate	7.0	2.5
Potential growth	2.5	3.7
Exchange rates		
Real exchange rate depreciation	0	
Nominal exchange rate depreciation	2.5	
Country risk premium	2	

Monetary policy in South Africa operates within an inflation target range of between 3 and 6 per cent. However, such a range cannot be implemented within the structure of the QPM. Instead, a point target is required. Recent MPC statements have indicated a preference by the MPC to focus on the mid-point of the 3 – 6 per cent target range over the medium to longer term, by stating in its July 2017 MPC statement that "[t]he MPC would prefer [inflation] expectations to be anchored closer to the mid-point of the target range". As such, the inflation target in the QPM is aligned with this preference and set at 4.5 per cent.

For foreign inflation, interest rates and the exchange rate, the G3 economies are used. The inflation target for these combined economies is 2 per cent. To identify the steady state neutral real interest rates of the G3 economies is slightly more complicated. Here we use the last 30 years as a proxy for the long run. Over this period real interest rates in the G3 economies have averaged around 0.5 per cent.

In order to determine South Africa's steady-state neutral real interest rate ($\bar{r}$) from Equation (43), we require estimates for equilibrium real exchange rate depreciation and the country risk premium in steady state. Equilibrium exchange rate estimates of De Jager (2012) indicate that the real exchange rate experienced trend appreciation during much of the early 2000s, from higher productivity growth in SA, compared with economies such as the US and euro area, and improving fiscal balances. This trend reversed following the global financial crisis. From 2010 onwards, SA experienced equilibrium depreciation in the real effective exchange rate (REER) as productivity differentials worsened more quickly in SA compared with the US. These two distinct periods of SA history complicate the choice of steady state value for the real effective exchange rate. On average, over the inflation targeting period, equilibrium depreciation of the REER has been closer to zero than any clear depreciation or appreciation trend. South Africa's long-run steady state growth rate is also likely to be similar to the US making the likelihood of trend movements in the REER low. As a consequence, the steady state real exchange rate depreciation is set to zero per cent in the QPM. By implication of Equation (42), the nominal exchange rate will therefore depreciate in steady state by the inflation-target differential (domestic vs. foreign) of 2.5 per cent, per year.

For South Africa's country risk premium, we look at a number of market measures including the spread between South Africa's USD denominated 10-year government bond and a US equivalent, the 5-year CDS spread, as well as the EMBI+ spread. In general all measures co-move well, with the average levels around 2 per cent.

With the foreign neutral real interest rate at 0.5 per cent, equilibrium depreciation of 0 per cent and a risk premium of 2 per cent, Equation (43) suggests that South Africa's neutral real interest rate should equal 2.5 per cent in steady state. This top-down approach to determining the neutral real interest rate is corroborated by the data, as South Africa's real interest rate has averaged around 2.5 per cent over the same 30 year sample for which the foreign real interest rate has averaged 0.5 per cent.

We use an export-weighted potential growth measure for the US, euro area, Japan, China and other advanced and emerging market economies to determine a global output gap used in this model.[20] Export-weighted global potential growth is assumed to be 3.7 per cent.

5.2.2 Potential growth

Potential growth, the rate of growth that puts no upward or downward pressure on inflation, is a fundamental construct in New Keynesian models such as the QPM. Potential growth is not estimated within the model but rather imposed from an alternate model, in line with the suite of models approach adopted at the SARB. The model used is an augmented version of Anvari et al. (2014) (Botha, Ruch and Steinbach, forthcoming). A brief description of the modelling choices and outcomes are provided here for ease of reference.

In general views of the output gap (see Fedderke et al., 2016; IMF, 2014; and Kemp, 2015) treat potential growth as a slow and smoothly evolving process. However, this view does have far reaching consequences for the output gap. By implication, all sudden changes to actual GDP, regardless of their origin, will be reflected in the output gap. Recent examples where the output gap would have widened due to sudden changes in GDP would be the platinum sector strike of 2014, and the 2015/2016 drought. The Bank's current methodological approach to potential growth suffers this fate.

For the 2014 strike, it could be argued that the unavailability of labour in the mining sector *temporarily* shifted South Africa's long-run aggregate supply curve (i.e. potential output) to the left. To better reflect these realities, Botha et al.(forthcoming) adopts an approach that reflects shorter-run idiosyncratic events that shift potential growth. It allows for an output gap that more accurately represent the demand side of the economy, i.e. that which the Bank can actually respond to.[21]

Let $\bar{y}_t$ be the (log) level of potential output, while g_t is the *smooth* quarter-on-quarter annualised

[20]Potential growth estimates for these economies are based on an in-house version of the Global Projection Model.

[21]This methodology dates back to Clark (1987), who applied it to US data within a highly parsimonious structure consisting of a minimal number of equations.

potential growth rate of the economy.[22] g_t can be thought of as underlying trend growth. From one quarter to the next, the (log) level of potential output then simply evolves as its level recorded in the previous quarter plus the potential growth rate:

$$\bar{y}_t = \bar{y}_{t-1} + g_t \tag{44}$$

In order to allow for idiosyncratic (or peculiar) events such as droughts and strikes to affect the level of potential, i.e. temporary deviations from the smooth underlying growth process g_t, an additional source of variability is required in Equation 44. This is achieved by allowing for quarter-on-quarter changes in the level of potential output to deviate from the underlying potential growth rate g_t through the inclusion of the residual term $e_t^{\bar{y}}$. Equation 44 then becomes:

$$\bar{y}_t = \bar{y}_{t-1} + g_t + e_t^{\bar{y}} \tag{45}$$

where $e_t^{\bar{y}}$ acts as a shock to the level of potential. This shock will average zero over time such that potential growth evolves around the underlying trend growth rate. Hereafter, an equation is needed that describes the evolution of the underlying potential growth process, g_t:

$$g_t = \rho_g g_{t-1} + (1-\rho_g)g^{ss} + e_t^g, \quad \rho_g < 1 \tag{46}$$

Equation 46 shows that g_t is a function of its history, a long-run steady state growth rate g^{ss}, and a residual term e_t^g. Shocks to the underlying potential growth rate are captured by the residual e_t^g, and lead to persistent deviations of underlying potential growth from its long-run steady state growth rate g^{ss}. In the absence of such shocks, the underlying trend growth rate eventually converges to its long run steady state growth rate.

Figure 5: Potential growth

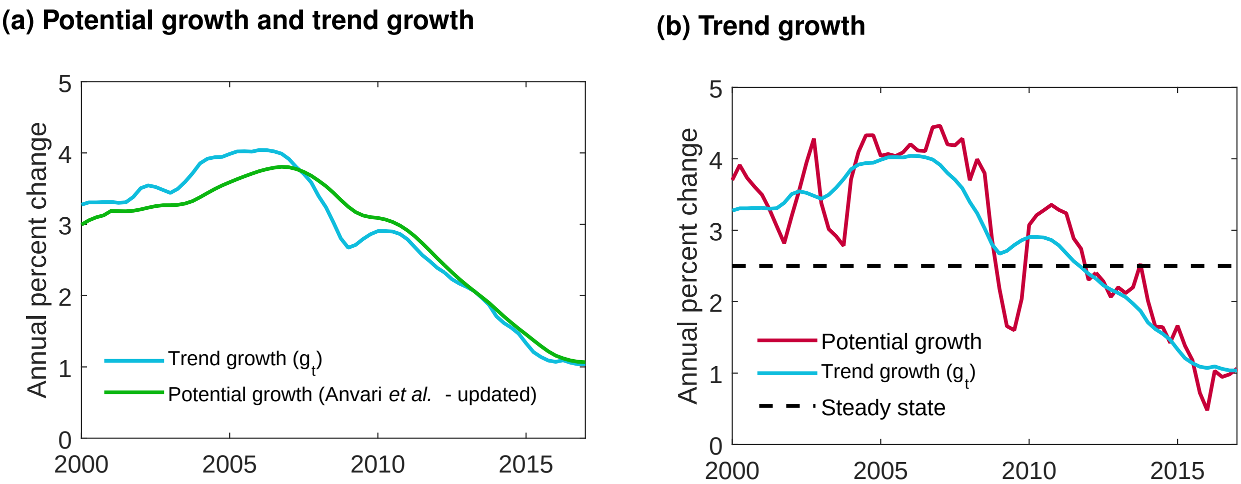


Figure 5b plots the evolution of g_t, the underlying trend growth rate that the economy can achieve, against the annual change in potential growth, $\bar{y}_t$. Underlying trend growth does not differ substantially from the previous estimates in Anvari et al. (2014) as seen in 5b. The story behind the

[22]Note that $\bar{y}_t = \log(\bar{Y}_t) * 100$, where $\bar{Y}_t$ is the level of real GDP. This log transformation of the level of GDP conveniently simplifies the model specification, as the first difference of $\log(\bar{Y}_t) * 100$, i.e. $\bar{y}_t - \bar{y}_{t-1}$, approximates quarter-on-quarter *percentage* change in non-annualised terms. Since g_t represents *annualised* potential growth, it is de-annualised by $g_t/4$ for Eq. 44 to hold.

underlying trend growth remains unchanged – underlying trend growth accelerated through the early 2000s reaching rates close to 4 per cent in the mid-2000s, but has since been on a downward trajectory, reaching lows of 1.1 per cent in 2016. Around this underlying trend are movements in actual potential growth (the red line in 5a), which ultimately determines the output gap.

5.2.3 Real equilibrium exchange rate

The equilibrium real effective exchange rate is a combined outcome of the Kalman filter step which uses the calibrated model structure, observed data and other defined equilibriums to determine the outcome. The real exchange rate in the QPM is based on the cross-rates between the rand and the US dollar, euro, and Japanese yen, deflated using the respective consumer price indices.[23]. Figure 6a plots the log level of the real effective exchange rate against its equilibrium. When the real effective exchange rate is above the equilibrium level, the rand is considered undervalued in real terms, and *vice versa*.

Figure 6: Real effective exchange rate

(a) Equilibrium

(b) Real effective exchange rate gap

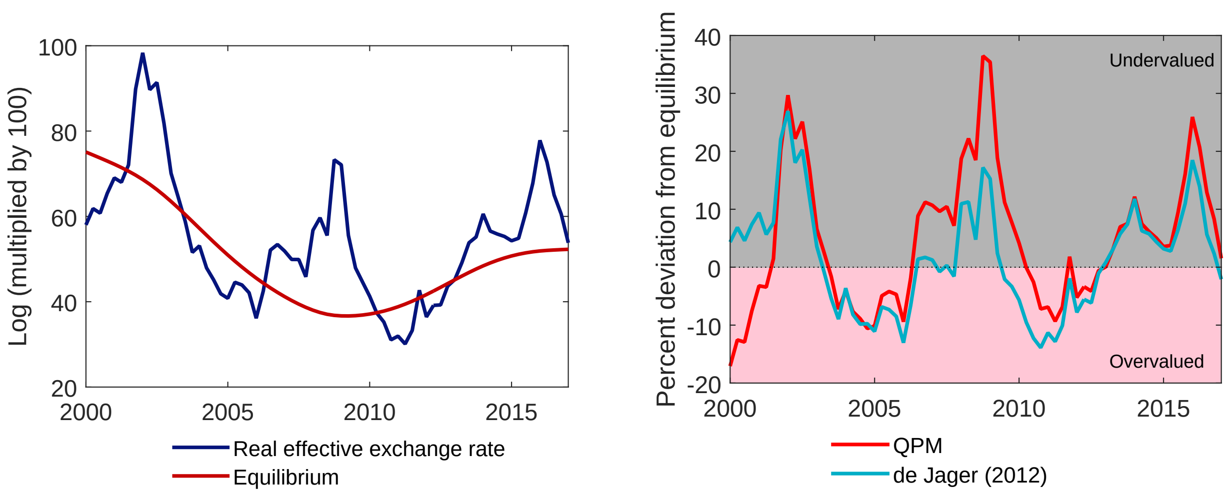


Figure 6b plots the resultant real exchange rate gap which drives cyclical outcomes in the model, and compares it to the real effective exchange rate as determined by De Jager (2012). There are three distinct periods of rand undervaluation including 2001-03, 2008-09 and 2015-16. In each case the rand was considered anywhere between 25 and 36 per cent undervalued at its peak, taking just under two years to return to fair value. Currently the rand is considered to be slightly undervalued at 2017Q1 levels in real terms. The QPM filtered equilibrium compares favourably to the De Jager (2012) work which determines the equilibrium real exchange rate based on productivity differentials with the United States and the stance of fiscal policy among others. Compared with De Jager (2012), the QPM sees a more undervalued exchange rate through the two most recent rand depreciation episodes. The more undervalued rand means that the QPM expects more inflationary pressure due to the rand depreciation, compared with De Jager (2012).

[23] The real exchange rate in the QPM has a narrower definition, when compared to the real effective exchange rate published by the SARB in the Quarterly Bulletin, which is based on 20 trading partners and is deflated using producer price indices

5.2.4 Rest of world: potential growth and neutral real interest rate

South Africa is a small open economy relying on economic conditions in the rest of the world to drive demand, provide savings to account for the domestic shortfall, and import goods and services. These global conditions also effect equilibrium outcomes particularly for the domestic neutral real interest rate, trend depreciation and growth outcomes. This section provides details on some of the foreign equilibriums used in the QPM. An in-house version of the IMF's Global Projection Model (GPM) is used to estimate foreign potential growth, real interest rates, consumer inflation, and the nominal exchange rate. These are then used directly in the QPM, as in the case for SA's potential growth, or as part of the information set the model uses to estimate the equilibrium, as is the case for the neutral real interest rate and real exchange rate.

Figure 7a plots the foreign neutral real interest rate against model real interest rates and the Laubach and Williams (2003) measure of the neutral. The foreign neutral real interest rate is below the Laubach and Williams (2003) measure, as the neutral real interest rates in euro area and Japan are lower and negative currently. In 2017Q1, the neutral real interest rate in the US is close to zero, where in the euro area and Japan it is likely between -0.5 and -1.0 per cent. The foreign neutral real interest rate has declined from around 1.5 per cent in the early 2000s to 0.5 per cent in 2007 prior to the GFC and negative since. The foreign neutral remains below its steady-state value.

Figure 7: Foreign monetary policy

(a) Foreign neutral real interest rate **(b) Foreign real interest rates**

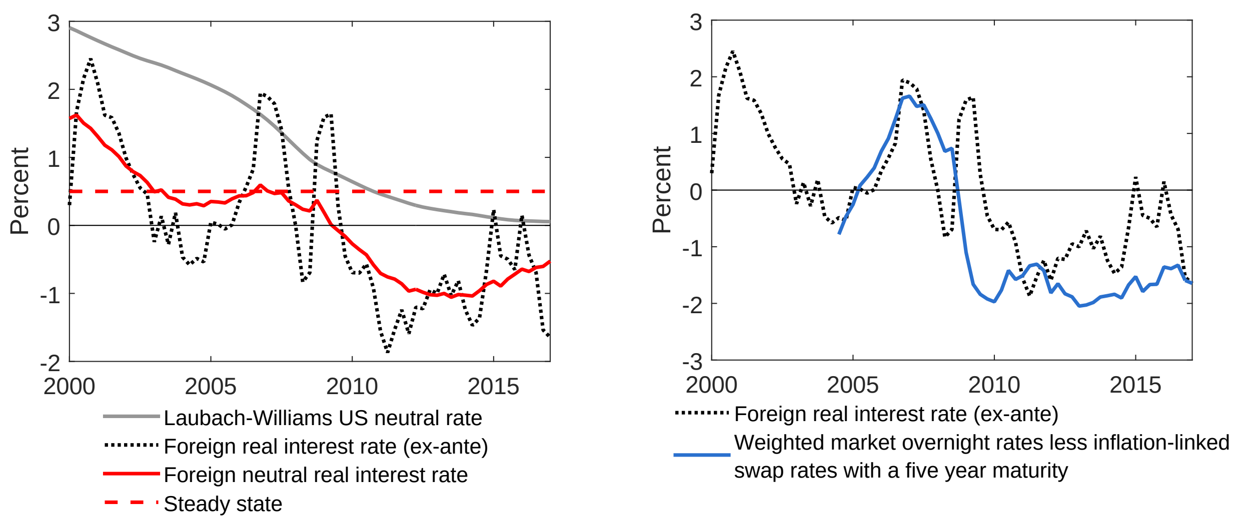
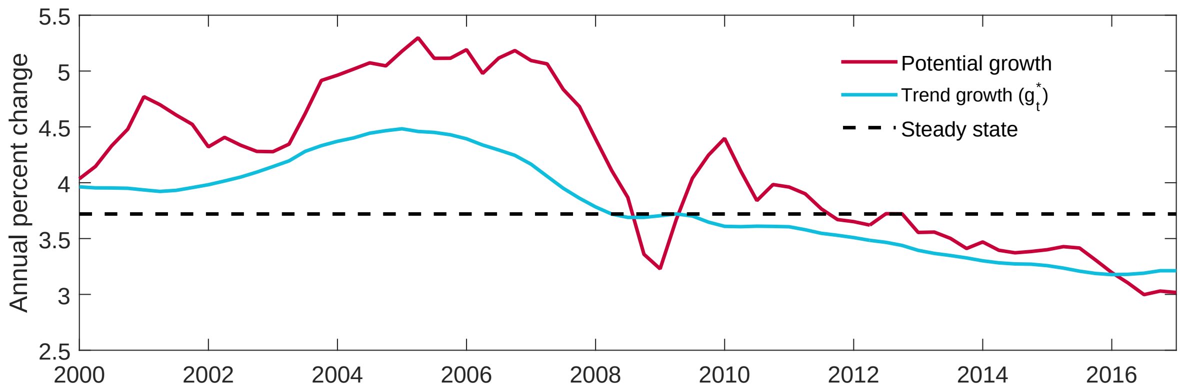


Figure 7b plots the model real interest rate against a market measure of the real interest rate. The market measure is a weighted average of the US, euro area, and Japan overnight interest rates less a measure of inflation expectations using a five-years ahead swap rate, five-years ahead. The figure indicates that the model estimate is similar to the market outcome but higher from around 2009. The model real interest rate averages -0.3 since 2004Q2, compared with -0.9 for the market measure.

Figure 8 plots the potential growth estimate from the GPM calculated as the export weighted average for the US, euro area, Japan, China and other advanced and emerging market economies. Global potential growth averaged just below 5 per cent from 2000 to 2007, before slowing to 3.6

per cent in the years following the great recession in 2008. Global potential growth is estimated around 3 per cent at the start of 2017, on a South African export weighted basis.

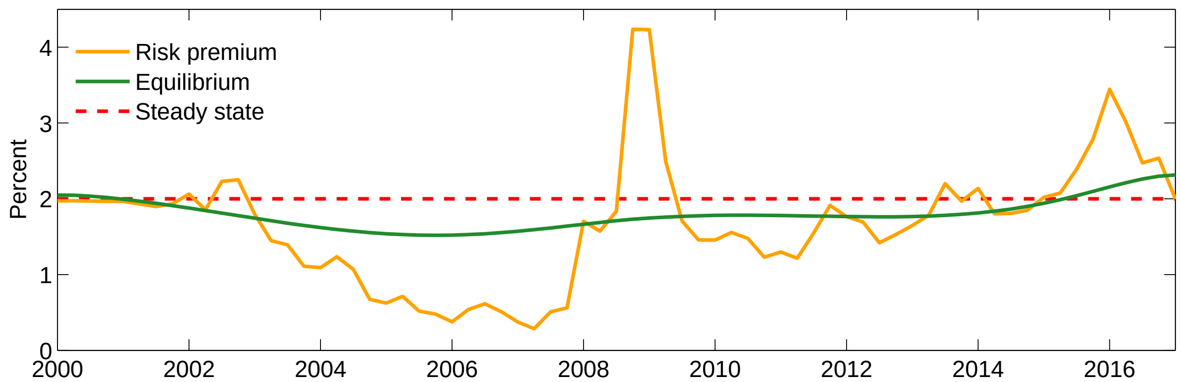
Figure 8: Foreign potential growth



5.2.5 Country risk premium

The equilibrium country risk is a key component of the UIP condition, as it ultimately has bearing on the domestic neutral real interest rate estimate. Given the lack of structural fundamentals that would explain country risk in the model, we measure its equilibrium as a smoothed (Hodrick-Prescott) version of the SA 5-year CDS spread – the series used to proxy country risk in the model.

Figure 9: Country risk in equilibrium



As can be seen from Figure 9, equilibrium country risk has been gradually rising since 2006, ending above its steady state value of 2 per cent by the start of 2017.

5.2.6 Neutral real interest rate

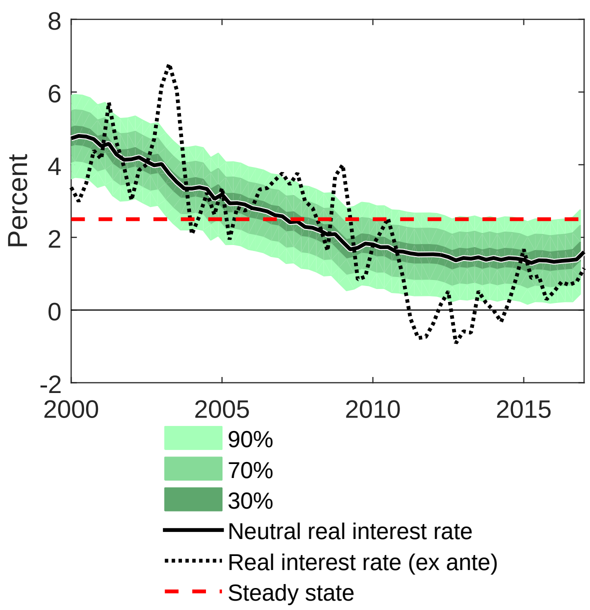
The neutral real interest rate is the rate at which inflation is stable and output is operating at its equilibrium, and a key variable to determine the appropriate stance of monetary policy. As the economic landscape changed significantly over the past decade, the appropriate stance of monetary policy has also changed. An example of the significant change in neutral rates can be seen in Laubach and Williams (2016) and Holston et al. (2017) where neutral interest rates in the United

States have fallen from 2-3 per cent, in the early to mid-2000s, to effectively zero in 2017. Estimates for the euro area from Holston et al. (2017) are between 0 and -1 in 2015. The QPM uses an interest parity condition in equilibrium where South Africa's neutral real interest rate is determined by the foreign neutral real interest rate (for the US, euro area, and Japan), the country risk premium, and expected equilibrium real exchange rate depreciation. Figure 10a plots the Kalman filtered results for the neutral real interest rate, its uncertainty, as well as model consistent real interest rates [24].

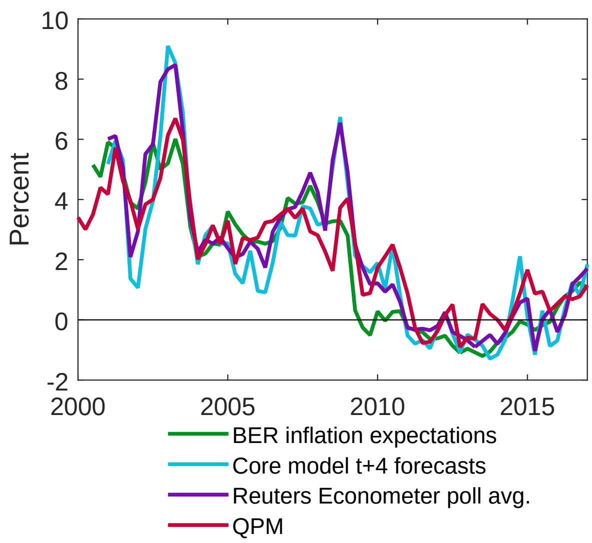
The neutral real interest rate has fallen from rates around 4-5 per cent in the early 2000s to rates closer to 1.5 per cent in 2016-17. The fall in domestic neutral real interest rates is in line with the fall in foreign neutral rates from 1.5 per cent in the early 2000s to between -0.5 and -1 per cent in 2016-17. Country risk premiums have remained relatively stable, if rising, over the latter part of the sample period.

Figure 10: Neutral real interest rate

(a) Neutral real interest rate



(b) Comparison of ex-ante real interest rates



The estimates of domestic neutral real interest rates suggest that policy remains mildly accommodative in 2017Q1, but less so compared with 2010-2014, as policy rates began to tighten in response to rising actual and forecasted inflation. Caution is needed when interpreting point estimates of the neutral real interest rate given the high degree of uncertainty surrounding these estimates, suggesting that these serve only as one of a few broad indicators of the stance of monetary policy. The current estimate does provide a useful macro-economic story for the stance of South African policy.

The filtered neutral real interest rate is a function of the model consistent real interest rate. To ensure that what the model estimates is appropriate, figure 10b compares the QPM outcome to other ex-ante real interest rates including from BER inflation expectations data, Reuters average

[24] The QPM uses a fisher equation of inflation expectations one-period ahead to define real interest rates. This differs, but is well correlated, from real interest rates determined by core model or market participants forecasts of inflation.

inflation forecasts, and the Core model. There is strong co-movement between all measures of real interest rates with correlations between 0.88 and 0.9. All measures highlight the fall in real interest rates following the great recession, as well as a more recent pick-up in real interest rates due to less accommodative policy from 2014 onwards.

5.3 Calibration

The QPM model is calibrated, rather than estimated, to account for a number of factors. First, the objectives of a model of this nature is more than just forecasting accuracy, and they do not fit neatly into a likelihood-based estimation procedure. Second, the entire model's properties must align to the empirics of the South African economy. Third, estimation is complicated by simultaneity, expectations, and real-time data problems. Fourth, extensive estimation work over a number of years, from various sources, goes into this model, as well as the underlying economic relationships, thus making a single estimation procedure inappropriate. Calibrated parameters are based, in part, on previous work on QPM models for South Africa by De Jager (2007) and De Jager, Johnston, and Steinbach (2015). Other empirical work used in calibration include Steinbach, Mathuloe, and Smit (2009), Parsley (2012), De Jager (2012), Aron, Farrell, Muellbauer, and Sinclair (2014), Anvari, Ehlers, and Steinbach (2014), du Plessis, Smit, and Steinbach (2014), Ruch and du Plessis (2015), Kabundi, Schaling, and Some (2016), Kabundi and Mbelu (2016), Fedderke, Liu, et al. (2016), and Fedderke and Mengisteab (2017). This approach is not uncommon at other central banks, for example Benes et al. (2016).

Table 2 provides the parameter values for the main equations in the model.

Table 2: Calibrated parameters

Real economy		Goods inflation		Taylor rule	
a_1	0.15	b_{21}	0.25	f_1	0.79
a_2	0.6	b_{22}	0.04	f_2	1.57
a_3	0.15	b_{23}	0.25	f_3	0.54
a_4	0.95	b_{24}	0.9	Term Structure	
a_5	0.15	b_{25}	0.43	α_1	0.4
a_6	0.01	Food inflation		α_2	0.3
Services inflation		b_{31}	0.65	α_3	0.2
b_{11}	0.2	b_{32}	0.012	α_4	0.1
b_{13}	0.2	b_{33}	0.16	h_0	0.8
b_{14}	0.85	b_{34}	0.8	h_4	0.8
b_{15}	0.61	b_{35}	0.5	h_8	0.05
Wage inflation		b_{36}	0.033	h_{20}	0.1
w_1	0.2	Exchange rate		h_{40}	0.3
w_2	0.08	e_1	0.9		
w_3	0.55	e_2	0.6		

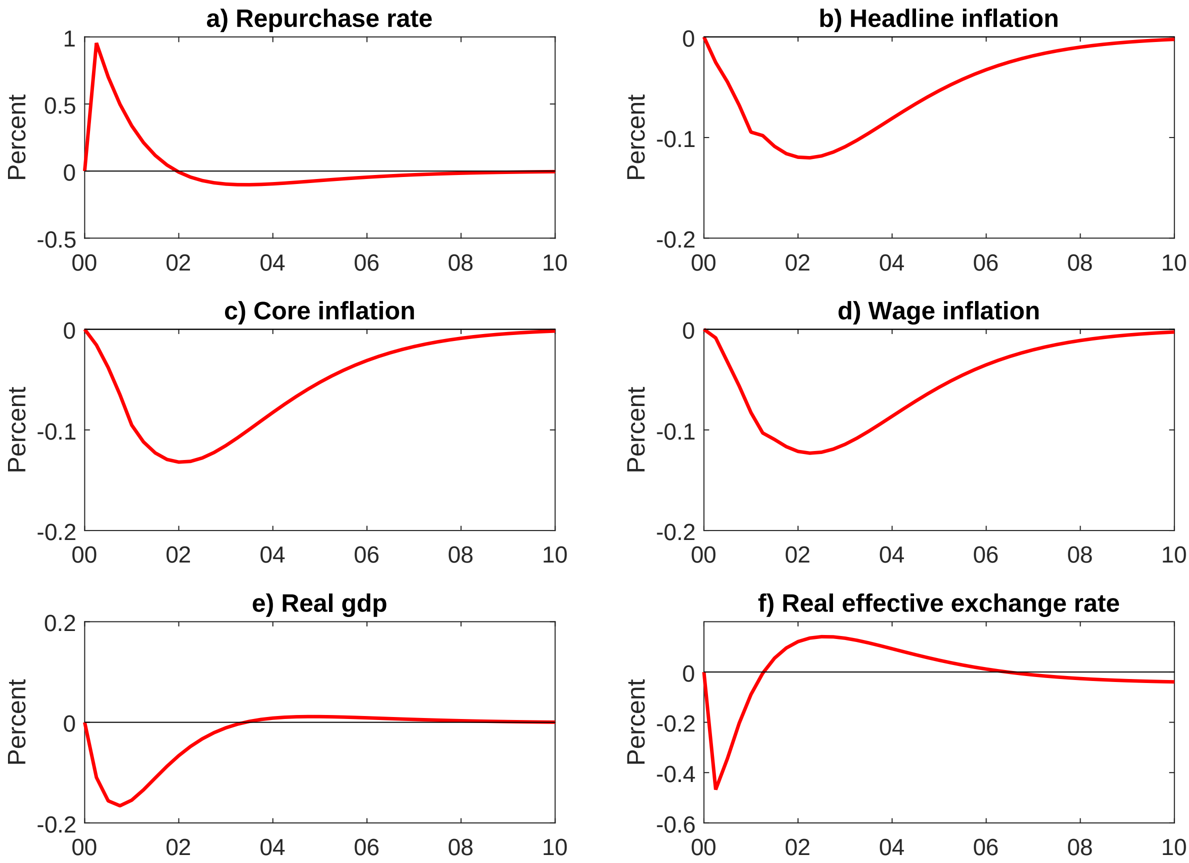
5.4 Impulse response analysis

To study the model properties this section presents impulse response functions (IRFs) to shocks to the policy rate (repurchase rate), output gap (demand), and the exchange rate. All shocks are performed in reference to the model's equilibrium or steady state, so that the starting values of all variables and gaps are set at zero when the shock takes place – i.e. their steady states.

5.4.1 Policy shock

Figure 11 presents a one per cent unexpected shock to the repurchase rate, focusing on the response of targeted inflation, real GDP growth, and the real exchange rate. The horizontal axis shows years. A repo shock is relatively persistent, reflecting the smoothing of the policy rate undertaken by the MPC, taking about a year to unwind (figure 11a). In response to tighter policy, targeted inflation declines by 0.12 percentage points two years after the shock (figure 11b), partly reflecting a more appreciated real exchange rate as well as lower growth. Real GDP growth is 0.17 percentage points lower on an annual basis with the peak impact two to three quarters after the shock. The real exchange rate also reacts immediately to the policy shock with the real exchange rate initially appreciating by 0.45 per cent.

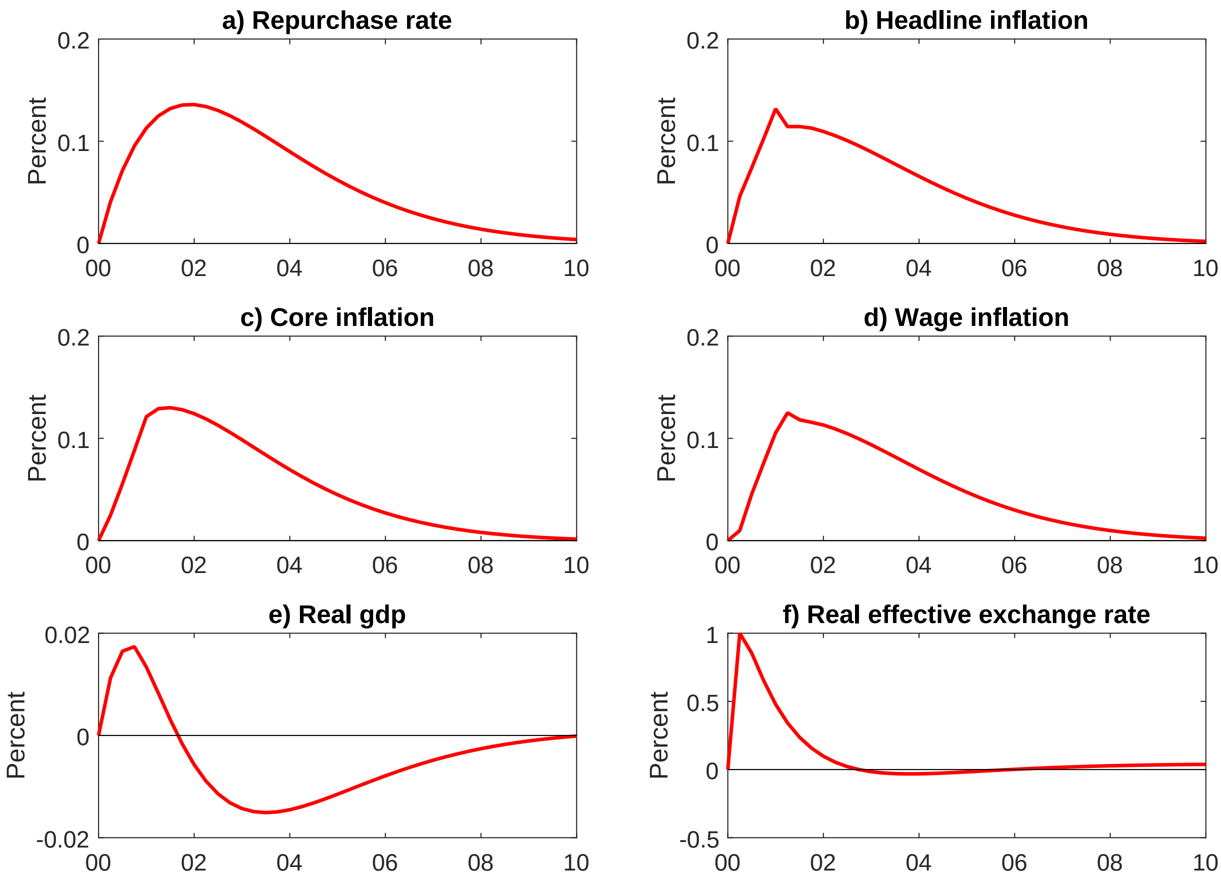
Figure 11: One per cent policy rate shock



5.4.2 Exchange rate shock

Figure 12 represents a one per cent unanticipated shock to the real effective exchange rate. The depreciation of the currency increases headline consumer inflation at its peak by 0.13 per cent. In response to higher inflation the repo rate is raised by 0.15 per cent.

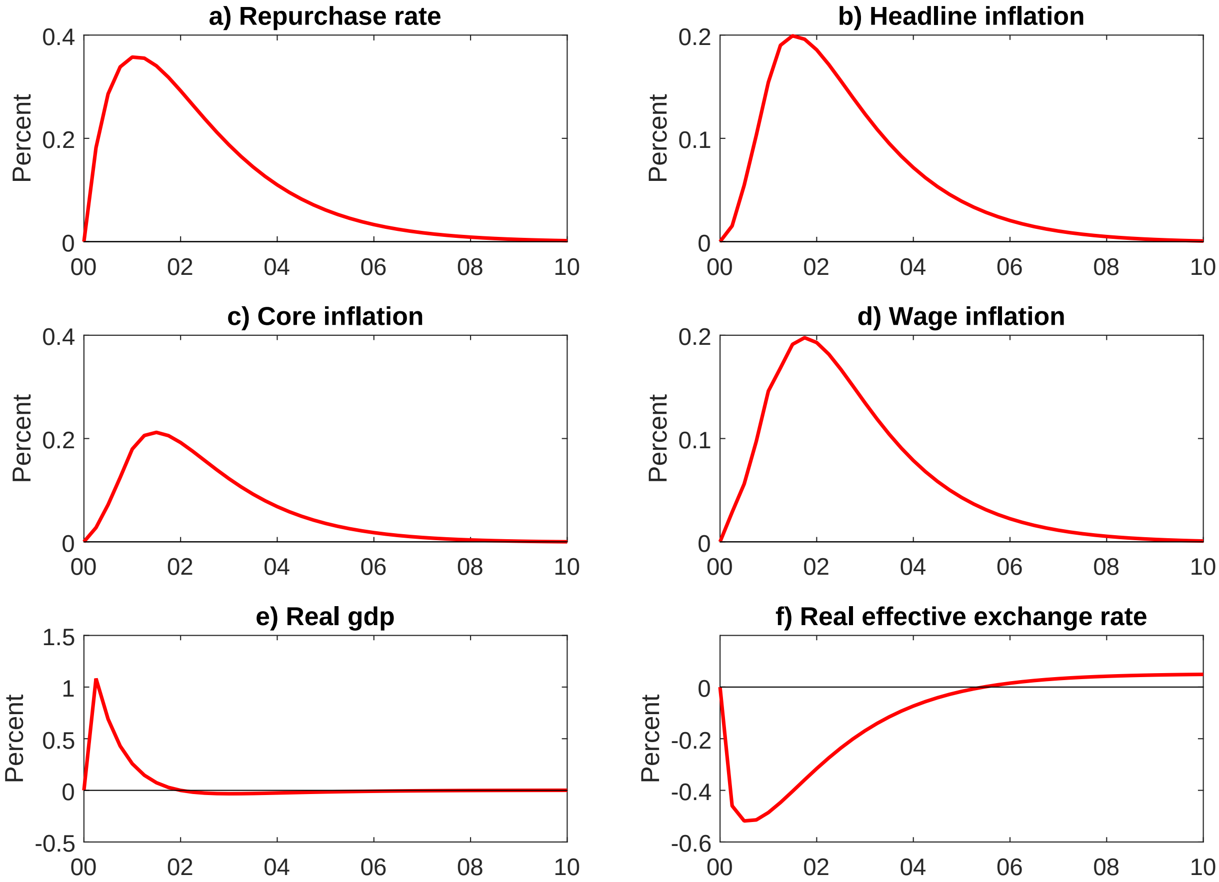
Figure 12: One per cent real effective exchange rate shock



5.4.3 Output gap shock

Figure 13 plots the IRFs from a one per cent shock to the output gap. Persistence in the output gap means that the shock only dissipates after seven quarters. In response to higher demand all components of inflation, with the exception of petrol which decreases in the first year due to the appreciation of the currency, increase. As a consequence headline inflation rises by 0.2 percentage points 6 quarters after the shock. The policy response is significantly stronger as policy is responding to both higher inflation and a larger positive output gap. At its peak the repo rate rises by 0.36 per cent four quarters after the shock. The higher expected policy leads to an appreciation of the real exchange rate.

Figure 13: One per cent GDP shock



5.5 Historical decompositions

In order to understand the model's economic story over time, this section looks at how the model properties and data decompose each variable based on its determinants (from the equation) and shocks. We provide two types of decompositions. First, each variable is decomposed into its explanatory variables by solving recursively for the lagged operator in each equation. This links each variable to the other variables in the model. Second, we use shock decompositions for each variable into all the shocks in the model. Shock decompositions are common in the literature, and are based on the premise that all the deviations from equilibrium that occur in the economy – the gaps – can ultimately be ascribed to shocks. The shock decomposition requires the grouping of the 56 shocks in the model into interpretable groups. Table A2 in the appendix provides details of how the models shocks are combined. This step requires some judgment and may affect the overall storyline. The two decompositions are complimentary, however, and together provide a model-consistent interpretation of historical drivers of inflation and growth.

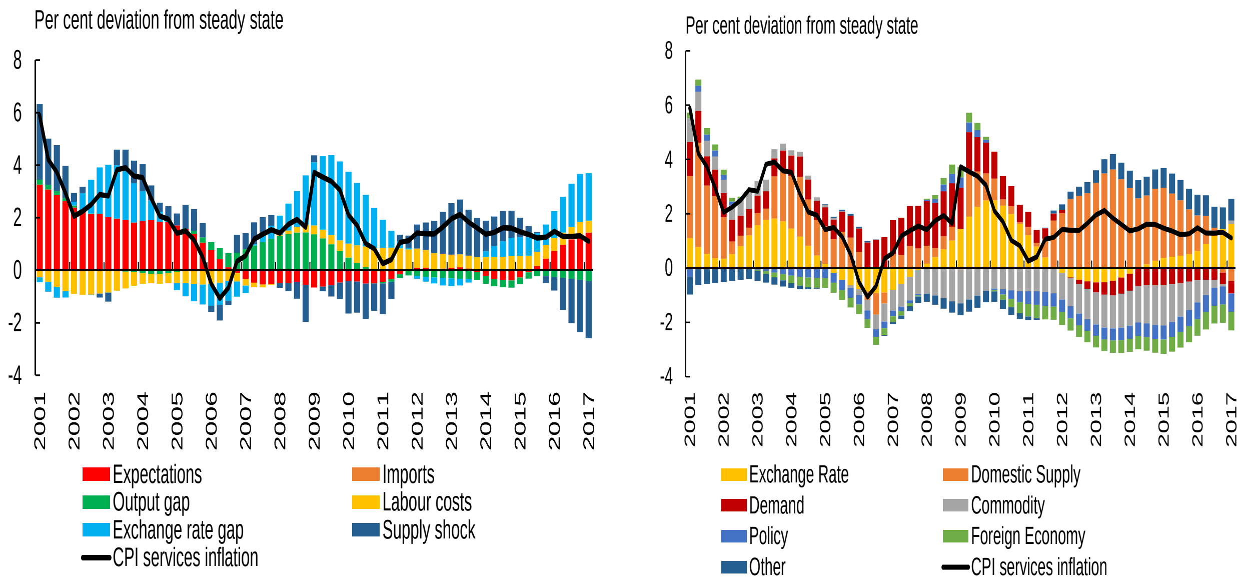
5.5.1 Inflation

In general the model correctly interprets the economic story over time for inflation components. It highlights the relative importances of the exchange rate (through intermediate inputs) and demand in the production process for services, compared with core goods. Similarly, it shows the impact of the exchange rate through the higher weight of imported goods in core goods inflation and food (labelled imports in the respective graphs).

Figure 14 shows the decompositions of services inflation, as a percentage point deviation from its equilibrium, from 2001 to the first quarter of 2017. The equation decomposition (14a) shows the main drivers of services inflation including the exchange rate (the real exchange rate gap determines the per cent of over/undervaluation), inflation expectations, demand, real labour costs, and direct imports. Services inflation is mainly driven by the real marginal cost of production and inflation expectations. The real marginal costs in the production of services includes the real exchange rate gap, real labour costs and the output gap, which explain the majority of outcomes. During periods of exchange rate undervaluation, the rand had added significantly to inflation developments particularly in 2001/02, 2008/09, and 2014 onwards. The QPM also shows how real labour costs rose following the financial crisis as nominal wages remained sticky despite falling inflation. Finally, insufficient demand has put downward pressure on inflation outcomes since 2010. Inflation expectations are also an important driver of services outcomes. Figure 14a shows how inflation expectations rose significantly following the 2001 depreciation of the exchange rate, but also as monetary policy worked to anchor inflation expectations with the introduction of inflation targeting. Inflation expectations have risen again since 2015 as overall inflation rose.

Figure 14: CPI: Services inflation

(a) Equation decomposition **(b) Shock decomposition**

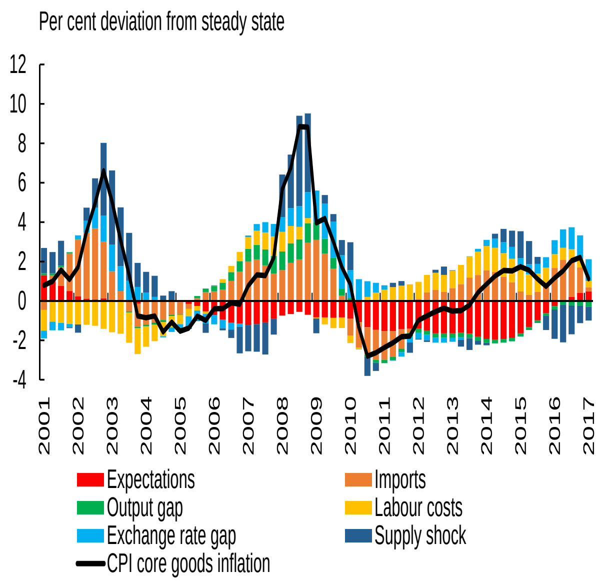


The shock decomposition of services inflation (figure 14b) highlights that exchange rate shocks are the most significant driver of higher inflation outcomes, followed by domestic supply shocks. Demand conditions played a role in raising services inflation over much of the 2000s, but since 2010 has contributed to lowering services inflation outcomes. Foreign inflation and demand outcomes have also contributed to containing service inflation outcomes since 2010.

Figure 15 plots the equation (15a) and shock (15b) decomposition of core goods inflation. The same drivers of services inflation are relevant for core goods, however, core goods also have a large direct imported component which is driven by the global inflation and exchange rate outcomes. The share of imported inflation, compared with the intermediate imports, is significant and explains on average 0.85 percentage points of core goods inflation since 2001, double the size of the exchange rate's role in intermediate inputs. The shock decomposition (figure 15b) is also similar to services inflation with exchange rate shocks explaining about the same amount of each outcome, and demand shocks increasing inflation through most of the 2000s, before shifting to a deflationary force after the great recession.

Figure 15: CPI: Core goods inflations

(a) Equation decomposition



(b) Shock decomposition

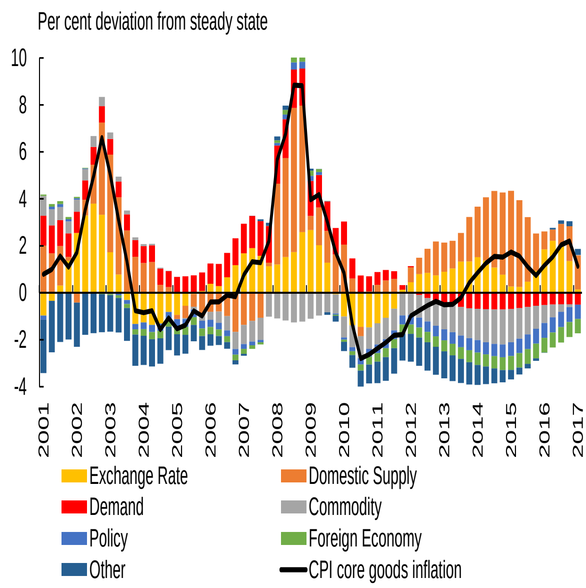
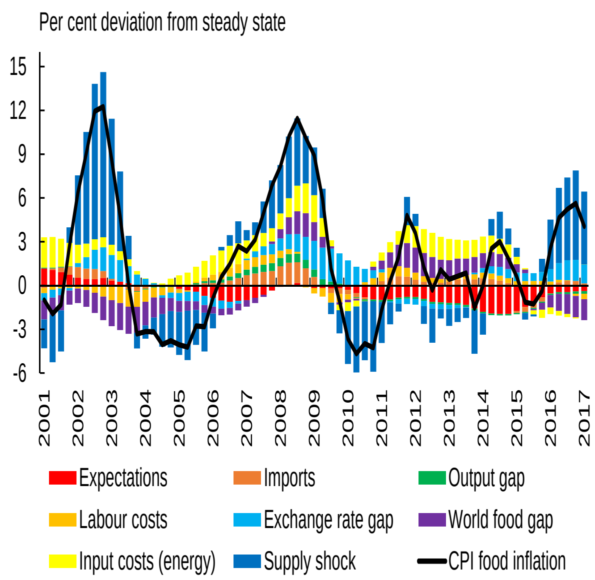


Figure 16 graphs the shock (16b) and equation (16a) decompositions for the CPI category of food and non-alcoholic beverages. Food inflation is a function of the direct imports, the rand price of international food products, as well as the domestic production costs including intermediate imports, the cost of labour, energy costs, and demand. Finally there is an unexplained component (labelled supply shock) which includes outcomes such as the recent drought. The international drivers of food play an important role in domestic food price outcomes with direct imports on average explaining 0.27 percentage points of food inflation since 2000. This is particularly the case during periods of exchange rate depreciation such as during 2001/02, and 2008/09. The rand value of global food prices similarly are important to the food outcome. Since 2015, the lower global price of food stuffs has helped to counteract some of the pressure on food prices due to the recent drought. Energy costs, used in the production and transport, have in general contributed positively to the cost of food averaging 0.8 percentage points since 2001. The collapse of oil prices in 2015 also counteracted some of the pressure of higher food prices due to the drought.

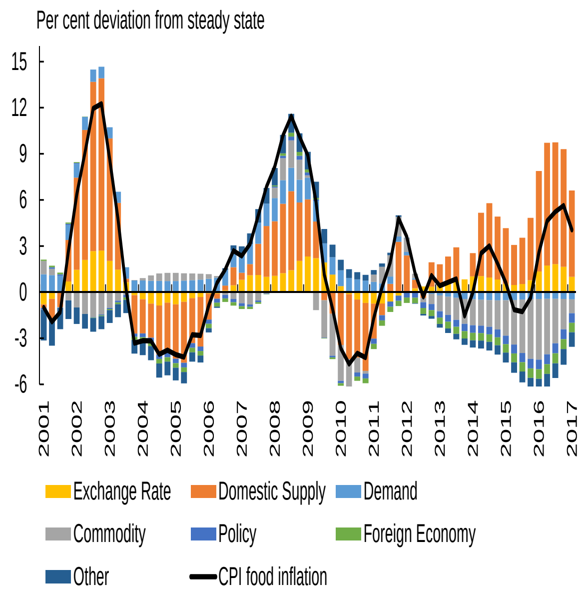
The shock decomposition of food (figure 16b) in general tells a similar story to the equation decomposition, however, highlights the role of domestic supply shocks, both to food directly from drought but also to the rise in marginal costs due to overall inflation outcomes. The group labelled 'Commodity' shows the impact of international food developments (but also included international oil, and industrial and precious metals). Low international food prices has helped to contain rising prices due to drought. In this decomposition the aggregate impact of the exchange rate can be seen through the group labelled 'Exchange rate'. The advantage of the shock decomposition is the ability to distinguish the overall impact of exchange rate developments, whereas in the equation decomposition, the exchange rate impact is distributed into a number of comportments.

Figure 16: CPI: Food inflation

(a) Equation decomposition



(b) Shock decomposition



5.5.2 Output Gap

Figure 17 shows the decomposition of the output gap based on the equation decomposition (17a) and the shock decomposition (17b). In figure 17a, the output gap is driven by developments in the exchange rate, foreign demand, commodity prices (labelled terms of trade), the policy stance (the real interest rate from its natural rate), and demand shocks. A significant driver of the output gap in recent times has been outcomes in the rest of the world, with the global output gap being negative since the great recession. On average, it contributes −0.4 percentage points to the South African output gap since 2009. Commodity prices also moved significantly lower from 2011, compounding the widening of the output gap over 2014 and 2015, before increasing once again in 2016 and early 2017. The largest contributor to the widening of the the output gap into the start of 2017 is demand shocks, the part of the output gap that is not explained by the model's variables. These shocks, in part, most likely reflect the impact of declining business and consumer confidence.

There have been some factors which, on the net, have helped to narrow the output gap since 2009. Prolonged periods of rand weakness, specifically during and following the great recession, and more recently since 2014, have contributed on average 0.1 percentage points. This indicates the importance of the rand as a shock absorber. The expansionary stance of monetary policy has also contributed 0.15 percentage points to the output gap since 2009, being particularly accommodative from 2011 to 2014. During this period forecasts of consumer inflation were well contained within the target range, and policy at its peak contributed 0.4 percentage points to the gap. In 2014, following a period of prolonged exchange rate depreciation, with core inflation creeping up and inflation outcomes breaching the upper end of the target range, the monetary policy committee started tightening policy. The QPM interprets monetary policy as still somewhat accommodative, in order to address the persistently negative output gap.

Figure 17: Output gap

(a) Equation decomposition **(b) Shock decomposition**

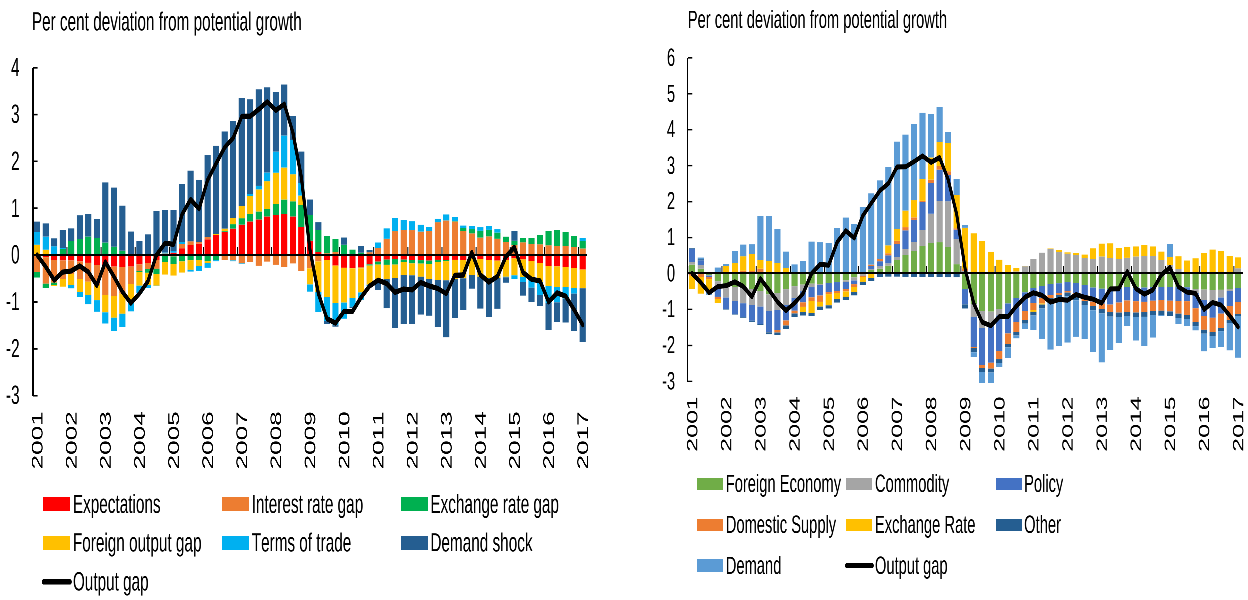
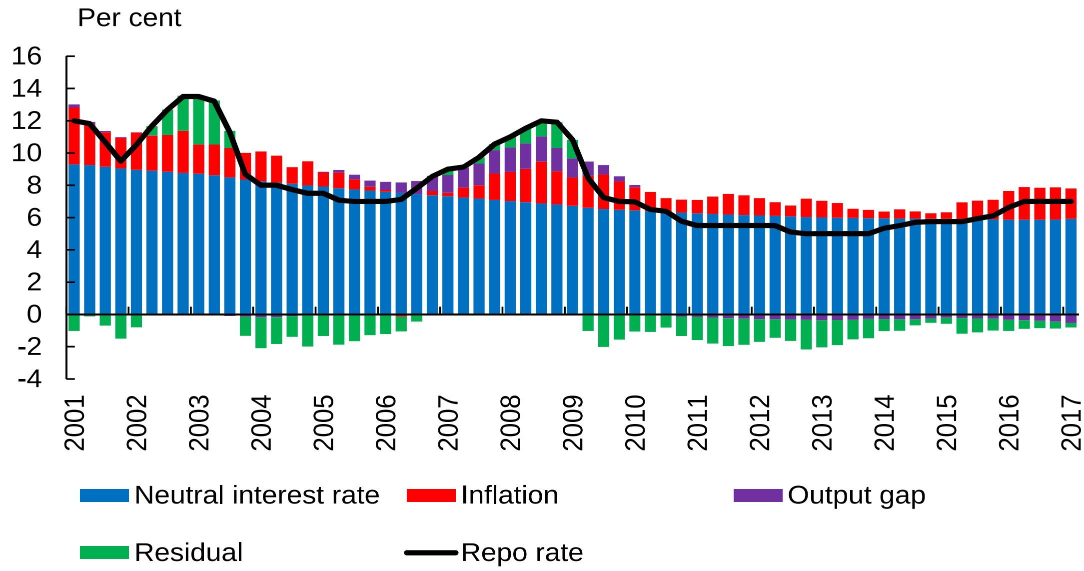


Figure 17b interprets the output gap in similar ways as above, with the global output gap being a drag on domestic outcomes since the great recession, and supporting growth prior. Commodity prices (representing internation food, oil, and industrial and precious metals) also follow a similar trend, contributing to better growth outcomes prior to the great recession, and from 2011-2014. Commodity price shocks turned positive in 2017Q1, as commodity prices, particularly industrial metals, rebounded. The exchange rate contribution from an over/under-valued currency is also consistent with figure 17a. One advantage of the shock decomposition is the ability to see how higher inflation outcomes undermine growth (group labelled 'Domestic supply'). This is clear during 2001 and more recently since 2014, where higher inflation explains −0.2 percentage points to the output gap.

5.5.3 Policy rate

The QPM model is closed with a Taylor rule that links changes in the policy rate to the Bank's objectives of inflation at target and output at its equilibrium. Figure 18 plots the equation decomposition of the repurchase rate into the neutral nominal policy rate, inflation from its target, and the output gap. The neutral nominal policy rate is the sum of the inflation target and the equilibrium real interest rate. Since 2000, the neutral real interest rate has been declining, driving down the nominal neutral as well. The neutral policy rate implies the rate at which monetary policy is neither expansionary or contractionary (*i.e.*, exerting no pressure on the inflation rate to rise or fall), or the output gap to open. From this neutral rate, monetary policy responds to both change to inflation from its target and the output gap. Inflation has generally been above the mid-point of the target and hence policy has had to respond to this higher inflation. However, policy has also responded to the output gap with periods of large positive output gaps driving up the policy rate and negative output gaps driving policy rates down.

Figure 18: Policy rate equation decomposition



The best example of a strong policy response to an overheating economy with rising inflation occurred from 2006. With inflation significantly above target, the Taylor rule implied policy tightening of over 250 basis points from neutral in 2008Q3, while a significantly positive output gap added an additional 150 basis points. More recently, the Bank has had to tighten policy in response to rising inflation, while the degree of tightening was muted by the widening of the negative output gap. However, it is important to keep in mind that this decomposition does not account for real-time data – the inflation forecast and output gap estimate – that the Bank would have used at the point in time when it made the specific policy decision. For example, revisions to the output gap over much of the post-crisis period have narrowed it (less negative), compared with what the magnitude of the output gap had been estimated to be at the time. The less negative output gap ex-post changes how the Taylor rule interprets the easing of policy that occurred.

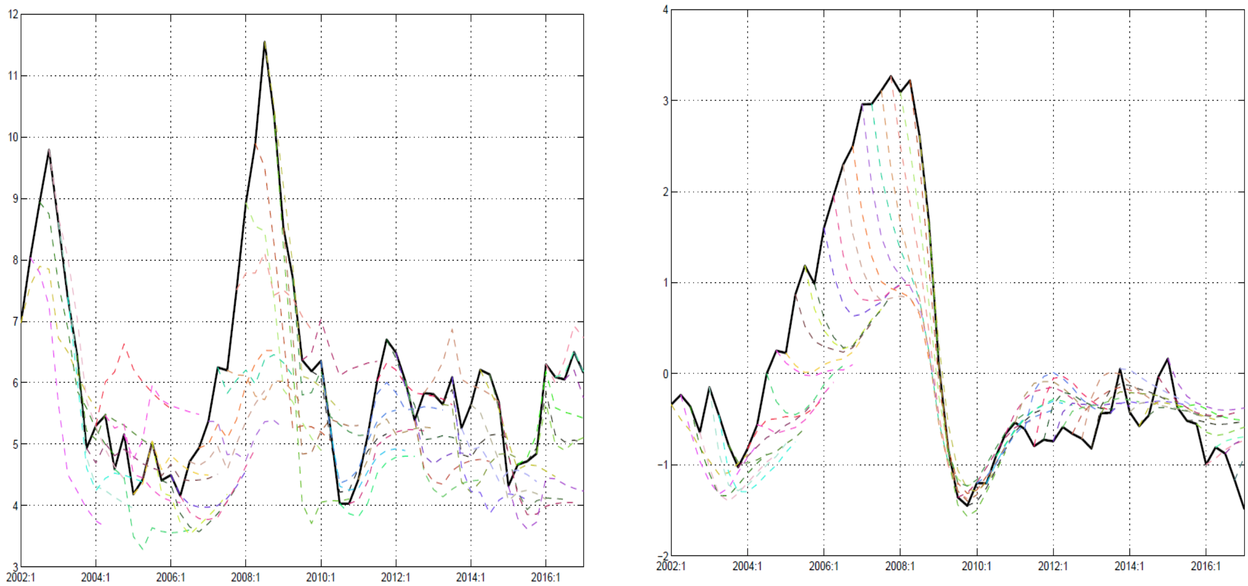
5.6 Forecast comparison

The QPM model, like the DSGE models it is based on, is designed to do both policy analysis and forecasting, and it is therefore not necessarily the best at forecasting over shorter horizons. This section looks at the out-of-sample forecasting performance of the QPM model.

Figure 19 plots the 8-step-ahead out-of-sample forecasts from 2002Q1 to 2017Q1 for targeted inflation (19a) and the output gap (19b). The QPM tends to underestimate inflation and output gap outcomes over this period, unable to forecast the significant rise in targeted inflation and the output gap during the boom years of the mid-2000s. The mean error (forecast less actual) of targeted inflation is negative over all horizons, partly as a result of the steady state value of inflation being at the mid-point of the target while actual targeted inflation averaged closer to 6 per cent during this period. The output gap forecast also tends to be biased downwards, however, the mid-2000s dominate the mean error calculation. More recently, the model has overestimated the output gap. Caution is needed when analysising the output gap since there is a significant difference between the real-time gap and the ex-post outcome as modelled here.

Figure 19: Historical out-of-sample simulations

(a) Headline CPI inflation **(b) The output gap**



To assess the relative performance of the QPM model, Table 3 shows the one to eight quarters-ahead forecast error statistics for nine variables of interest, relative to that of a naive forecast from a random walk model, over the period 2002Q1 to 2017Q1.[25] The QPM model outperforms a random walk model in all but three instances - the near-term forecasts of the output gap. The QPM also tends to improve significantly on a random walk model over longer horizons with the outperformance anywhere from 30 to 50 per cent.

[25]Table 3 contains the ratios of the root mean squared errors (RSME) of the QPM forecasts to the RMSEs from a random walk. The naive (random walk) forecasts assume that the variable of interest remains at its last known value over the entire forecast horizon. Therefore, a ratio of less than one indicates that the QPM model has a smaller forecast error (RMSE) than the naive forecast for that specific variable and forecast horizon.

Table 3: Forecast performance: Root mean squared error relative to random walk

Variable	Forecast horizon							
	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8
Headline inflation, yy	0.79	0.83	0.84	0.89	0.78	0.69	0.66	0.64
Core goods inflation, yy	0.83	0.90	0.89	0.87	0.69	0.57	0.54	0.53
Services inflation, yy	0.87	0.89	0.95	0.99	0.91	0.92	0.90	0.89
Food inflation, yy	0.54	0.58	0.60	0.62	0.56	0.55	0.53	0.51
Petrol inflation, yy	0.79	0.75	0.70	0.68	0.65	0.70	0.72	0.70
Electricity inflation, yy	0.90	0.85	0.84	0.79	0.71	0.69	0.69	0.68
Real GDP, qqa	0.99	0.71	0.59	0.57	0.56	0.57	0.56	0.56
Output gap (per cent of potential GDP)	1.26	1.12	1.00	0.88	0.79	0.72	0.66	0.62
Average nominal wages, yy	0.79	0.82	0.88	0.90	0.89	0.85	0.76	0.68

6 Conclusion

The Quarterly Projection Model is an integral part of the modelling and forecasting process at the South African Reserve Bank. Its comparative advantage lies in the forward-looking nature of the model, and the recognition that the actions of monetary policy affect expectations and hence the behaviour of consumers and firms. The QPM also simplifies the communication of monetary policy decisions, while anchoring economic outcomes to well-defined equilibriums.

This paper provides details of the structure and functioning of the QPM model, with particular focus on the four most important gaps – the output gap, real exchange rate gap, real interest rate gap, and inflation gap (or inflation from target). Moreover, the model is used to decompose these four gaps in order to tell a coherent story of South Africa's macroeconomic dynamics since the inception of inflation targeting.

From the perspective of the policy maker, the QPM provides a tool that quantifies the consequences of its actions on the economy, while adequately highlighting the trade-offs that are faced in the process. Most importantly, the model can be used as a platform through which the inflation expectations of economic agents can ultimately be anchored to the Bank's desired inflation target. This is achieved through transparent communication around the likely path that the policy rate will be required to follow over the forecast horizon in order to align inflation with its targeted objective.

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Appendices

A Data

Table A1: Data

	Variable	Transformation	Source
Prices			
	CPI: services	Index (2016Q4 = 100)	Statistics South Africa
	CPI: core goods	Index (2016Q4 = 100)	Statistics South Africa
	CPI: petrol	Index (2016Q4 = 100)	Statistics South Africa
	CPI: food	Index (2016Q4 = 100)	Statistics South Africa
	CPI: electricity	Index (2016Q4 = 100)	Statistics South Africa
	CPI: services weight	Level	Statistics South Africa
	CPI: core goods weight	Level	Statistics South Africa
	CPI: food weight	Level	Statistics South Africa
	CPI: electricity weight	Level	Statistics South Africa
	CPI: petrol weight	Level	Statistics South Africa
	Average wages	Current rand millions	Statistics South Africa
	Foreign consumer prices	Weighted US, euro area, Japan	Global projection model
	Eq. commodity prices	HP-filter with measurement error	Own calculations
	Commodity price index	Weighted export commodities (2010 = 100)	Bloomberg and own calculations
	International food price index	Index (Dec 2012 =100)	Food and Agricultural Organization of the United Nations
	Brent crude oil price index	Index (2016 = 100)	Bloomberg
	Basic fuel price	Rand	Department of Energy and Central Energy Fund
	Taxes and margins on petrol	Rand	National Treasury and Department of Energy
Real economy			
	Real GDP	Constant rand millions	Statistics South Africa
	Potential growth	Per cent	Auxiliary Model
	Underlying potential growth	Per cent	Auxiliary Model
	Foreign output gap	Per cent of potential GDP	Global Projection Model
	Foreign potential growth	Per cent	Global Projection Model
	Employment	Millions	Quarterly Employment Survey
Financial			
	Repurchase rate	Per cent	South African Reserve Bank
	Yield on government bonds	Per cent	South African Reserve Bank
	Yield on government bonds	Per cent	South African Reserve Bank
	Yield on government bonds	Per cent	South African Reserve Bank
	Nominal exchange rate	Weighted of cross-rates with US dollar, euro, and Yen	South African Reserve Bank
	Country risk premium	Per cent	Bloomberg
	Foreign policy rate	Weighted policy rates of US, euro and Japan	Global Projection Model

B Shock decomposition

Table A2 shows the groupings of shocks used to create the shock decomposition graphs. Groupings are attempts to combine similar concepts which are interpretable. An simple example is the domestic demand shock which includes only the shock to the output gap. Shocks are commonly referred to name of the residuals in each equation. The decomposition graphs also include an 'other' category which includes shocks to the relative prices, equilibriums, and all other shocks that do not assist in an economic narrative.

Table A2: Shock decomposition groupings

Group	Shocks
Demand	$\varepsilon_t^{\hat{y}}$
Domestic supply	$\varepsilon_t^{\pi^{services}}$; $\varepsilon_t^{\pi^{coregoods}}$; $\varepsilon_t^{\pi^{food}}$; $\varepsilon_t^{\pi^{electricity}}$; $\varepsilon_t^{\pi^{petrol}}$; $\varepsilon_t^{\Delta pettax}$;
Commodity	$\varepsilon_t^{\widehat{rcom}}$; ε_t^{food}; and ε_t^{oil}
Exchange rate	ε_t^{S}; ε_t^{prem}; $\varepsilon_t^{term^8}$; $\varepsilon_t^{term^{20}}$; and $\varepsilon_t^{term^{40}}$
Policy	ε_t^{i}
Foreign economy	$\varepsilon_t^{\hat{y}^*}$; $\varepsilon_t^{\pi^m}$; $\varepsilon_t^{\pi^*}$; $\varepsilon_t^{\pi^*}$; and $\varepsilon_t^{i^*}$

C Variables names and naming convention

x	Log-level variable multiplied by 100 (except in the case of interest rates and the like).
x^*	Foreign (trading partner's) variable.
$\overline{x}$	Equilibrium variable of x.
$\widehat{x}$	Gap variable of x (deviation from equilibrium).
x^{ss}	Steady state value of variable x.
Δx	Quarter-on-quarter change (annualised log % in case of log variables).
$\Delta_4 x$	Year-on-year change (log % in case of log variables).
y	Log-level of output (GDP) multiplied by 100.
g	Quarterly growth in GDP underlying trend (in % pa).
p	Log-level of headline consumer price index (SA) multiplied by 100.
p^x	Log-level of CPI component x (SA) multiplied by 100.
p^m	Log-level of import prices multiplied by 100.
$relp^x$	Relative price of component x to headline CPI.
π^x	Quarter-on-quarter inflation (annualised log %).
$\pi^{4,x}$	Year-on-year inflation (log %).
$wage$	Log-level of average salaries multiplied by 100.
$rwage$	Wages deflated by headline CPI (real wage).
emp	Log-level of employment multiplied by 100.
s	Log-level of the nominal effective exchange rate multiplied by 100.
z	Log-level of the real effective exchange rate multiplied by 100 (CPI deflated).
$prem$	Country risk premium (in % pa).
i	Nominal short-term (policy) interest rate.
r	Real short-term (policy) interest rate.
lr	Real long-term (policy) interest rate.
rmc^x	Real marginal costs of good x.
$food$	Log-level of international food prices multiplied by 100.
$rfood$	International food prices deflated by foreign prices (CPI).
com	Log-level of international commodity prices multiplied by 100.
$rcom$	International commodity prices deflated by foreign prices (CPI).
oil	Log-level of international oil prices multiplied by 100.
$roil$	International oil prices deflated by foreign prices (CPI).