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Joint estimation of liquidity and credit risk premia in bond prices with an application*

Jens H. E. Christensen† and Daan Steenkamp‡

Abstract

This paper introduces a novel arbitrage-free dynamic term structure model that jointly accounts for liquidity and credit risk premia in panels of bond prices. While liquidity risk is bond-specific, credit risk is common across bonds and follows a square-root process to ensure nonnegativity and econometric identification. A simulation study confirms the separate identification of liquidity and credit risk. We apply the model to South African government bond prices and document the existence of large and weakly correlated liquidity and credit risk premia. This underscores that liquidity and credit stresses are distinct risks to bond investors.

JEL classification

D84, E31, E43, E44, E47, E52, E58, G12

Keywords

Affine arbitrage-free term structure model, financial market frictions, emerging market economy, sovereign debt

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1 Introduction

The debt capacity of governments in emerging market economies (EMEs) is widely perceived to be more limited than that of governments in advanced economies. As a consequence of the resulting smaller outstanding volumes, the government bond markets in EMEs are also commonly considered to be less liquid than their advanced economy counterparts. Hence, liquidity risk, which is uncertainty about the ability to trade a given asset, is likely a nonnegligible component in the pricing of EME sovereign bonds. At the same time, credit risk, which is uncertainty about an issuer's ability to make promised payments in a timely manner, is likely to be a separate, potentially important component in EME sovereign bond prices. However, from the corporate credit literature, we know that liquidity and credit risks are interrelated and very challenging to disentangle, as liquidity naturally dries up if investors fear a bankruptcy is looming (see Driessen (2005) and De Jong and Driessen (2012), among many others).[1]

In this paper, we achieve separate identification of the liquidity and credit risk components in panels of bond prices using a novel state-of-the-art arbitrage-free dynamic term structure model. The identification of the liquidity risk component relies on the crucial assumption that liquidity is security-specific, meaning that at its core liquidity risk is about an investor's ability – at a random point in the future when hit with a liquidity shock – to sell a specific bond, namely the one owned, back to the market without incurring any major discounts relative to the general level of bond prices prevailing at that time. In contrast, credit risk is the opposite: it is not specific to any particular security, but rather applies to all debt issued by the legal entity in question. Ultimately, this distinction is linked to the bankruptcy code, according to which a missed payment on *any* owed debt puts *all* owed debt into a state of default. Technically, this implies that credit risk should be priced equally across all outstanding bonds of the same seniority.[2]

Guided by these fundamental observations, the identification of the liquidity risk factor in our model comes from its unique loading for each individual security, as in Andreasen, Christensen and Riddell (2021). To make this operational, our analysis relies on the prices of individual bonds rather than the more usual input of yields from fitted synthetic curves. The underlying mechanism assumes that, over time, an increasing proportion of the outstanding inventory is locked up in buy-and-hold portfolios. Given forward-looking investor behaviour, this lock-up effect means that a particular bond's sensitivity to the marketwide liquidity risk factor will vary depending on how seasoned and how close to maturity the bond is. In a careful study of nominal United States (US) Treasuries, Fontaine and Garcia (2012) find a pervasive liquidity factor that affects all bond prices, with loadings that vary with the maturity

[1] For corporations, the somewhat infrequent risk associated with rolling over maturing debt may play an integral role in the dynamic interaction between the liquidity and credit risk of the outstanding debt (see He and Xiong (2012)). For sovereign debt issuers, the rollover risk is more continuous in nature thanks to the large outstanding amounts of short-term bills that need to be refinanced on an ongoing basis.

[2] Although our focus is on sovereign bonds of similar seniority, the presented framework can easily accommodate differences in the losses at default across bonds.

and age of each bond. By observing a cross-section of bond prices over time – each with a different time since issuance and time to maturity – we can identify the overall liquidity risk factor and each bond's loading on that factor. Finally, the credit risk factor in our model is assumed to be a conventional slope factor, as is standard in the vast literature on corporate bond credit risk (see Duffee (1999) and Longstaff, Mithal and Neis (2005) for classic examples). Importantly, its econometric identification is ensured by letting it follow a square-root process (see Cox, Ingersoll and Ross (1985)). This also preserves its nonnegativity.

This is a general framework that can be combined with any existing arbitrage-free dynamic term structure model of the risk-free short rate r_t. To facilitate the empirical implementation, we choose to use the realistic dynamic arbitrage-free Nelson-Siegel (AFNS) model introduced in Christensen, Diebold and Ruderbusch (2011). This implies that the *frictionless* yields within our model, that is, whose that would prevail without any financial frictions or credit risk components, have the well-known Nelson and Siegel (1987) level, slope and curvature factor structure.[3]

To demonstrate the model's empirical tractability, we apply it to the South African government bond market, which is ideal for our purposes. First, the South African government has a long history of issuing a variety of government bonds with maturities of up to 35 years and traded in relatively liquid markets – by emerging bond market standards.[4] This provides us with the requisite long sample of a sufficient set of high-quality bond prices needed to empirically implement our model. Second, although liquid, we stress that the South African government bond market is not nearly as liquid as most government bond markets in advanced economies. Hence, market liquidity is likely to be a material risk factor for investors in these bonds. Lastly and importantly, the South African government has increased its amount of debt substantially over the last 15 years. While government debt-to-gross domestic product (GDP) in South Africa reached a record low of 28% back in 2008, it stood at 73% by the end of 2023, which is high by EME standards. Thus, while credit risk may have been a minor component in the pricing of South African government bonds early on in our sample, it is likely to play a more prominent role towards the end of our sample.

Our results can be summarised as follows. First, our novel five-factor term structure model, which accounts for both liquidity and credit risk premia in the bond prices, produces a very tight fit to the data, with a root-mean-squared error smaller than 4 basis points for all bonds combined when measured in yield deviations.

Second, the estimated liquidity premia average 55 basis points over our sample with significant variation and a high standard deviation of 62 basis points. Moreover, outside of short-lived sharp spikes around the global financial crisis in 2009 and the COVID-19 pan-

[3] Econometrically, through the shared λ parameter between the slope and curvature factor in the AFNS model, this choice also clearly sets the frictionless slope factor apart from the credit risk slope factor despite both being latent factors in our model framework.

[4] Many studies on emerging bond markets have shorter samples (see Beauregard et al. (2024) or Cardozo and Christensen (2024) for evidence from Mexico and Colombia respectively).

demic in early 2020, the estimated liquidity premium series is characterised by a notable upward trend over our sample. As a consequence, liquidity premia in the South African government bond market are highly elevated by the end of our sample period. The estimated liquidity premia are robust to a variety of implementation choices.

Third, the estimated credit risk premia average 118 basis points, with a standard deviation of 39 basis points. While they are mostly relatively stable during our sample, the credit risk premia experienced a large outsized spike during 2023 that left them well above 300 basis points at the end of our sample in February 2024. This high credit risk premium coincided with the government debt-to-GDP ratio being well above 70%, as noted earlier. Investors appear to have become increasingly concerned about the high level of government debt in South Africa during the last year of our sample.

Finally and importantly, in comparing the two risk premium series, the average credit risk premium series has a low negative correlation of -19% with the average liquidity premium series. We take these results to indicate the existence of large and weakly correlated liquidity and credit risk premia in this market. This finding also proves that liquidity and credit stresses are indeed distinct risks to bond investors, as also emphasised in the corporate credit literature. The absence of a positive correlation between liquidity and credit risk premia in the South African government bond market may reflect the fact that sovereign debt is less lumpy than corporate debt, as short-term bills are continuously maturing. Unlike in corporate bond markets, where large approaching rollover risks can give rise to fears about an issuer's ability to refinance and related negative speculative market dynamics (i.e. credit and liquidity risk spiking simultaneously), sovereign bond markets are much less at risk of experiencing such speculative market dynamics. Based on our findings and this logic, we speculate that liquidity and credit risk premia are likely also weakly correlated in other EME sovereign bond markets, but we leave it for future research to confirm this conjecture.

To better understand the determinants of the estimated liquidity and credit risk premia, we use regression analysis with them as the dependent variables and a large number of control variables. The results show that the average liquidity premium series is significantly negatively correlated with both the share of the market held by buy-and-hold investors, as proxied through the holdings of pension funds, and the share owned by foreigners. Assuming that pension funds and foreign investors are both sophisticated in their trading strategies, these findings suggest that the frictions to trading in this market are lowered when these two groups represent a larger share of the investor universe. Moreover, the results show that increases in the stock of bonds scaled by nominal GDP tend to put upward pressure on the liquidity risk premia in this market. Given that the estimated liquidity risk premium series is characterised by an upward trend during our sample period, our regression results show that the increased debt issuance during this period has more than offset the positive effects from the increased foreign presence in the South African government bond market.

As for the estimated credit risk premia, it is only the foreign share, which preserves a consistently negative coefficient, that is mostly statistically significant. This points to some positive correlation between the foreign presence and the debt sustainability of the South African government, but the causality could run in the opposite direction as well, whereby foreigners are mostly drawn to South African government bonds at times when priced credit risk is low. Beyond this interesting finding, this set of regressions is characterised by notable parameter instability and poorer fit than observed in the liquidity risk premium regressions. Given the global and systematic nature of our explanatory variables, this suggests that the credit risk of the South African government appears to be largely idiosyncratic. For the same reason, our estimation results could be taken to imply that a default by the South African government would most likely be a non-systemic event from the vantage point of global financial markets. This seems reasonable given that South Africa is a small open economy located at the southernmost tip of the vast African continent.

Overall, these results provide strong evidence of an important role for foreigners in the South African government bond market. Ultimately, it may be less relevant whether it is an increased share of foreigners alone that lowers the liquidity and credit risk premia or some unobserved characteristic that drives them down and then attracts foreign investors to this market. Either way, based on our findings, any policies or strategies that can help increase the foreign presence in the South African bond market would seem to offer tangible benefits in the form of lowering the liquidity and credit risk premia in the bond prices.

In a final exercise, we design a simulation study using our estimated South African model as the true model to further study and document its ability to separately identify the liquidity and credit risk factors and distinguish them from the frictionless level, slope and curvature factors. Based on $N = 100$ simulated samples identical in bond composition structure to our South African bond sample,[5] we find that, indeed, the extended Kalman filter estimation is able to both accurately filter all five state variables, including the separate liquidity and credit risk factors, and deliver unbiased estimates of all risk-neutral $\mathbb{Q}$-related model parameters used for pricing, while we see the usual upward bias in the mean-reversion parameters under the real-world objective $\mathbb{P}$-dynamic (see Bauer, Rudebusch and Wu (2012) for a detailed discussion). The accuracy is particularly high when measurement noise is low, but crucially it remains satisfactory for moderate measurement noise at the level observed in our South African bond sample. This supports our recommendation to apply the presented model to both other emerging sovereign bond markets and corporate bond markets in advanced economies whenever liquidity and credit risk premia are material and both components merit careful consideration.[6]

[5] Here, identical structure means that the simulated samples are of the exact *same length* as our South African sample and contain the *same number* of bonds as our South African sample; each has the *same coupon* as in our South African sample and appears in the data at the exact *same time* as the corresponding bond in our South African sample. This matters because multiple bonds in the data have occasional missing observations, a sign of the lower liquidity in this market relative to advanced sovereign bond markets.

[6] This contrasts with models of sovereign bond prices from most advanced economies, where both liquidity

The remainder of the paper is organised as follows. Section 2 describes our South African government bond market data, while section 3 introduces the novel no-arbitrage term structure model we use and details the estimation results. Section 4 analyses the liquidity and credit risk premia embedded in the South African government bond prices, while section 5 details our simulation study and its results. Finally, section 6 concludes.

2 South African government bond market data

In this section, we first describe the South African government bond data we use in the model estimation before we examine of the major market participants and their holdings, the bid-ask spreads in the markets for these bonds and the involved level of credit risk.

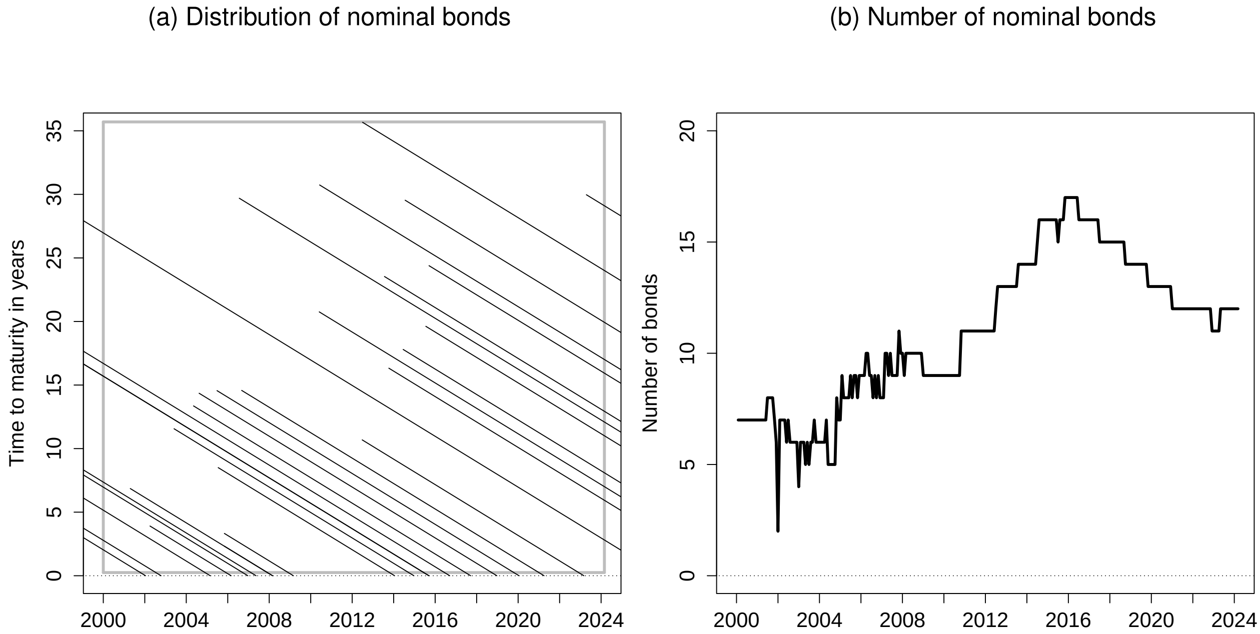
2.1 South African government bond data

The available universe of individual South African government fixed-coupon bonds is illustrated in Figure 1. Each bond is represented by a solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. These bonds are all marketable non-callable bonds denominated in South African rands that pay a fixed rate of interest semi-annually. We track the entire universe of bonds issued since January 2000. In addition, we include a few bonds outstanding at the start of our sample period. In general, the South African government has issued a diverse set of bonds, but with a clear preference for issuing long-term bonds with maturities of up to 35 years. This contrasts with most other sovereign bond markets on the African continent, where bonds with 20 years or longer to maturity are issued much less frequently, if at all.[7] For our analysis, the main point to note is that there is a wide variety of bonds with different maturities and coupon rates in the data throughout our sample. This variation provides the foundation for the econometric identification of the factors in the yield curve models we use.

and credit risk considerations are frequently omitted.

[7] This contrasts with Latin American countries where such long-term debt is quite common (see Beauregard et al. (2024) and Ceballos, Christensen and Romero (2024) for evidence from Mexico and Chile, respectively).

Figure 1: Overview of the South African government bond data



Note: Panel (a) shows the maturity distribution of the South African government fixed-coupon bonds considered in the paper. The solid gray rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on 31 January 2000, and end on 29 February 2024, and limited to bond prices with more than three months to maturity after issuance. Panel (b) reports the number of outstanding bonds at a given point in time.

Table 1 shows the contractual characteristics of all 31 bond securities in our sample. The number of monthly observations for each bond using three-month censoring before maturity is also shown in the table.

Table 1: Sample of South African government bonds

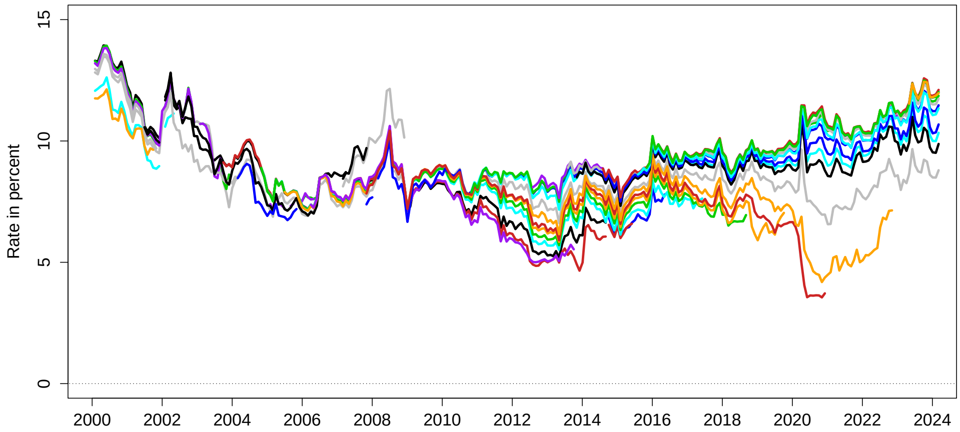
Fixed-coupon bonds	No. obs.	Issuance		Total notional amount
		Date	Amount	
(1) 12% 2/28/2005	51	5/2/1989	6.09	7.50
(2) 13.5% 9/15/2015	175	10/24/1991	0.03	27.28
(3) 13.5% 9/15/2016	21	10/24/1991	0.03	25.89
(4) 13.5% 9/15/2015	9	10/24/1991	0.03	6.30
(5) 9% 10/15/2002	28	4/15/1994	0.00	3.49
(6) 9.5% 5/15/2007	53	4/15/1994	0.10	22.91
(7) 12.5% 1/15/2002	21	4/3/1995	0.00	23.12
(8) 12.5% 12/21/2006	63	4/4/1996	0.03	9.00
(9) 10.5% 12/21/2026	288	5/21/1998	1.00	23.12
(10) 10% 2/28/2008	69	4/20/2001	0.08	12.61
(11) 12% 2/28/2006	21	4/2/2002	6.09	27.01
(12) 8.75% 12/21/2014	133	5/30/2003	0.60	11.37
(13) 8.25% 9/15/2017	152	5/7/2004	0.35	65.58
(14) 8% 12/21/2018	167	8/13/2004	0.20	23.76
(15) 7.25% 1/15/2020	172	6/24/2005	0.25	23.76
(16) 7.5% 1/15/2014	91	7/15/2005	0.25	47.77
(17) 10% 2/28/2009	22	11/1/2005	0.04	55.62
(18) 10% 2/28/2008	9	11/1/2005	0.04	58.48
(19) 6.25% 3/31/2036	197	7/21/2006	0.30	103.59
(20) 6.75% 3/31/2021	158	9/1/2006	0.25	378.17
(21) 7% 2/28/2031	161	5/28/2010	0.30	363.17
(22) 6.5% 2/28/2041	161	6/4/2010	0.40	191.12
(23) 7.75% 2/28/2023	125	6/22/2012	0.50	310.81
(24) 8.75% 2/28/2048	140	6/29/2012	0.50	310.63
(25) 8.5% 1/31/2037	128	7/19/2013	0.65	104.26
(26) 8% 1/31/2030	125	10/4/2013	0.30	284.39
(27) 8.25% 3/31/2032	117	6/13/2014	0.35	240.60
(28) 8.75% 1/31/2044	116	7/18/2014	1.05	93.45
(29) 8.875% 2/28/2035	104	7/17/2015	0.25	254.32
(30) 9% 1/31/2040	101	9/11/2015	0.25	400.60
(31) 11.625% 3/31/2053	11	4/11/2023	1.30	37.76

Note: The table reports the characteristics, first issuance date and amount, total number of auctions and total amount issued in billions of South African rand, for the available universe of South African government fixed-coupon bonds in the sample. Also reported are the number of monthly observation dates for each bond during the sample period from 31 January 2000 to 29 February 2024.

Figure 1 shows the distribution across time of the number of bonds included in the sample. With the exception of the first few years of our sample, the number of bonds has fluctuated between 10 and 15 for most of our sample. Combined with the cross-sectional dispersion in the maturity dimension observed in Figure 1, this implies that our panel of bond prices is very well-balanced.

Figure 2 shows the time series of the yields to maturity implied by the observed South African government bond prices. First, we note that the general yield level in South Africa trended down between 2000 and 2005 and remained fairly stable between then and the onset of the COVID-19 pandemic in early 2020. By the end of our sample, however, there has been a notable reversal that has left South African long-term government bond yields

Figure 2: Yield to maturity of South African government bonds



NOTE: This figure shows the yields to maturity implied by the South African government fixed-coupon bond prices. The data are monthly, covering the period from 31 January 2000 to 29 February 2024, and censor the last three months for each maturing bond.

by the end of our sample close to where they started in the early 2000s. This contrasts with government bond yields in advanced economies, which have declined significantly during this period (see Holston, Laubach and Williams (2017) and Christensen and Rudebusch (2019), among many others). Second, as in US Treasury yield data, there is notable variation in the shape of the yield curve. At times, as in early 2006, yields across maturities are relatively compressed. At other times, the yield curve is steep, with long-term bonds trading at yields 400–500 basis points above those of shorter-term securities – as in 2013 and again in 2021. These characteristics are the practical motivation behind our use of a three-factor model for the frictionless part of the South African yield curve, adopting an approach similar to the standard for US and United Kingdom (UK) nominal yield data (see Christensen and Rudebusch (2012)).

To support that choice more formally, we note that researchers have typically found that three factors are sufficient to model the time variation in the cross-section of US Treasury yields (e.g. Litterman and Scheinkman (1991)). To perform a similar analysis based on our sample of South African government bond prices, we construct synthetic zero-coupon bond yields by fitting the flexible Svensson (1995) yield curve to the set of bond prices observed for each observation date.[8] To have a yield panel representative of the underlying bonds in our sample, we include yields for eight constant maturities: 1, 2, 3, 5, 7, 10, 20 and 30 years. The data series are daily, covering the period from 3 January 2000 to 29 February 2024.

The result of a principal component analysis of the yield panel is shown in Table 2. The top panel shows the eigenvectors that correspond to the first three principal components.

[8] Technically, we proceed as described in Andreasen, Christensen and Rudebusch (2019). We stress that these synthetic zero-coupon yields are used solely to shed light on the factor structure in the South African bond price data. As explained in the main text, all other yield analysis in the paper uses the coupon bond prices directly.

Table 2: Factor loadings of South African government bond yields

Maturity in months	First PC	Second PC	Third PC
12	0.70	0.29	-0.63
24	0.39	0.17	0.24
36	0.31	0.08	0.42
60	0.29	-0.06	0.39
84	0.28	-0.17	0.31
120	0.26	-0.30	0.19
240	0.16	-0.55	-0.09
360	0.09	-0.68	-0.27
% explained	55.30	27.50	15.58

NOTE: The top rows show the eigenvectors corresponding to the first three principal components (PCs). Put differently, they show how bond yields at various maturities load on the first three principal components. The proportion of all bond yield variability explained by each PC is shown in the final row. The data are daily South African zero-coupon government bond yields from 3 January 2000 to 29 February 2024, a total of 6 200 observations for each yield series.

The first principal component accounts for 55.3% of the variation in the bond yields, and its loading across maturities is uniformly positive. Similar to a level factor, a shock to this component changes all yields in the same direction irrespective of maturity. The second principal component accounts for 27.5% of the variation in these data and has sizeable positive loadings for the shorter maturities and sizeable negative loadings for the long maturities. Similar to a slope factor, a shock to this component steepens or flattens the yield curve. Finally, the third component, which accounts for 15.6% of the variation, has a hump-shaped factor loading as a function of maturity, which is naturally interpreted as a curvature factor. These three factors combined account for 98.4% of the total variation. This motivates our choice to focus on the Nelson and Siegel (1987) model with its level, slope and curvature factors for modelling this sample of South African bond prices. However, for theoretical consistency, we use the arbitrage-free version of this class of models derived in Christensen, Diebold and Rudebusch (2011). To explain the remaining variation in the bond yield data not accounted for by the level, slope and curvature factors, we augment the model with both a liquidity and a credit risk factor, as detailed in section 3. Finally, we stress that the estimated state variables in our model are *not* identical to the principal component factors discussed here but are estimated through Kalman filtering.[9]

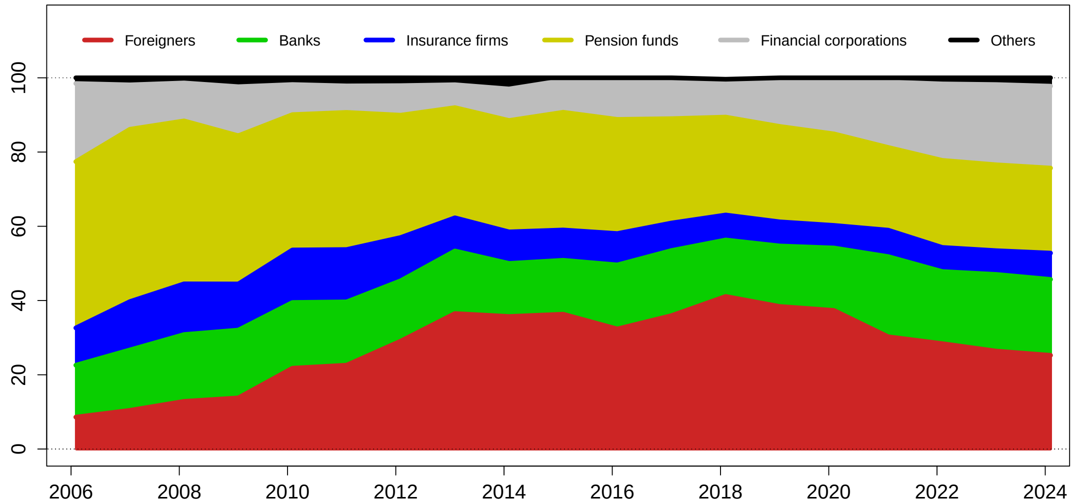
2.2 South African government bond holdings

In this section, we provide details on the investor groups holding South African government bonds. The data we use have been collected by the South African Reserve Bank (SARB) since 2006 to track market activity in the South African sovereign bond markets. Importantly, the data break down investor holdings into multiple groups, which include banks, insurance companies, pension funds and foreigners.

[9] A number of recent papers use principal components as state variables. Joslin, Singleton and Zhu (2011) is an early example.

Figure 3 shows the relative share of the nominal fixed-rate government bond market held by domestic residents and foreigners each January, starting in 2006. Note that there has been a significant increase in the foreign-held share since 2010, which implies that foreigners have become the largest investor group, accounting for about one-third of the market by the end of our sample. This expansion of the foreign role has come at the expense of the participation of pension funds, while the holdings of the other domestic investor groups have changed little on net.

Figure 3: Holdings of South African government bonds



In recent years, elevated fiscal risk has been associated with declining foreign demand for new issuances of South African government bonds. The modestly declining role of foreign investors in the domestic bond market since their share peaked in 2018 appears to have contributed to a decline in market liquidity (see SARB 2023). The related increased holdings of government bonds by the domestic financial sector have raised concerns about whether government fiscal risk could jeopardise financial stability in South Africa (see SARB (2021) and Diesel et al. (2022)). We evaluate these conjectures as part of our analysis.

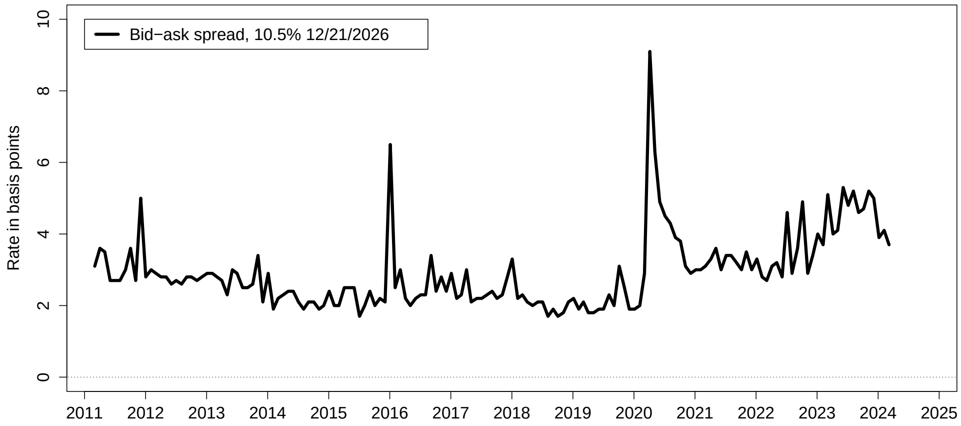
Overall, we take this evidence to show that there is an active and diverse market for South African government bonds.

2.3 Bid-ask spreads of South African government bonds

To measure bond market liquidity conditions, bid-ask spreads are widely used (see Amihud and Mendelsohn (1991) for a classic reference). Following that tradition, Figure 4 shows the bid-ask spread for a key reference bond (10.5% 12/21/2026) monitored by the SARB.[10] The plotted series starts in 2011 when the data become available.

[10] This is bond number 9 in our sample and also serves as our benchmark bond in the model estimation (see Section 3.2 for details).

Figure 4: Bid-ask spread of benchmark South African government bond



We note that this bid-ask spread series follows the conventional pattern also observed in other bond markets, with stable periods followed by sudden sharp but short-lived spikes. Moreover, it has an upward trend that seems to have started in 2019 before the pandemic and continued since then. At the end of our sample, liquidity conditions in the South African government bond market thus appear to be poorer than at any time the past 13 years, outside of a few short-lived spikes.

The bid-ask spreads in the South African government bond market are orders of magnitude larger than the bid-ask spreads in the US Treasury market.[11] Thus, although liquid by the standards of emerging bond markets, South African government bonds are notably less liquid than those of the US Treasury. Consequently, we want to account for the liquidity risk in this market in our analysis.

Finally, in mapping this evidence to the analysis to come, we emphasise that bid-ask spreads reflect *current* market liquidity conditions, while the liquidity risk premia we aim to capture with our model represent investors' expectations about *future* market liquidity conditions and how they vary with investors' anticipated and unanticipated liquidity shocks. These shocks determine the compensation investors demand for assuming the liquidity risk of these bonds.

2.4 The credit risk of South African government bonds

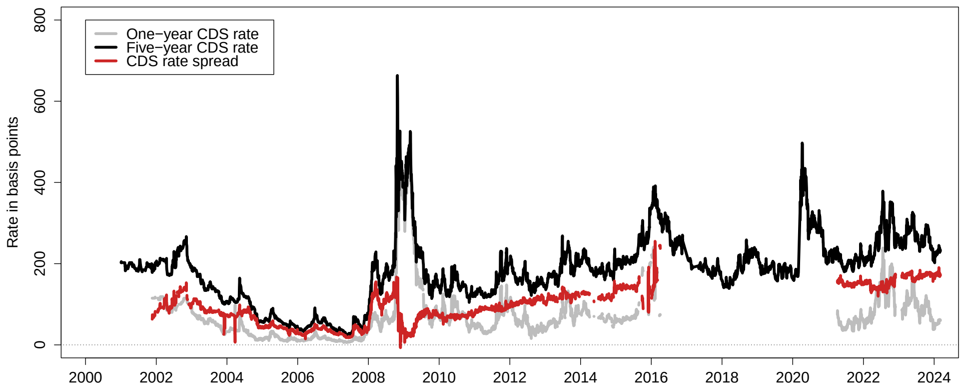
To gauge whether there are any material credit risk issues to consider in modelling the South African government bond prices, we consider rates on so-called credit default swap (CDS) contracts. These rates reflect the annual rate investors are willing to pay to buy protection against default-related losses on South African government bonds over a fixed period of time stipulated in the contract. Such contracts have been used to price the credit

[11] In their footnote 2, Andreasen, Christensen and Riddell (2021) report bid-ask spreads for 10-year US Treasury notes that are well below 1 basis point.

risk of many countries, including South Africa, since the early 2000s.

In Figure 5, we plot the available series for the one- and five-year South African CDS rate with solid gray and black lines respectively. The spread between the two CDS rates is shown with a solid red line. We note that the five-year CDS rate has fluctuated in a fairly narrow range between 100 and 200 basis points, except for a few brief episodes – including the global financial crisis in 2008–2009, when South African CDS rates temporarily spiked above 400 basis points, and the early stages of the COVID-19 pandemic. This is a level of credit risk on par with most investment-grade firms in the US, and its variation is mostly very gradual. Still, we do see an upward trend in the data that seems to correlate with the increase in the amount of outstanding government debt. Thus, we want to account for the credit risk component in the bond prices within our analysis.

Figure 5: South African CDS rates



3 Model estimation and results

In this section, we first detail our model which serves as the benchmark in our analysis, before we describe the restrictions imposed to achieve econometric identification of the model. We end the section with a summary of the estimation results.[12]

3.1 The AFNS-L-C Model

To capture the fundamental or frictionless factors operating on the South African government bond yield curve, we choose to focus on the tractable affine dynamic term structure model introduced in Christensen, Diebold and Rudebusch (2011). In this arbitrage-free Nelson-Siegel (AFNS) model, the state vector is denoted by $\overline{X}_t = (L_t, S_t, C_t)$, where L_t is a level factor, S_t is a slope factor and C_t is a curvature factor. The instantaneous risk-free rate is defined as

$$r_t = L_t + S_t. \tag{1}$$

[12] Full description of our regression results and the analytical derivation of the analytical bond price formulas of our novel model can be obtained from the authors.

The risk-neutral (or $\mathbb{Q}$-) dynamics of the state variables used for pricing are given by the following system of stochastic differential equations:[13]

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \end{pmatrix}, \quad (2)$$

where Σ is the constant covariance (or volatility) matrix.[14] Based on this specification of the $\mathbb{Q}$-dynamics, the frictionless zero-coupon bond yields preserve the Nelson and Siegel (1987) factor loading structure as

$$y_t(\tau) = L_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A(\tau)}{\tau}, \quad (3)$$

where $\frac{A(\tau)}{\tau}$ is a deterministic so-called yield-adjustment term that ensures absence of arbitrage (see Christensen, Diebold and Ruderbusch (2011) for details).

Next, we follow Andreasen, Christensen and Riddell (2021) and augment the frictionless model outlined above with a liquidity risk factor to account for the bond-specific liquidity risk premia embedded in the South African government bond prices. The bond prices are subsequently sensitive to liquidity pressures, and their pricing is performed with a discount function that accounts for the liquidity risk:

$$\bar{r}^i(t, t_0^i) = r_t + \beta^i (1 - e^{-\lambda^{L,i}(t - t_0^i)}) X_t^L, \quad (4)$$

where t_0^i denotes the date of issuance of the bond in question and β^i is its sensitivity to the variation in the liquidity risk factor X_t^L, with $\lambda^{L,i}$ being the associated decay parameter. Since β^i and $\lambda^{L,i}$ have a nonlinear relationship in the bond pricing formula, it is possible to identify both empirically. Finally, we stress that equation 4 can be included in any dynamic term structure model to account for security-specific liquidity risks.

The inclusion of the issuance date t_0^i in the pricing formula is a proxy for the phenomenon that as time passes, an increasing fraction of a given security is typically held by buy-and-hold investors. This limits the amount of the security available for trading and drives up the liquidity premium. Rational and forward-looking investors will take this dynamic pattern into consideration when they determine what they are willing to pay for a security at any given point in time between the date of issuance and the maturity of the bond. This dynamic pattern is built into the model structure we use.

To make this operational, we let X_t^L be a separate Ornstein-Uhlenbeck process under the

[13] As discussed in Christensen, Diebold and Ruderbusch (2011), with a unit root in the level factor, the model is not arbitrage-free with an unbounded horizon; therefore, as is often done in theoretical discussions, we impose an arbitrary maximum horizon.

[14] As per Christensen, Diebold and Ruderbusch (2011), Σ is a lower triangular matrix, and $\theta^{\mathbb{Q}}$ is set to zero without loss of generality.

pricing measure

$$dX_t^L = \kappa_L^{\mathbb{Q}}(\theta_L^{\mathbb{Q}} - X_t^L)dt + \sigma_{44} dW_t^{X^L,\mathbb{Q}}. \tag{5}$$

Finally, the credit risk of South African government bonds is modelled using the reduced-form credit risk modeling approach (see Lando (1998)). Specifically, the default intensity process in its general form is assumed to be given by

$$\lambda_t = \lambda_L L_t + \lambda_S S_t + X_t^{\lambda}, \tag{6}$$

where λ_L and λ_S represent the sensitivity of the default intensity to variation in the risk-free rate factors, while the default intensity risk factor itself is assumed to have the following dynamics:

$$dX_t^{\lambda} = \kappa_{\lambda}^{\mathbb{Q}}(\theta^{\mathbb{Q}} - X_t^{\lambda})dt + \sigma_{55} \sqrt{X_t^{\lambda}} dW_t^{X^{\lambda},\mathbb{Q}}. \tag{7}$$

Combining the reduced-form credit risk modelling approach with the recovery of market value (RMV) assumption studied in Duffie and Singleton (1999) implies that the discounting of the cash flows from South African government bonds is performed with a risk-adjusted rate given by

$$\begin{aligned} r_t^{\lambda} &= r_t + s_t && (8)\\ &= r_t + L^{\mathbb{Q}}\lambda_t && (9)\\ &= (1 + L^{\mathbb{Q}}\lambda_L)L_t + (1 + L^{\mathbb{Q}}\lambda_S)S_t + L^{\mathbb{Q}}X_t^{\lambda}, && (10) \end{aligned}$$

where $L^{\mathbb{Q}}$ is the market-implied loss rate, which we fix at 0.5 across all bonds in our sample. Under the RMV assumption, $L^{\mathbb{Q}}$ is not separately identifiable (as demonstrated by Duffie and Singleton (1999) and Houweling and Vorst (2005), among others). If, instead, recovery is a fraction of face value, $L^{\mathbb{Q}}$ can in principle be separately identified, provided a full term structure of bond prices is observed over a sufficiently long period (as shown by Pan and Singleton (2008)).

The technical contribution of this paper is to combine the liquidity and credit risk components into a single discount function, given by:

$$\bar{r}^{\lambda,i}(t,t_0^i) = r_t^{\lambda} + \beta^i(1 - e^{-\lambda^{L,i}(t-t_0^i)})X_t^L. \tag{11}$$

Let $X_t = (L_t, S_t, C_t, X_t^L, X_t^{\lambda})$ denote the state vector of this five-factor AFNS-L-C model, where L denotes the liquidity risk adjustment and C refers to the credit risk augmentation. Although our benchmark implementation is to assume a diagonal structure for the volatility Σ matrix, we note that the maximally flexible risk-neutral $\mathbb{Q}$-dynamics of the state variables used for pricing that we consider are given by[15]

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^L \\ dX_t^\lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & \kappa_L^{\mathbb{Q}} & 0 \\ 0 & 0 & 0 & 0 & \kappa_\lambda^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_L^{\mathbb{Q}} \\ \theta_\lambda^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^L \\ X_t^\lambda \end{pmatrix} \right] dt$$
$$+ \begin{pmatrix} \sigma_{11} & 0 & 0 & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 & 0 & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & 0 & 0 \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{55} \end{pmatrix} \begin{pmatrix} \sqrt{1} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{1} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{1} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{1} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{X_t^\lambda} \end{pmatrix} \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \\ dW_t^{X^L,\mathbb{Q}} \\ dW_t^{X^\lambda,\mathbb{Q}} \end{pmatrix}.$$

To stay well-defined, the square-root process X_t^λ must be prevented from assuming negative values, so the parameters $\kappa_{5,1}^{\mathbb{Q}}$, $\kappa_{5,2}^{\mathbb{Q}}$, $\kappa_{5,3}^{\mathbb{Q}}$ and $\kappa_{5,4}^{\mathbb{Q}}$ in the fifth row of the mean-reversion matrix $K^{\mathbb{Q}}$ must be zero. For the same reason, the parameters σ_{51}, σ_{52}, σ_{53} and σ_{54} in the lower triangular volatility matrix Σ must be fixed at zero as well. Moreover, we fix the parameters $\kappa_{1,5}^{\mathbb{Q}}$, $\kappa_{2,5}^{\mathbb{Q}}$, $\kappa_{3,5}^{\mathbb{Q}}$ and $\kappa_{4,5}^{\mathbb{Q}}$ in the fifth column of the mean-reversion matrix $K^{\mathbb{Q}}$ to zero to ensure that the bond price formula has an analytical closed-form solution, which is needed to empirically implement this model. For the same reason, we do not allow X_t^λ to generate stochastic volatility in the four other state variables, even though that would be admissible to stay within the affine model class. The structure above hence provides the maximally flexible specification of the AFNS-L-C model that is both fully identified econometrically *and* preserves analytical bond price formulas.

As shown in the online supplementary appendix, the net present value of one unit of currency paid by bond i at time $t+\tau^i$ has the following exponential-affine form:

$$\begin{aligned} P_t^i(t_0^i,\tau^i) &= E_t^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i} \bar{r}^{\lambda,i}(s,t_0^i)ds}\Big] \qquad (12) \\ &= \exp\Big(B_1^i(\tau^i)L_t + B_2^i(\tau^i)S_t + B_3^i(\tau^i)C_t + B_4^i(t_0^i,t,\tau^i)X_t^L + B_5^i(\tau^i)X_t^\lambda + A^i(t_0^i,t,\tau^i)\Big). \end{aligned}$$

Now consider the whole value of bond i issued at time t_0^i with maturity at $t+\tau^i$ that pays an annual coupon C^i semi-annually. Its price is given by[16]

$$P_t^i(t_0^i,\tau^i,C^i) = C^i(t_1-t)E_t^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\bar{r}^{\lambda,i}(s,t_0^i)ds}\Big] + \sum_{j=2}^{N}\frac{C^i}{2}E_t^{\mathbb{Q}}\Big[e^{-\int_t^{t_j}\bar{r}^{\lambda,i}(s,t_0^i)ds}\Big] + E_t^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i}\bar{r}^{\lambda,i}(s,t_0^i)ds}\Big]. \quad (13)$$

[15] This model belongs to the $A_1(5)$ class of affine dynamic term structure models in the notation of Dai and Singleton (2000). Although the model is not formulated using their canonical form, it can be viewed as a restricted version of the corresponding canonical $A_1(5)$ model.

[16] This is the clean price, which does not account for any accrued interest and maps to our observed bond prices.

To complete the model description, we need to specify the risk premia that connect the factor dynamics under the risk-neutral $\mathbb{Q}$-measure to the dynamics under the real-world objective $\mathbb{P}$-measure, where we use the extended affine risk premium specification described in Cheridito, Filipovic and Kimmel (2007). In our model framework, this specification implies that the resulting unrestricted five-factor AFNS-L-C model has maximally flexible $\mathbb{P}$-dynamics given by:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^L \\ dX_t^\lambda \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} & \kappa_{14}^{\mathbb{P}} & \kappa_{15}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & \kappa_{24}^{\mathbb{P}} & \kappa_{25}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} & \kappa_{34}^{\mathbb{P}} & \kappa_{35}^{\mathbb{P}} \\ \kappa_{41}^{\mathbb{P}} & \kappa_{42}^{\mathbb{P}} & \kappa_{43}^{\mathbb{P}} & \kappa_{44}^{\mathbb{P}} & \kappa_{45}^{\mathbb{P}} \\ 0 & 0 & 0 & 0 & \kappa_{55}^{\mathbb{P}} \end{pmatrix} \left(\begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \\ \theta_4^{\mathbb{P}} \\ \theta_5^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^L \\ X_t^\lambda \end{pmatrix} \right) dt$$
$$+ \begin{pmatrix} \sigma_{11} & 0 & 0 & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 & 0 & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & 0 & 0 \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{55} \end{pmatrix} \begin{pmatrix} \sqrt{1} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{1} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{1} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{1} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{X_t^\lambda} \end{pmatrix} \begin{pmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \\ dW_t^{X^L,\mathbb{P}} \\ dW_t^{X^\lambda,\mathbb{P}} \end{pmatrix}.$$

This is the transition equation in the Kalman filter estimation.

Note that to preserve absence of arbitrage within the extended affine risk premium specification, the square-root process X_t^λ must remain strictly positive. This is ensured by imposing the following Feller conditions on this process under both the $\mathbb{P}$- and the $\mathbb{Q}$-measure:

$$\kappa_{55}^{\mathbb{P}} \theta_5^{\mathbb{P}} > \frac{1}{2}\sigma_{55}^2 \quad \text{and} \quad \kappa_\lambda^{\mathbb{Q}} \theta_\lambda^{\mathbb{Q}} > \frac{1}{2}\sigma_{55}^2. \tag{14}$$

Finally, in our benchmark implementation, we shut down the ability of the risk-free frictionless factors to affect the default intensity – that is, we impose the restrictions $\lambda_L = \lambda_S = 0$. This has the added advantage that the default intensity is a strictly positive process.

3.2 Model estimation and econometric identification

Owing to the non-linear relationship between state variables and bond prices in equation 13, the model cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012) (see Christensen and Rudebusch (2019) for details). To make the fitted errors comparable across bonds of various maturities, we scale each bond price by its duration. Thus, the measurement equation for the bond prices takes the following form

$$\frac{P_t^i(t_0^i, \tau^i)}{D_t^i(t_0^i, \tau^i)} = \frac{\widehat{P_t^i}(t_0^i, \tau^i)}{D_t^i(t_0^i, \tau^i)} + \varepsilon_t^i. \tag{15}$$

Here, $\widehat{P}_t^i(t_0^i, \tau^i)$ is the model-implied price of bond i, $D_t^i(t_0^i, \tau^i)$ is its duration, which is calculated before estimation, and ε_t^i represents independent and Gaussian distributed measurement errors with mean zero and a common standard deviation σ_ε. See Andreasen, Christensen and Rubebusch (2019) for evidence supporting this formulation of the measurement equation. Owing to the non-Gaussian factor dynamics, the estimation based on the extended Kalman filter is quasi maximum likelihood (QML).

Since the liquidity factor is a latent factor that we do not observe, its level is not identified without additional restrictions. We thus let the ninth bond in our sample have a unit loading on the liquidity risk factor X_t^L, that is, the bond issued on 21 May 1998 with maturing on 21 December 2026, and a coupon rate of 10.5% has $\beta^i = 1$.[17] This choice implies that the β^i sensitivity parameters measure liquidity sensitivity relative to that of the 2026 bond.

We note that the $\lambda^{L,i}$ parameters can be difficult to identify if their values are too large or too small. As a consequence, we impose the restriction that they fall within the range from 0.0001 to 10, which is without practical consequences, as demonstrated by Christensen, Fischer and Schultz (2021). For numerical stability during the model optimisation, we impose the restriction that the β^i parameters fall within the range from 0 to 250, which turns out not to be a binding constraint at the optimum.

We assume that all bond price measurement equations have $i.i.d.$ fitted errors with zero mean and standard deviation σ_ε.

Finally, the state variables are assumed to be stationary, as is standard in the finance literature. This allows us to start the Kalman filter at their unconditional mean.

3.3 Results

In this section, we describe the estimation results for the AFNS-L-C model. To demonstrate the impact of adding liquidity and credit risk factors to the AFNS model, we first estimate the standard AFNS model, then augment it with the liquidity and credit risk factors separately, denote the AFNS-L model and AFNS-C model respectively, and finally estimate the full AFNS-L-C model. In all cases, we consider the most parsimonious specification with diagonal mean-reversion $K^{\mathbb{P}}$ and volatility Σ matrices.

Table 3 shows the summary statistics for the fitted errors across all four model estimations. First, we note that we get a decent fit with the plain-vanilla AFNS model with its level, slope and curvature factors. The root mean-squared pricing errors (RMSE) of all yields combined is 11.75 basis points. Although somewhat elevated, this result for the overall RMSE is consistent, with the result from the principal component analysis in section 2, which showed that the three first principal components only account for 98.4% of the variation in the yield data. Adding the credit risk factor in the AFNS-C model only provides a modest improvement in the overall model fit, to 9.21 basis points. Instead, it is really the bond-specific

[17] The bid-ask spread of this bond is analysed in section 2.3.

Table 3: Summary statistics of fitted errors of South African government bond yields

Fixed-coupon bonds	AFNS		AFNS-L		AFNS-C		AFNS-L-C	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
(1) 12% 2/28/2005	1.27	9.42	-0.56	5.32	2.68	7.57	-0.03	1.51
(2) 13.5% 9/15/2015	4.20	11.97	0.27	4.11	0.26	8.45	0.66	3.12
(3) 13.5% 9/15/2016	-7.34	10.55	-0.61	2.89	-2.98	8.80	0.07	3.07
(4) 13.5% 9/15/2015	-4.62	5.81	1.13	4.71	5.67	6.28	-0.12	1.07
(5) 9% 10/15/2002	-8.86	15.74	-0.89	4.82	-7.86	12.55	-0.36	3.50
(6) 9.5% 5/15/2007	3.01	14.25	1.93	11.00	3.30	13.10	-2.31	6.02
(7) 12.5% 1/15/2002	9.33	14.05	0.82	2.07	4.88	7.84	-0.90	5.10
(8) 12.5% 12/21/2006	5.54	11.53	1.99	6.40	5.14	10.46	1.77	4.21
(9) 10.5% 12/21/2026	-7.18	12.55	0.54	5.29	-2.40	7.53	0.60	4.06
(10) 10% 2/28/2008	-9.67	16.63	-0.49	8.51	-3.79	10.46	0.78	3.47
(11) 12% 2/28/2006	-1.05	8.80	1.14	2.52	-5.97	9.05	-0.47	1.08
(12) 8.75% 12/21/2014	-6.70	14.90	-1.92	5.96	-5.81	12.92	-1.05	4.09
(13) 8.25% 9/15/2017	6.13	9.91	1.73	5.33	1.82	7.29	1.80	4.49
(14) 8% 12/21/2018	4.31	8.20	0.20	3.79	1.24	4.67	-0.14	3.33
(15) 7.25% 1/15/2020	5.73	9.48	0.78	3.94	3.65	6.31	0.60	3.54
(16) 7.5% 1/15/2014	1.25	8.67	1.72	6.34	5.14	8.99	0.37	3.02
(17) 10% 2/28/2009	4.38	7.57	0.13	2.73	5.24	8.14	0.34	1.18
(18) 10% 2/28/2008	-2.71	6.06	0.33	3.13	-6.18	6.68	-0.44	1.27
(19) 6.25% 3/31/2036	-9.78	14.52	1.34	6.09	-10.16	15.38	0.72	4.38
(20) 6.75% 3/31/2021	0.86	11.19	0.65	4.95	-0.74	7.11	-0.44	4.41
(21) 7% 2/28/2031	-2.10	7.65	0.66	4.34	-0.76	5.71	0.13	3.53
(22) 6.5% 2/28/2041	-5.87	15.06	0.69	5.12	-8.10	13.64	0.50	5.38
(23) 7.75% 2/28/2023	-3.75	12.48	-1.23	5.67	0.72	6.69	-0.45	3.95
(24) 8.75% 2/28/2048	3.50	14.68	1.65	5.78	2.06	8.75	1.10	4.06
(25) 8.5% 1/31/2037	7.59	10.23	-0.09	4.54	5.94	8.07	0.58	3.64
(26) 8% 1/31/2030	0.24	9.45	2.14	6.57	2.97	8.20	1.60	5.26
(27) 8.25% 3/31/2032	4.10	8.77	0.82	4.19	4.03	6.76	1.28	3.90
(28) 8.75% 1/31/2044	4.93	10.94	1.04	4.19	4.55	8.12	0.77	3.42
(29) 8.875% 2/28/2035	7.78	11.50	-0.08	4.92	6.06	9.20	0.91	3.44
(30) 9% 1/31/2040	8.39	11.67	0.81	4.34	7.55	9.77	0.94	3.75
(31) 11.625% 3/31/2053	4.55	7.19	1.08	3.86	10.11	11.51	0.28	2.74
All bonds	0.30	11.75	0.61	5.30	0.25	9.21	0.48	3.98

Note: This table shows the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) for various models estimated on the sample of South African government bond prices. The pricing errors are reported in basis points and computed as the difference between the implied yield on the coupon bond and the model-implied yield on this bond. The data are monthly and cover the period from 31 January 2000 to 29 February 2024.

liquidity risk factor that allows the AFNS-L model to significantly improve on the fit of the AFNS model, with RMSEs of all yields combined at 5.30 basis points. Finally, having the liquidity and credit risk factors combined produces the best fit with an overall RMSE of 3.98 basis points, which represents a really tight fit to the entire cross-section of bond prices.

Table 4: Estimated bond-specific risk parameters

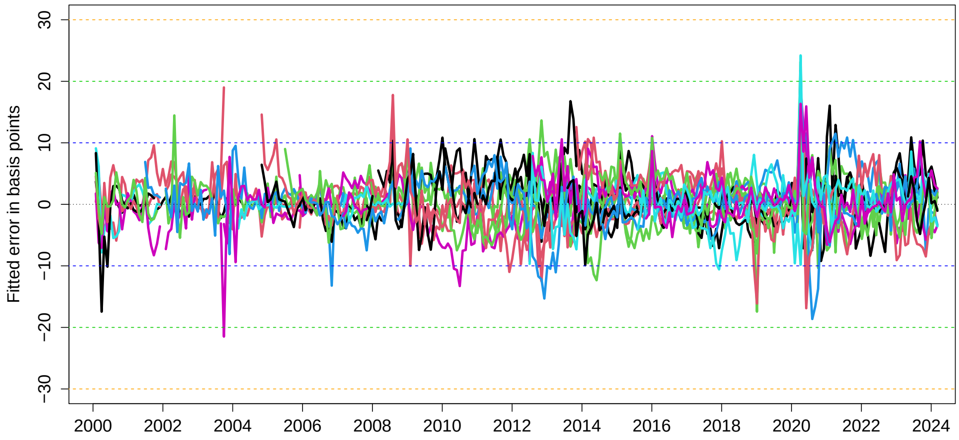
Fixed-coupon bonds	AFNS-L				AFNS-L-C			
	β^i	SE	$\lambda^{L,i}$	SE	β^i	SE	$\lambda^{L,i}$	SE
(1) 12% 2/28/2005	1.3786	0.1370	8.9466	5.7444	0.6612	0.1578	8.5266	5.7307
(2) 13.5% 9/15/2015	63.9261	0.9421	0.0009	0.0001	73.8717	0.6331	0.0009	0.0000
(3) 13.5% 9/15/2016	7.6524	1.4030	0.0039	0.0015	85.5231	0.9100	0.0010	0.0000
(4) 13.5% 9/15/2015	0.8131	0.1027	9.7590	5.7361	2.3287	0.1684	9.4485	5.7225
(5) 9% 10/15/2002	2.2776	0.2850	9.5296	5.7334	1.8362	0.2307	9.0573	5.7198
(6) 9.5% 5/15/2007	49.8242	0.9413	0.0017	0.0002	2.0021	0.1404	9.4296	5.7171
(7) 12.5% 1/15/2002	244.1522	0.9823	0.0022	0.0004	220.0146	0.6436	0.0011	0.0002
(8) 12.5% 12/21/2006	0.9618	0.1094	9.2288	5.7252	1.8626	0.1363	9.6055	5.7117
(9) 10.5% 12/21/2026	1	n.a.	9.9980	5.7225	1	n.a.	9.9997	5.7090
(10) 10% 2/28/2008	0.6850	0.0942	9.9972	0.9434	1.0470	0.0936	2.1835	0.6202
(11) 12% 2/28/2006	25.7404	1.0971	0.0035	0.0014	0.2709	0.2622	9.9995	5.7036
(12) 8.75% 12/21/2014	173.1399	0.9601	0.0010	0.0001	128.3567	0.6425	0.0015	0.0001
(13) 8.25% 9/15/2017	144.1265	1.1524	0.0012	0.0001	3.7711	0.4644	0.0627	0.0102
(14) 8% 12/21/2018	4.5680	0.8623	0.0424	0.0098	2.6639	0.3585	0.0941	0.0194
(15) 7.25% 1/15/2020	2.6809	0.3138	0.0911	0.0166	1.9653	0.1262	0.1607	0.0201
(16) 7.5% 1/15/2014	1.2826	0.0912	0.7137	0.1031	1.7187	0.1196	0.4438	0.0656
(17) 10% 2/28/2009	222.9387	1.0514	0.0016	0.0002	248.9202	0.7161	0.0020	0.0002
(18) 10% 2/28/2008	0.6370	0.1074	10.0000	1.2485	0.9773	0.1094	9.9969	0.8676
(19) 6.25% 3/31/2036	0.9651	0.0554	9.9968	1.0084	4.4883	0.6603	0.0326	0.0071
(20) 6.75% 3/31/2021	1.5270	0.0660	0.3192	0.0374	1.1538	0.0311	9.9979	0.7178
(21) 7% 2/28/2031	1.4186	0.5835	0.0908	0.0751	4.2809	0.7710	0.0337	0.0080
(22) 6.5% 2/28/2041	1.6182	0.4657	0.0910	0.0572	9.8418	0.7594	0.0208	0.0024
(23) 7.75% 2/28/2023	1.4009	0.0410	4.1091	0.7258	1.1274	0.0247	9.9999	0.7638
(24) 8.75% 2/28/2048	1.5961	0.1689	0.2691	0.1590	55.4975	0.8239	0.0046	0.0003
(25) 8.5% 1/31/2037	1.4343	0.1975	0.2951	0.2117	3.1906	0.3525	0.1518	0.0414
(26) 8% 1/31/2030	1.0198	0.0280	0.9058	0.6814	2.0705	0.2630	0.1260	0.0337
(27) 8.25% 3/31/2032	1.1255	0.0422	1.0584	1.3105	1.9622	0.1225	0.2943	0.0892
(28) 8.75% 1/31/2044	1.5823	0.1795	0.3525	0.2693	6.3086	0.8121	0.0665	0.0141
(29) 8.875% 2/28/2035	1.2984	0.0574	0.7122	0.5595	2.3856	0.1245	0.4084	0.1248
(30) 9% 1/31/2040	1.5196	0.1147	0.4640	0.2534	4.0897	0.3965	0.1418	0.0264
(31) 11.625% 3/31/2053	26.1384	5.3814	0.0138	0.0035	4.1541	0.2665	10.0000	1.8437

NOTE: This table shows the estimated bond-specific parameters β^i and $\lambda^{R,i}$ for each bond in the AFNS and AFNS-R models estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ. Standard errors (SE) are not available (n.a.) for the normalised value of β^9.

Figure 6 shows the individual fitted yield error series within the AFNS-L-C model. Except for a very limited number of short-lived spikes, the error series essentially remain within the 10-basis-point error band. This underscores the very tight fit of the AFNS-L-C model. It also shows that there is no material omitted factor buried in the residuals. This suggests that the AFNS-L-C model fully accounts for all relevant systematic risk factors in our panel of South African bond prices.

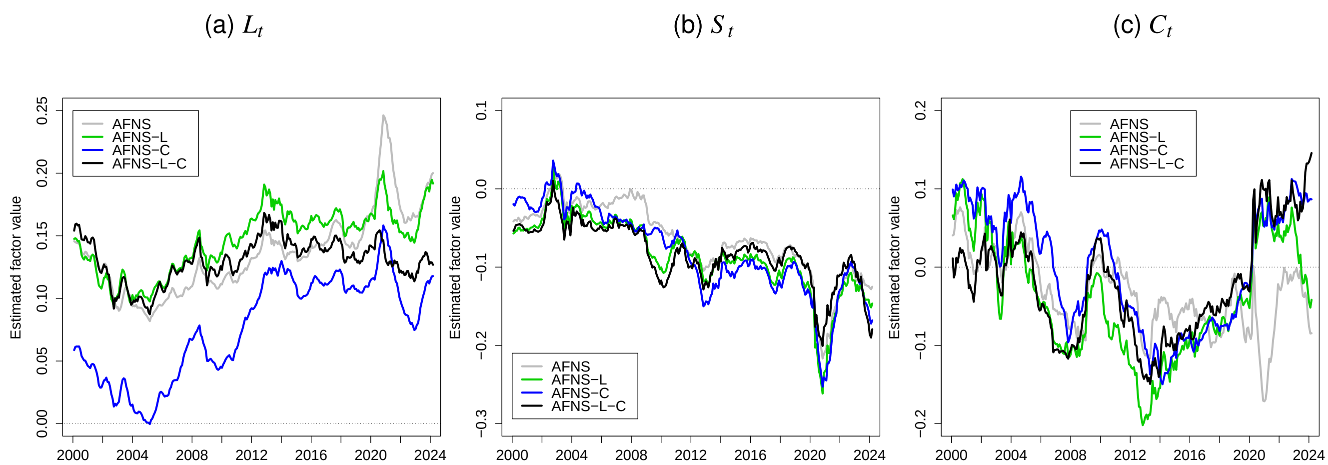
Figure 7 shows the estimated frictionless risk factors from all four model estimations. One notable thing is that, in the standard AFNS model, the pandemic spike in yields in early 2020 is accounted for by a spike in the frictionless level factor and partly offset with a matching sharp drop in the frictionless curvature factor. However, both of these gyrations come across as somewhat excessive relative to the filtered histories until that point. In contrast, the AFNS-L-C model produces a more balanced decomposition in terms of its estimated frictionless factors during this period. For reference, we note that the observed yield series shown in Figure 2 largely exhibit changes during this period that match the historical experience.

Figure 6: Fitted errors of South African government bond yields



Note: Illustration of the fitted errors of South African government bond yields to maturity implied by the AFNS-L-C model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ. The data are monthly and cover the period from 31 January 2000 to 29 February 2024.

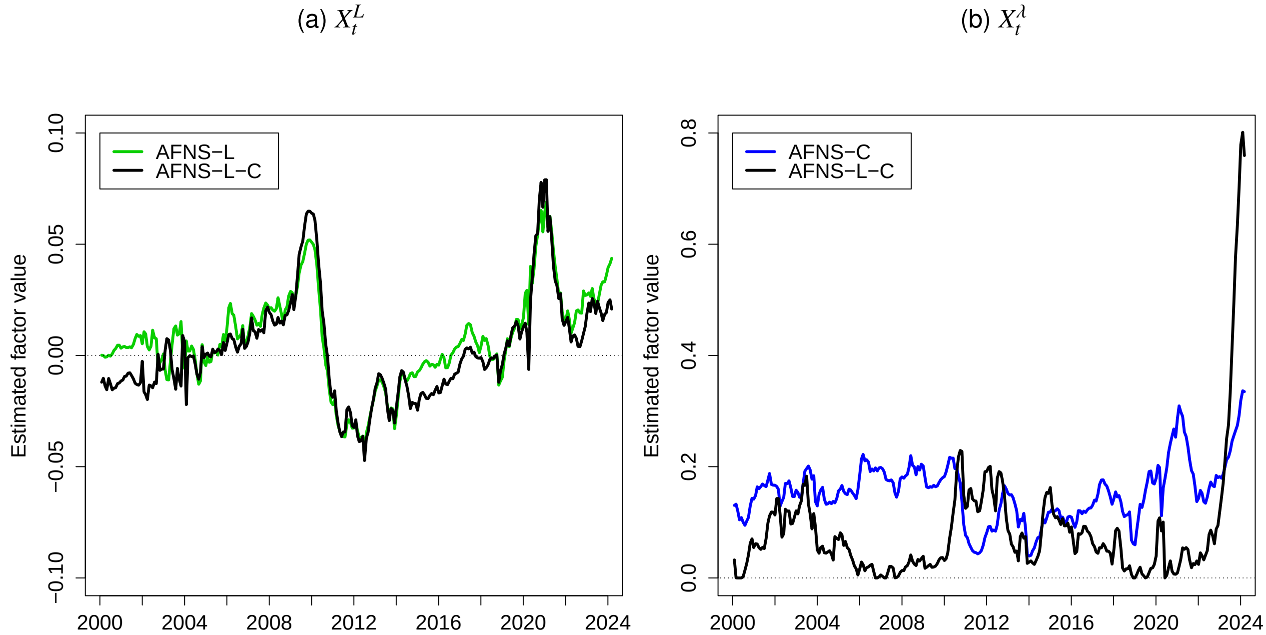
Figure 7: Estimated frictionless risk factors



As for the estimated liquidity and credit risk factors shown in Figure 8, we note that the liquidity risk factor X_t^L appears to be very well-identified given that its estimated path is very similar across the AFNS-L and AFNS-L-C models. In comparison, the credit risk factor seems to be less well-identified given the different estimated paths for the X_t^{λ}-factor across the AFNS-C and AFNS-L-C models, with a modest positive correlation of 0.31.

Table 5 contains the estimated dynamic parameters from all four model estimations and Table 4 the estimated bond-specific risk parameters. For the frictionless level factor, we

Figure 8: Estimated liquidity and credit risk factors



see a clearly trending pattern in panel (a) in Figure 7 for its estimated path in the AFNS-C model. This contrasts with a seemingly stationary path for L_t in the AFNS-L-C model. These differences are also reflected in the associated estimated mean-reversion rate $\kappa_{11}^{\mathbb{P}}$, which is very low and equal to 0.0332 in the AFNS-C model and higher at 0.2859 in the AFNS-L-C model, consistent with a less persistent dynamic pattern. These differences in the filtered state variables are also reflected in the associated mean parameter $\theta_1^{\mathbb{P}}$.

For the frictionless slope factor S_t, we see very similar estimated paths for all four models in panel (b) of Figure 7. As a consequence, the associated mean-reversion and mean parameters, $\kappa_{22}^{\mathbb{P}}$ and $\theta_2^{\mathbb{P}}$, are also relatively similar across all four models.

In terms of the frictionless curvature factor C_t, the main difference across the four models relates to the post-pandemic period, where the standard AFNS model produces negative estimated values for C_t, while the AFNS-C and AFNS-L-C models produce high positive estimated values. C_t is consequently more persistent with a lower estimated value of $\kappa_{33}^{\mathbb{P}}$ and a higher estimated mean $\theta_3^{\mathbb{P}}$ in the latter two models.

In the AFNS-L and AFNS-L-C models, which include the liquidity risk factor, the latter is estimated to have fairly similar dynamic properties in terms of $(\kappa_{44}^{\mathbb{P}}, \sigma_{44}, \theta_4^{\mathbb{P}}, \kappa_L^{\mathbb{Q}}, \theta_L^{\mathbb{Q}})$ under both the real-world $\mathbb{P}$-measure and the risk-neutral $\mathbb{Q}$-measure, consistent with the very similar filtered paths shown in panel (c) of Figure 7.

As for the credit risk factor X_t^λ included in the AFNS-C and AFNS-L-C models, there is a notably wider wedge between the estimated real-world $\mathbb{P}$-dynamics and the estimated risk-neutral $\mathbb{Q}$-dynamics – that is, between $(\kappa_{55}^{\mathbb{P}}, \sigma_{55}, \theta_5^{\mathbb{P}})$ and $(\kappa_\lambda^{\mathbb{Q}}, \theta_\lambda^{\mathbb{Q}})$ – within the AFNS-L-C model relative to the AFNS-C model. It thus seems that by adding the liquidity risk factor, the AFNS-L-C model is better positioned to exploit the degrees of freedom offered by the

Table 5: Estimated dynamic parameters

Parameter	AFNS		AFNS-L		AFNS-C		AFNS-L-C	
	Est.	SE	Est.	SE	Est.	SE	Est.	SE
$\kappa_{11}^{\mathbb{P}}$	0.0338	0.0457	0.0681	0.0783	0.0332	0.0429	0.2859	0.1350
$\kappa_{22}^{\mathbb{P}}$	0.0703	0.0862	0.0871	0.0917	0.0395	0.0699	0.0315	0.1240
$\kappa_{33}^{\mathbb{P}}$	0.3004	0.1745	0.2432	0.1448	0.0752	0.0818	0.1206	0.1503
$\kappa_{44}^{\mathbb{P}}$	-	-	0.2290	0.2214	-	-	0.3851	0.2061
$\kappa_{55}^{\mathbb{P}}$	-	-	-	-	0.3577	0.2211	0.1593	0.2352
σ_{11}	0.0088	0.0002	0.0113	0.0004	0.0091	0.0004	0.0125	0.0004
σ_{22}	0.0182	0.0007	0.0236	0.0010	0.0214	0.0013	0.0244	0.0015
σ_{33}	0.0406	0.0010	0.0466	0.0022	0.0313	0.0013	0.0427	0.0020
σ_{44}	-	-	0.0152	0.0014	-	-	0.0197	0.0019
σ_{55}	-	-	-	-	0.1338	0.0179	0.2912	0.0353
$\theta_1^{\mathbb{P}}$	0.1586	0.0494	0.1707	0.0329	0.0950	0.0364	0.1345	0.0089
$\theta_2^{\mathbb{P}}$	-0.0622	0.0603	-0.0870	0.0688	-0.0619	0.0988	-0.0396	0.1254
$\theta_3^{\mathbb{P}}$	-0.0302	0.0255	-0.0231	0.0400	0.0599	0.0815	0.0464	0.1039
$\theta_4^{\mathbb{P}}$	-	-	0.0140	0.0216	-	-	0.0031	0.0169
$\theta_5^{\mathbb{P}}$	-	-	-	-	0.1809	0.0641	0.2661	0.3625
λ	0.2067	0.0025	0.1856	0.0036	0.1252	0.0045	0.1803	0.0039
$\kappa_L^{\mathbb{Q}}$	-	-	0.9323	0.0420	-	-	1.0987	0.0788
$\theta_L^{\mathbb{Q}}$	-	-	0.0056	0.0004	-	-	0.0041	0.0003
$\kappa_\lambda^{\mathbb{Q}}$	-	-	-	-	1.0364	0.0496	2.3793	0.2074
$\theta_\lambda^{\mathbb{Q}}$	-	-	-	-	0.0675	0.0204	0.0178	0.0084
σ_ε	0.0012	8.00×10^{-6}	0.0006	4.87×10^{-6}	0.0009	6.62×10^{-6}	0.0004	4.52×10^{-6}
Max $\mathcal{L}^{EKF}$	15,715.57		17,274.99		16,259.45		17,815.90	

Note: The table shows the estimated dynamic parameters for the AFNS, AFNS-L, AFNS-C and AFNS-L-C models, each estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ.

extended affine risk premium specification. We take this as another sign that the AFNS-L-C model is overall better specified than the AFNS-C model.

For the estimated value of λ, we note that it is notably smaller in value than typically found in other markets. Mechanically, the lower estimated values for this parameter imply that all four models put more weight on fitting long-term yields relative to what is normally observed in US and UK government bond markets (see Christensen and Rudebusch (2012)). This fits well with the healthy number of long-term bonds in our South African bond sample, as also highlighted in section 2.

Finally, regarding the measurement error standard deviations σ_ε, we note that their estimated values are consistent with the respective overall RMSEs reported in Table 3.

4 The estimated bond risk premia

In this section, we first describe how we quantify the liquidity risk premia based on the estimated AFNS-L-C model, including an assessment of their robustness, followed by an analysis of their determinants. We then proceed to a description of the calculation of the credit risk premia based on the estimated AFNS-L-C model, and an examination of their behaviour and a separate analysis of their determinants.

4.1 The estimated bond liquidity risk premia

We now use the estimated AFNS-L-C model to extract the bond-specific liquidity risk premia in the South African government bond market. To compute these premia, we first use the estimated parameters and the filtered states $\left\{X_{t|t}\right\}_{t=1}^{T}$ to calculate the fitted bond prices $\left\{\hat{P}_t^i\right\}_{t=1}^{T}$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ by solving the fixed-point problem

$$\begin{aligned}\hat{P}_t^i &= C(t_1-t)\exp\left\{-(t_1-t)\hat{y}_t^{c,i}\right\}+\sum_{k=2}^{n}\frac{C}{2}\exp\left\{-(t_k-t)\hat{y}_t^{c,i}\right\} \\ &\quad +\exp\left\{-(T-t)\hat{y}_t^{c,i}\right\},\end{aligned} \tag{16}$$

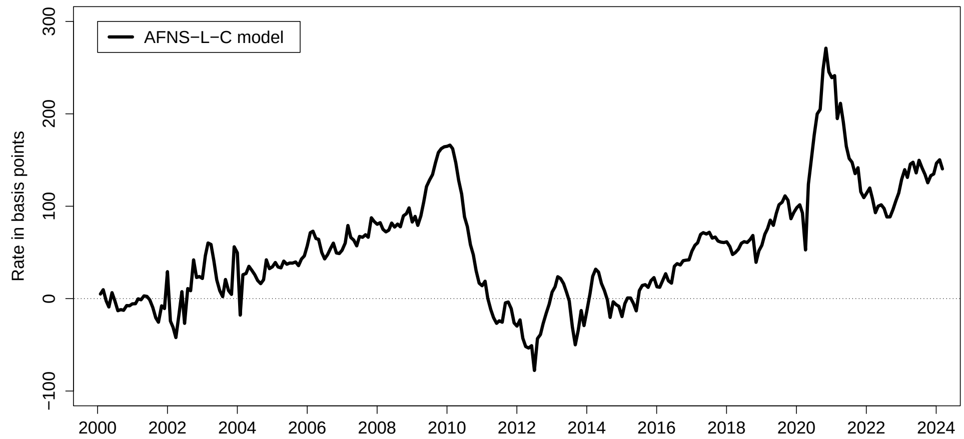
for $i=1,2,...,n_t$, meaning that $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ is approximately the rate of return on the ith bond if held until maturity (see Sack and Elsasser (2004)). To obtain the corresponding yields with correction for the liquidity risk premium, a new set of model-implied bond prices is computed from the estimated AFNS-L-C model, but turning off the liquidity risk factor by using the constraints that $X_{t|t}^L=0$ for all t as well as $\sigma_{44}=0$ and $\theta_L^Q=0$. These prices are denoted $\left\{\tilde{P}_t^i\right\}_{t=1}^{T}$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using equation 16. They represent estimates of the prices that would prevail in a world without any financial frictions. The liquidity risk premium for the ith bond is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i}-\tilde{y}_t^{c,i}. \tag{17}$$

Figure 9 shows the average estimated liquidity risk premium $\bar{\Psi}_t$ across the outstanding bonds at each point in time. The average estimated bond liquidity risk premium clearly varies notably over time, with a maximum of 271 basis points achieved at the peak of the pandemic and a low of -78 basis points in early 2012. For the entire period, it has an average of 54.36 basis points with a standard deviation of 62.19 basis points.

For Mexican government bonds, Christensen, Fischer and Schultz (2021) report average estimated liquidity premia of 50 basis points with a standard deviation of 18 basis points for the 2007–2019 period, while Cardozo and Christensen (2024) report estimated liquidity premia for Colombian government bonds that average 41 basis points over the 2005–2020 period with a standard deviation of 28 basis points. Our estimates for the South African government bond market thus appear to be comparable to other estimates reported in the literature, although more volatile.

Figure 9: South African government bond liquidity risk premia

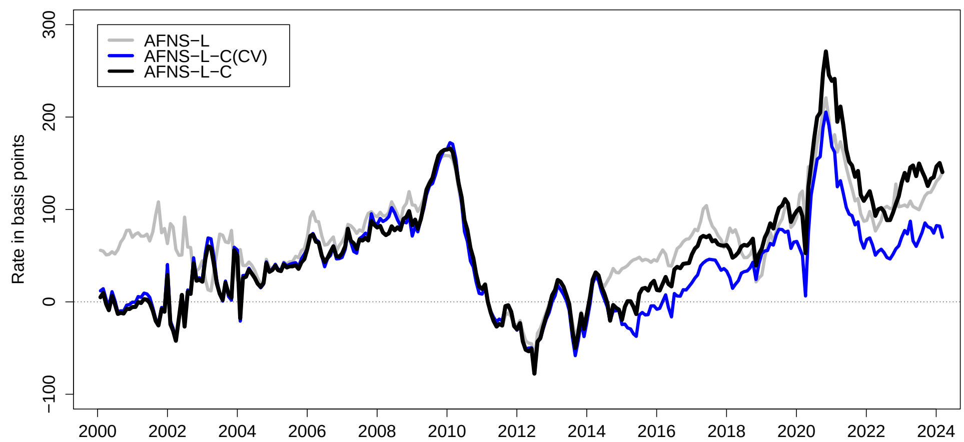


Note: This figure illustrates the average estimated liquidity premium of South African government bond yields to maturity for each observation date implied by the AFNS-L-C model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ. The liquidity premia are measured as the estimated yield difference between the fitted yield to maturity of individual bonds and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The data are monthly and cover the period from 31 January 2000 to 29 February 2024.

4.1.1 Robustness of the estimated liquidity risk premia

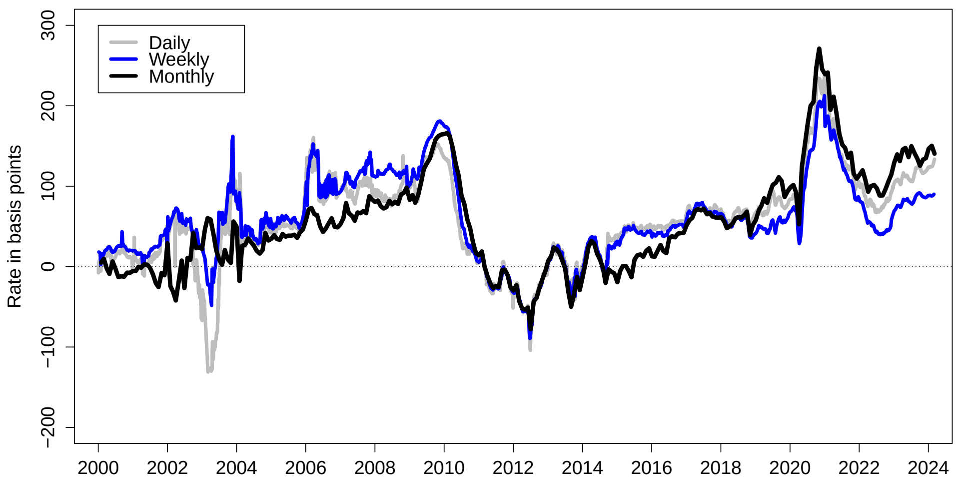
To assess the sensitivity of the estimated liquidity risk premia to the assumed model structure, we calculate the average estimated liquidity risk premia from the AFNS-L model, which is a simplified version of the AFNS-L-C model without the credit risk factor, and from a modified version of the AFNS-L-C model in which the credit risk factor is assumed to be a Gaussian process with constant volatility, labelled the AFNS-L-C(CV) model. Figure 10 shows the average estimated liquidity premium series from these three estimations. The very similar results show that the estimated liquidity risk premia have little sensitivity to these model choices.

Figure 10: South African government bond liquidity risk premia: model sensitivity



To examine the sensitivity of the estimated liquidity risk premia to the data frequency, we estimate the AFNS-L-C model using daily and weekly data in addition to our benchmark monthly data. Figure 11 shows the resulting average estimated liquidity risk premia, which are very similar as well. The estimated liquidity risk premia are thus robust to alterations to the data frequency.

Figure 11: South African government bond liquidity risk premia: data frequency



4.1.2 Determinants of the estimated liquidity risk premia

In this section, we use regression analysis to assess the key factors driving the variation in the average estimated bond liquidity premium series in the AFNS-L-C model.

In a recent paper, Cardozo and Christensen (2024) argue that the trading of inflation-indexed bonds is likely to be dominated by patient domestic buy-and-hold investors for structural reasons tied to the fact that those bonds provide a natural hedge against inflation risks. Consequently, their liquidity risk premia should be large, while their trading should be dominated by patient domestic investors. Ceballos, Christensen and Romero (2024) confirm this conjecture for Chilean inflation-indexed bonds. Given our focus on nominal government bonds, we are interested in examining the role of this class of investors for the liquidity risk premia in the South African government bond market. To that end, we use the pension fund holdings share as a proxy for the holdings of all domestic buy-and-hold investors in South Africa. On the one hand, this class of investors tends to follow buy-and-hold strategies, which could lead to lower trading volumes and higher liquidity risk premia. On the other hand, the arguments stipulated above apply to inflation-indexed bonds that are protected against inflation risk and may not apply to nominal bonds, meaning that even patient buy-and-hold investors may see a need to trade nominal bonds when hit with inflation shocks. It is ultimately an empirical question as to whether or how this group of investors affects the perceived liquidity risk of these bonds as reflected in our average estimated liquidity premium series.

A related point is that the size of the bond market should matter for the liquidity risk premium investors demand for assuming the liquidity risk of these bonds. We conjecture that, all else being equal, if the market size increases, the liquidity premia should decline. To test this hypothesis, we use the monthly total outstanding amount of South African government bonds scaled by nominal GDP.

Finally, Christensen, Fischer and Schultz (2021) find that foreign holdings in the Mexican bonos market is positively correlated with the size of the liquidity risk premia in that market. Their results suggest that a heavy concentration among foreign investors increases the flight risk, as global investors can choose to move their money elsewhere at short notice. Investors are aware of this increased flight risk and demand a higher liquidity risk premium when foreign concentrations are elevated. We are interested in examining whether the foreign share plays a similar role for liquidity risk premia in the South African government bond market.

In Table 6, column 1 shows the result of using these three variables to explain the variation in the average estimated bond liquidity premium series. We find a significant *negative* coefficient on the pension fund share. Hence, the role of institutional investors for the level of liquidity risk premia may be very different in nominal bond markets relative to how they affect the trading dynamics of inflation-indexed bond markets as examined in Ceballos, Christensen and Romero (2024) for Chilean inflation-indexed bonds. We find that, as the market becomes more dominated by buy-and-hold investors, its steady state moves towards one of declining liquidity risk premia. At the same time, we obtain an insignificant negative coefficient on the bond market-to-GDP ratio. This means that an expansion of the bond market relative to GDP seems to matter little for bond liquidity premia based on this initial simple regression model. Finally, increased foreign participation tends to put downward pressure on the liquidity risk premia in this market, which contrasts with the findings of Christensen, Fischer and Schultz (2021). We speculate that foreign participation in South Africa, which has remained below 40% as evidenced in Figure 3, may not have reached the critical level where foreign participation is viewed as a potential material flight risk. For comparison, the foreign participation in the Mexican bonos market studied in Christensen, Fischer and Schultz (2021) reached 60% during their sample. Hence, at low to moderate participation levels, increased foreign shares appear to be a net positive where a more diverse and active investor base leads to improved trading conditions and lower liquidity risk premia.

To verify the robustness of these findings, we consider three groups of control variables.[18]

To begin, we are interested in the role of factors that are believed to matter for South African government bond market liquidity specifically or general bond market liquidity more broadly as they could matter for the estimated bond-specific risk premia. First, we add the bid-

[18] See the online appendix for a full description of the explanatory variables and the full regression results.

Table 6: Regression results for average estimated liquidity risk premium

	(1)	(2)	(3)	(4)	(5)
Pension fund holdings	-1 350.55***	-726.14***	-1 054.65***	-1 032.41***	-594.40***
	(36.17)	(205.88)	(295.80)	(362.77)	(198.55)
Outstanding bonds to GDP	-0.11	7.57***	1.92***	1.45***	5.76***
	(145.077)	(1.84)	(1.70)	(2.30)	(1.38)
Foreign holdings	-488.52***	-265.40**	-633.99***	-421.22***	-464.16***
	(355.368)	(133.60)	(139.96)	(128.80)	(104.22)
Liquidity controls		Yes			Yes
Risk sentiment controls			Yes		Yes
Domestic macro controls				Yes	Yes
Intercept	600.81***	405.44***	620.53***	550.72***	372.89***
	(178.60)	(117.39)	(132.57)	(169.21)	(113.23)
Adjusted R^2	0.702	0.872	0.819	0.805	0.916
Observations after adjustments	167	157	167	167	157

Note: The table shows the results of regressions with the average estimated bond-specific risk premium as the dependent variable and explanatory variables. Standard errors computed by the Newey-West estimator are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10%, 5% and 1% levels respectively.

ask spread of our benchmark bond shown in Figure 4. Second, we include the average bond age and the one-month realised volatility of the 10-year bond yield as proxies for bond liquidity, following the work of Houweling, Mentink and Vorst (2005). We note that the South African National Treasury's issuance of debt has been at long maturities relative to other major emerging markets. Finally, inspired by the analysis of Hu, Pan and Wong (2013), we also include a noise measure of bond prices to control for variation in the amount of arbitrage capital available in this market. Combining these four explanatory variables tied to market liquidity and functioning produces the results reported in regression (2) in Table 6. We note a high adjusted R^2 of 0.87. While pension fund holdings and the foreign share both preserve their statistically significant coefficients, the bond market size to GDP, although smaller in magnitude, now has a statistically significant *positive* coefficient. This suggests that increased debt issuance is associated with increasing market frictions, as reflected in our average estimated liquidity premium series.

After exploring the role of liquidity factors, we examine the effects of factors reflecting risk sentiment on the bond liquidity premium series. These variables include the VIX, the US Treasury 10-year on-the-run premium, the three-month 'TED' spread (difference between the interest rates on interbank loans and short-term US sovereign debt), the five-year CDS rate for South Africa, the Merrill Lynch Option Volatility Estimate (MOVE index), and the 10-year US Treasury yield. The results of the regression with these six added control variables are reported in regression (3) in Table 6, which has an adjusted R^2 equal to 0.82. Importantly, our three key explanatory variables preserve their significant coefficients, as in the first control regression.

In the final group, we assess the role played by standard domestic macro variables for the liquidity risk premium series. These include the year-over-year change in the South African consumer price index, the South African monetary policy overnight rate and the JP Morgan Emerging Market Bond Index (known as the 'EMBI plus index') for South Africa. The results of the regression with these three macroeconomic control variables are reported in regression (4) in Table 6. It produces a slightly lower adjusted R^2 of 0.81, but our three key explanatory variables remain statistically significant.

To assess the robustness of the results from the first four regressions, we include all variables with the results reported in column 5 in Table 6. This joint regression produces a high adjusted R^2 of 0.92. More importantly, our three key explanatory variables remain statistically significant and preserve both their sign and approximate magnitudes, as in the earlier control regressions. We thus consider our findings to be robust.

The main takeaway from the regression analysis is that increased holdings among both pension funds and foreigners tend to put downward pressure on our estimated liquidity premium series and hence improve the functioning of this market. In contrast, increased debt issuance as measured by the debt-to-GDP ratio is associated with higher liquidity risk in this market. We speculate that this latter finding may reflect increasing investor fear of not being able to unwind their holdings in an environment of increasing debt without negatively affecting bond prices.

Based on our regression results, the combination of increased debt issuance and a declining foreign market share is a problematic combination from the point of view of market functioning as it entails significantly higher liquidity risk premia – which is evident in Figure 9 for the period since 2018. Our results thus seem to suggest that the growing fiscal burden of the South African government may be negatively affecting the dynamics and general resilience of the South African government bond market. This in turn could affect not only the transmission of monetary policy onto the government's costs of borrowing, but also the soundness of the banking system and individual banks' balance sheets. This could ultimately jeopardise the country's financial stability more broadly. This underscores the importance of measuring and monitoring liquidity risk in the government bond market and understanding the factors driving those risks, and we see the analysis in this section as a significant contribution to that vital end goal.

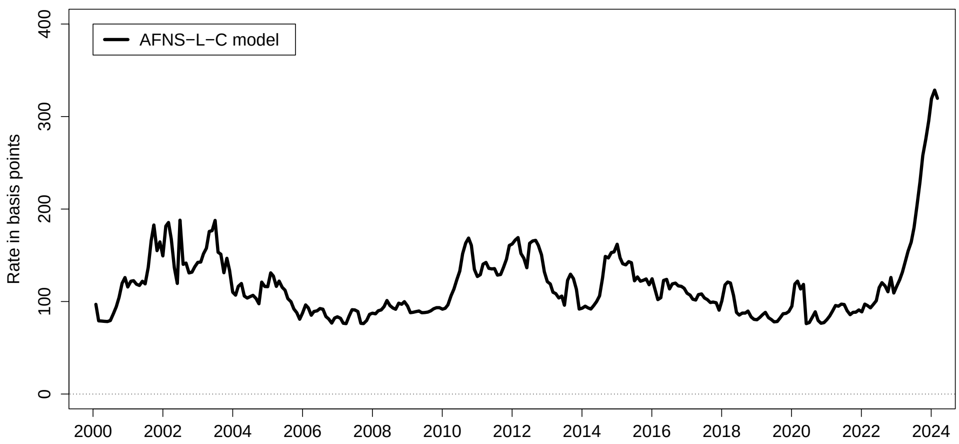
4.2 The estimated bond credit risk premia

We now proceed to using the estimated AFNS-L-C model to extract the credit risk premia in the South African government bond market. To compute these premia, we first use the estimated parameters and the filtered states $\left\{X_{t|t}\right\}_{t=1}^{T}$ to calculate the fitted bond prices $\left\{\hat{P}_t^i\right\}_{t=1}^{T}$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ by solving the fixed-point problem in equation 16 for $i = 1,2,...,n_t$. To obtain the corresponding yields with correction for the credit risk premium, a new set of model-implied bond prices is computed from the estimated AFNS-L-C model, but turning off the credit risk factor by using the constraints that $X^{\lambda}_{t|t} = 0$ for all t as well as $\sigma_{55} = 0$ and $\theta^{Q}_{\lambda} = 0$. These prices are denoted as $\left\{\overline{P}^{i}_{t}\right\}^{T}_{t=1}$ and converted into yields to maturity $\overline{y}^{c,i}_{t}$ using equation 16. They represent estimates of the prices that would prevail in a world without any credit risk. The credit risk premium for the ith bond is then defined as

$$\zeta^{i}_{t} \equiv \hat{y}^{c,i}_{t} - \overline{y}^{c,i}_{t}. \tag{18}$$

Figure 12 shows the average estimated credit risk premium $\bar{\zeta}_t$ across the outstanding bonds at each point in time. The average estimated credit risk premium is mostly stable during our sample period, but it spikes sharply during the last year of our sample and ends our sample in February 2024 above 300 basis points. It has an average of 117.68 basis points over the entire period, with a standard deviation of 38.83 basis points and a maximum value of 329 basis points, reached in January 2024 and a minimum value of 76 basis points, assumed in October 2020.

Figure 12: South African government bond credit risk premia

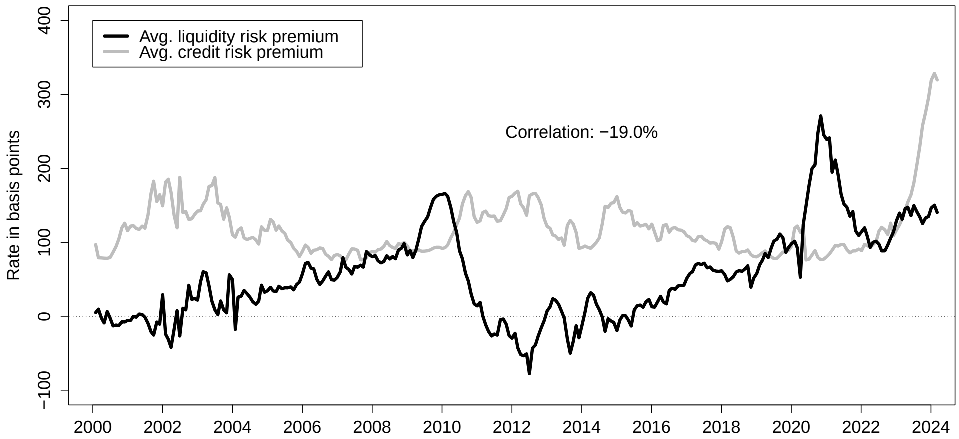


NOTE: This figure illustrates of the average estimated credit risk premium of South African government bond yields to maturity for each observation date implied by the AFNS-L-C model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ. The credit risk premia are measured as the estimated yield difference between the fitted yield to maturity of individual bonds and the corresponding credit risk-free yield to maturity with the credit risk factor turned off. The data are monthly and cover the period from 31 January 2000 to 29 February 2024.

We then compare the estimated liquidity and credit risk premium series, with both shown in Figure 13. In comparing the two risk premium series, we note that they have a low negative correlation of -19% and that their correlation in first differences is -29%. We take these results to document the existence of large and weakly correlated liquidity and credit risk premia in this market. This finding also indicates that liquidity and credit stresses are distinct risks to bond investors and each merits its separate compensation.

Which of these two risks is the more concerning changes over time. At times of general,

Figure 13: South African government bond liquidity and credit risk premia



broad-based – if not outright global – market illiquidity such as the global financial crisis or the COVID-19 pandemic, liquidity risk premia are likely to dominate, and our results for the South African government bond market align with that view. In calmer times with low, or even negligible, liquidity risk premia, the omnipresent credit risk component is likely to dominate.

Our results support the view that the financial market turmoil surrounding the global financial crisis and the COVID-19 pandemic and associated spikes in South African bond yields mainly reflected global financial market illiquidity and flight-to-safety or flight-to-cash effects and had little, if anything, to do with the ability of the South African government to honour its promised debt payments. In contrast, the recent spike in the credit risk premium seems tied to the increase in the South African government debt-to-GDP ratio. This higher perceived level of credit risk also seems to have influenced investors' perceptions about the future liquidity in the South African government bond market, causing them to demand notably higher liquidity risk premia. This interpretation is consistent with our regression results in Table 6.[19] Moreover, these results suggest that the steepening of South Africa's sovereign yield curve in recent years (documented in Soobyah and Steenkamp (2020b)) to a large extent reflects increasing liquidity and credit risk premia.

4.2.1 Determinants of the estimated credit risk premia

In this section, we use regression analysis to determine the key factors driving the variation in the average estimated credit risk premium series in the AFNS-L-C model.

As suggested by their name, credit default swap rates represent the premium investors are willing to pay to hedge credit risk. We would hence expect our estimated credit risk pre-

[19] This is in line with the findings of Havemann et al. (2022), who document flight-to-safety behaviour among domestic fixed-income funds with sovereign bond exposures in response to the emergence of the COVID-19 pandemic, while foreign investors withdrew from the South African sovereign debt market.

mium series to be positively correlated with the five-year CDS rate. That said, Soobyah and Steenkamp (2020a) show that about three quarters of the variation in South African CDS rates can be explained by global factors that have little to do with domestic economic developments in South Africa. Given that our credit risk premium series is estimated from domestic bond prices denominated in South African rands, this connection could be anticipated to be weak.

Relatedly, given that the credit risk of the South African government is ultimately a function of its outstanding debt level and its ability to service it, the size of the bond market relative to nominal GDP should be another key variable, similar to the liquidity premium regressions.

Finally, the role of foreigners in investors' assessment of the credit risk in emerging sovereign bond markets is an important topic relevant to all EMEs with globally integrated financial markets, of which South Africa is a prime example. The foreign share thus also remains a key explanatory variable in this set of regressions.

In Table 7, column 1 shows the result of using these three variables to explain the variation in the average estimated bond credit risk premium series.

First, our estimated credit risk premium series has a weak and unstable relationship with the five-year CDS rate for South Africa. This seems in line with the findings of Soobyah and Steenkamp (2020b) and supports the view that CDS rates contain significant global risk sentiment components that make them detached from the underlying credit risk for extended periods.[20] There are additional complications to consider regarding the comparison to the CDS rates. Typically, only bonds issued in external markets and denominated in one of the standard specified currencies such as the euro or US dollar are deliverable under the contract. Among this set of deliverable bonds, any bond is admissible. There is thus a cheapest-to-deliver option for the buyer of protection, which is reflected in the CDS rates. In contrast, our set of bonds is all issued in the domestic bond market and denominated in South African rands and would therefore be unlikely to qualify as deliverable bonds in CDS contracts. There are thus several tangible reasons why our estimated credit risk premia may differ from the rates quoted for South Africa in the global CDS market.

Second, the outstanding amount of bonds relative to nominal GDP has a small positive but insignificant coefficient in the regression. The amount of debt thus seems to matter little for the credit risk premia demanded by investors for holding South African government bonds.

Finally, the foreign share has a negative coefficient that is highly statistically significant. An increased foreign presence is associated with lower credit risk premia in this market, but the causation is unclear. Do foreigners come to this market when credit risk is lower? Does the increased demand from foreigners bid up the prices more broadly? If so, it will have to squeeze one or more of the risk premia in this market. Our results in Tables 6 and 7 taken together could be interpreted as reflecting such broad-based effects from increased foreign

[20] See also Gamboa-Estrada and Romero (2022) for similar evidence for a sample of Latin-American countries.

Table 7: Regression results for average estimated credit risk premium

	(1)	(2)	(3)	(4)	(5)
five-year CDS	-0.04	0.03	0.03	0.15	0.42***
	(0.09)	(0.10)	(0.06)	(0.14)	(0.14)
Outstanding bonds to GDP	0.01	-3.57***	-0.37	2.13	-3.08**
	(1.20)	(1.20)	(0.80)	(1.72)	(1.23)
Foreign holdings	-338.96***	-159.02	-356.21**	-464.35***	-188.59*
	(115.55)	(125.51)	(147.53)	(123.13)	(110.19)
Liquidity controls		Yes			Yes
Risk sentiment controls			Yes		Yes
Domestic macro controls				Yes	Yes
Intercept	243.21***	56.03***	204.14***	186.17***	-140.28
	(39.85)	(102.21)	(56.35)	(35.93)	(103.68)
Adjusted R^2	0.172	0.296	0.408	0.483	0.686
Observations after adjustments	167	157	167	167	157

NOTE: The table shows the results of regressions with the average estimated bond credit risk premium as the dependent variable and explanatory variables. Standard errors computed by the Newey-West estimator are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10%, 5% and 1% levels respectively.

presence. Alternatively, the results suggest that foreigners tend to be more drawn to the South African bond market when the priced credit risk is low.

To verify the robustness of these findings, we consider the same three groups of control variables as before.[21]

In regression (2) with the liquidity controls, there is a modest sign that the credit risk factor within the AFNS-L-C model may be picking up some residual liquidity components not fully captured by the liquidity risk factor.

In regression (3) with the risk sentiment controls, the increase in the adjusted R^2 suggests that our measure of the priced credit risk in the South African bond market is to some extent a function of investors' risk sentiment, as reflected in our set of risk sentiment control variables.

Regression (4) includes the domestic macroeconomic control variables. As one could expect, the priced credit risk series is also influenced by domestic macro variables.

In regression (5), we include all considered explanatory variables, producing a high adjusted R^2 of 0.69, well above the adjusted R^2s from the previous regressions. This suggests that our three sets of control variables are distinct and reflect separate information sets of importance in determining our credit risk premium series. Consequently, this represents our preferred regression model for the credit risk premium series.

Based on the preferred regression model, there is a positive connection between the five-

[21] See the online appendix for the full regression results.

year CDS rate and the credit risk premium series. There is also a counterintuitive *negative* connection between the outstanding amount of bonds relative to GDP and the estimated premium investors demand for assuming the credit risk of the bonds. Finally, there is relatively strong evidence for a negative relationship between the foreign-held share of the bond market and the priced credit risk premia, as discussed earlier.

4.3 Summary

To summarise, we document the existence of large time-varying liquidity and credit risk premia in the South African government bond market, both of which are robustly estimated based on our novel AFNS-L-C model. The two risk premia series are only weakly correlated. This shows that liquidity and credit risk are distinct material risks to investors in this market, each commanding separate compensation.

Based on our preferred regression with all explanatory variables included, we find that both risk premium series are significantly influenced by the foreign-held share with negative estimated coefficients, meaning that increased foreign participation correlates with lower liquidity and credit risk premia in the bond prices. In contrast, the outstanding amount of bonds relative to nominal GDP plays an unusual role in our regressions. Increased debt leads to higher liquidity risk premia but lower credit risk premia. This counterintuitive result is challenging to explain. One possibility is that the South African government tends to raise its debt issuance when it perceives priced credit risk to be low. We leave it for future research to examine this finding further.

5 Simulation study: Is the AFNS-L-C model identified?

The preceding analysis has shown that the AFNS-L-C model seems to produce robust estimates of both the filtered paths for all five state variables and their dynamic parameters for the South African bond price data. However, concern might remain whether the model's ability to distinguish the liquidity risk factor from the credit risk factor holds in general and extends to other bond samples, given its latent factor structure. Here, we aim to provide an answer to that important question by conducting a Monte Carlo study to analyse the finite-sample properties of estimating the AFNS-L-C model using bond price samples similar to that observed for the South African government bond market. This can also speak to the accuracy of this estimation approach more broadly and extend the results reported by Andreasen, Christensen and Rudebusch (2019) for dynamic term structure models (DTSMs) simpler than the AFNS-L-C model.

We first describe the formulation of the Monte Carlo study. The results for the estimated model parameters are reported in section 5.2, while the accuracy of the filtered states and the resulting estimated liquidity and credit risk premium series are explored in sections 5.3 and 5.4 respectively. Finally, section 5.5 provides a brief summary of our main findings.

5.1 Setup for the Monte Carlo study

To study the efficiency of the Kalman filter in estimating the AFNS-L-C model with stochastic volatility, we undertake a carefully orchestrated simulation study.

Unlike previous simulation studies in the literature, our Monte Carlo study is formulated at the level of individual coupon bonds to assess the accuracy of the estimated bond-specific risk premia within the AFNS-L-C model. To get a representative data-generating process for the South African bond market, we use the estimates of the AFNS-L-C model from Table 5. Based on these parameters, we first simulate $N = 100$ samples for the five states at a monthly frequency for 290 months, which corresponds to the number of monthly observations in our South African sample. To faciliate interpretation, sample paths for the states will be common across all exercises in the Monte Carlo study to facilitate the interpretation. The inputs for the estimation approach are constructed as follows.

For our one-step estimation approach, we use the simulated states to compute N panels of coupon-bond prices that match those observed in the South African sample in terms of available bonds and their characteristics. These bond prices are computed using the bond price formula in equation 13 in combination with the bond-specific discount function in equation 12. We then add measurement errors $\varepsilon_t^i \sim \mathcal{NID}\big(0, \sigma_\varepsilon^2\big)$ to the simulated bond prices and scale these errors by the duration of the simulated bond for consistency with equation 15.[22]

To study the role of the data quality, we consider two cases, where the standard deviation of the measurement errors σ_ε is either 1 or 10 basis points. The first case, with $\sigma_\varepsilon = 1$ basis point, represents an ideal setting, with hardly any noise in the bond prices, and helps to isolate the efficiency of the factor identification within the AFNS-L-C model. The second case, with $\sigma_\varepsilon = 10$ basis points, is included to describe a more realistic setting, as we find $\sigma_\varepsilon = 4$ basis points in our South African sample when estimating the AFNS-L-C model. These results speak to the kind of performance we can expect from the AFNS-L-C model in emerging bond markets with data quality somewhat below the level of our South African sample.

We now turn to the details of the simulation of the factor paths. The continuous-time $\mathbb{P}$-dynamics are, in general, given by

$$dX_t = K^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t)dt + \Sigma D(X_t)dW_t^{\mathbb{P}}.$$

For both restricted square-root processes and unconstrained processes, we approximate the continuous-time process using the Euler approximation (Thompson (2008) is an exam-

[22] Note that we also use the same set of simulated samples of ε_t^i throughout the Monte Carlo study to make the results as comparable as possible.

ple). To exemplify, for a restricted square-root process,

$$dX_t^i = \kappa_{ii}^{\mathbb{P}}(\theta_i^{\mathbb{P}} - X_t^i)dt + \kappa_{ij}^{\mathbb{P}}(\theta_j^{\mathbb{P}} - X_t^j)dt + \sigma_{ii}\sqrt{X_t^i}dW_t^{\mathbb{P},i},$$

the algorithm is

$$X_t^i = X_{t-1}^i + \kappa_{ii}^{\mathbb{P}}(\theta_i^{\mathbb{P}} - X_{t-1}^i)\Delta t + \kappa_{ij}^{\mathbb{P}}(\theta_j^{\mathbb{P}} - X_{t-1}^j)\Delta t + \sigma_{ii}\sqrt{X_{t-1}^i}\sqrt{\Delta t}z_t^i, \quad z_t^i \sim N(0,1).$$

We fix Δt at a uniform value of 0.0001, which is equivalent to approximately 27 shocks per day to each process through Brownian motion. As Feller conditions and other non-negativity requirements are imposed in the estimations performed with the observed bond prices, the parameter sets used in the simulations satisfy all non-negativity requirements, so the "true" underlying continuous-time process never becomes negative $\mathbb{P}$-a.s. However, for the discretely observed process above there is always a positive – but usually very small – probability that the approximation will become negative. Whenever this happens, we truncate the simulated square-root processes at 0, similar to the model estimations.

We ideally want to draw the starting point of the simulation algorithm, X_0, from the unconditional joint distribution of the five state variables. However, we do not know the unconditional distribution of $X_t = (L_t, S_t, C_t, X_t^L, X_t^\lambda)$ for the AFNS-L-C model. To overcome this problem, we take the estimated value of the five state variables at the end of the observed bond price sample and simulate the five state variables according to the algorithm above for 1 000 years and repeat this 100 times. This effectively gives us random draws from the joint unconditional distribution of $X_t = (L_t, S_t, C_t, X_t^L, X_t^\lambda)$. These starting values are identical for all simulated samples to make the results as comparable as possible.

In the final step, we use the 100 simulated samples from each exercise as input into a corresponding number of Kalman filter estimations, where we use the true parameters as the starting point for each optimisation. Estimating the true model in each case provides us with a clean read of the properties of the extended Kalman filter as a QML estimator of the AFNS-L-C model, not impacted by any errors related to model misspecification.

5.2 Accuracy of estimated parameters

In this section, we use the estimation results from the simulated bond price samples to examine the accuracy of the estimated parameters in the AFNS-L-C model.

Table 8 shows the summary statistics of the estimated dynamic parameters across the $N = 100$ simulated bond price samples. For the estimated mean-reversion rates in the diagonal of the $K^{\mathbb{P}}$ mean-reversion matrix, we note a pronounced upward bias in these parameter estimates. The state variables in the estimated model thus tend to have lower persistence than is the case in the true model. This is a well-documented problem in time series analysis and is discussed in detail in the context of dynamic term structure models in Bauer, Rudebusch and Wu (2012). For the same reason, these parameter estimates are associated with a fair amount of uncertainty, as reflected in their high empirical standard deviations.

Table 8: Accuracy of the estimated dynamic parameters in the AFNS-L-C model

Par.	True value	$\sigma_\varepsilon = 1$ basis point	
		Mean	Std. dev.
$\kappa_{11}^{\mathbb{P}}$	**0.2858**	0.4775	0.2324
$\kappa_{22}^{\mathbb{P}}$	**0.0317**	0.1511	0.1756
$\kappa_{33}^{\mathbb{P}}$	**0.1204**	0.2529	0.2052
$\kappa_{44}^{\mathbb{P}}$	**0.3844**	0.5124	0.2128
$\kappa_{55}^{\mathbb{P}}$	**0.1593**	0.2836	0.1595
σ_{11}	**0.0125**	0.0125	0.0001
σ_{22}	**0.0244**	0.0247	0.0012
σ_{33}	**0.0427**	0.0422	0.0015
σ_{44}	**0.0197**	0.0196	0.0007
σ_{55}	**0.2912**	0.2860	0.0097
$\theta_1^{\mathbb{P}}$	**0.1345**	0.1296	0.0096
$\theta_2^{\mathbb{P}}$	**-0.0397**	-0.0153	0.1349
$\theta_3^{\mathbb{P}}$	**0.0464**	0.0636	0.0799
$\theta_4^{\mathbb{P}}$	**0.0031**	0.0046	0.0122
$\theta_5^{\mathbb{P}}$	**0.2662**	0.2704	0.0917
λ	**0.1803**	0.1800	0.0029
$\kappa_L^{\mathbb{Q}}$	**1.0988**	1.0882	0.0361
$\theta_L^{\mathbb{Q}}$	**0.0041**	0.0041	0.0001
$\kappa_\lambda^{\mathbb{Q}}$	**2.3782**	2.3768	0.0102
$\theta_\lambda^{\mathbb{Q}}$	**0.0178**	0.0204	0.0055

NOTE: The table shows the mean estimate and the standard deviation (Std. dev.) of the sampling distribution for each of the estimated dynamic parameters in the AFNS-L-C model when using QML in the model estimation based on simulated data with low and high noise, respectively, both with simulated samples of length $T = 290$ and $N = 100$ repetitions.

In contrast, the parameters tied to the AFNS-L-C model's $\mathbb{Q}$-dynamics used for pricing, (σ_{11}, σ_{22}, σ_{33}, σ_{44}, σ_{55}, λ, $\kappa_L^{\mathbb{Q}}$, $\theta_L^{\mathbb{Q}}$, $\kappa_\lambda^{\mathbb{Q}}$, $\theta_\lambda^{\mathbb{Q}}$), are all estimated with great precision and little to no bias and with very small empirical standard deviations. This is another well-documented fact – namely that the risk-neutral $\mathbb{Q}$-dynamics used for pricing tend to be very accurately estimated (see Andreasen, Christensen and Rudebusch (2019) for evidence in the context of the standard AFNS model).

The estimated mean parameters appear to have relatively modest bias, but their estimates are somewhat uncertain, as reflected in their slightly elevated empirical standard deviations. Qualitatively similar results are reported by Andreasen, Christensen and Rudebusch (2019) in their study of simulated bond samples based on the standard AFNS model. Mechanically, there is a limit to the bias in the estimated mean parameters. Thanks to the high precision of the estimated $\mathbb{Q}$-related parameters, the state variables tend to be filtered with a matching high precision, as we document below in section 5.3. Consequently, the estimated mean parameters are only likely to notably deviate from their true value in the simulated samples where the simulated factors deviate materially from their mean. Given the relatively high persistence of S_t, C_t and X_t^λ in the true model, we would expect deviations in the estimated mean parameters to be larger and more frequent for these three state variables, and that is indeed what we observe in Table 8.

In Table 9, we report the estimated bond-specific loadings, β^i, on the liquidity risk factor. They are all estimated with very high precision, which is not surprising given that they are identified from the cross-section of bond prices similar to the other $\mathbb{Q}$-related parameters discussed above. The key takeaway is that the AFNS-L-C model and its estimation based on samples of bond prices and the extended Kalman filter are clearly able to distinguish between the individual β^i parameters across the bonds in our South African sample. The estimated standard deviations reported in Table 4 appear to be realistic compared to the corresponding empirical standard deviations reported in Table 9. Hence, they appear to be reliable as well.

In Table 10, we report the results for the estimated bond-specific decay parameters, $\lambda^{L,i}$, in the bond-specific factor loadings on the liquidity risk factor. We note a similarly high degree of precision in the estimation of these parameters. The takeaway thus remains that the AFNS-L-C model and its estimation based on samples of bond prices and the extended Kalman filter is truly able to distinguish between both the β^i and the $\lambda^{L,i}$ parameters across bonds in our South African sample. As a result, we expect the estimated bond-specific liquidity risk premia to be very close to their true simulated values.

Table 9: Accuracy of the estimated bond-specific sensitivities to the liquidity risk factor in the AFNS-L-C model

Par.	True value	$\sigma_\varepsilon = 1$ basis point	
		Mean	Std. dev.
β^1	**0.6612**	0.6461	0.0329
β^2	**73.8717**	74.3252	0.3488
β^3	**85.5231**	85.7376	0.3684
β^4	**2.3287**	2.3437	0.0728
β^5	**1.8362**	1.8078	0.0798
β^6	**2.0021**	2.0102	0.0346
β^7	**220.0146**	222.0166	2.7900
β^8	**1.8626**	1.8674	0.0298
β^{10}	**1.0470**	1.0496	0.0220
β^{11}	**0.2709**	0.2541	0.0583
β^{12}	**128.3567**	129.0600	0.5069
β^{13}	**3.7711**	3.7596	0.1422
β^{14}	**2.6639**	2.6650	0.0590
β^{15}	**1.9653**	1.9679	0.0260
β^{16}	**1.7187**	1.7467	0.0453
β^{17}	**248.9202**	249.7584	0.4610
β^{18}	**0.9773**	0.9825	0.0238
β^{19}	**4.4883**	4.5435	0.2603
β^{20}	**1.1538**	1.1545	0.0068
β^{21}	**4.2809**	4.4137	0.3155
β^{22}	**9.8418**	9.9941	0.7059
β^{23}	**1.1274**	1.1284	0.0062
β^{24}	**55.4975**	55.8491	0.2798
β^{25}	**3.1906**	3.2001	0.0443
β^{26}	**2.0705**	2.0871	0.0481
β^{27}	**1.9622**	1.9689	0.0191
β^{28}	**6.3086**	6.3948	0.1940
β^{29}	**2.3856**	2.3940	0.0222
β^{30}	**4.0897**	4.1108	0.0457
β^{31}	**4.1541**	4.1662	0.0774

Note: The table shows the mean estimate and the standard deviation (Std. dev.) of the sampling distribution for each of the estimated bond-specific sensitivities to the liquidity risk factor in the AFNS-L-C model when using QML in the model estimation based on simulated data with low and high noise, respectively, both with simulated samples of length $T = 290$ and $N = 100$ repetitions.

5.3 Accuracy of estimated states

In this section, we switch our focus to an analysis of the accuracy of the filtered states in the extended Kalman filter-based QML estimation of the AFNS-L-C model. Given the high precision of all the risk-neutral $\mathbb{Q}$-related parameters documented in the previous section, we expect the filtering of the state variables to be equally accurate.

These results are reported in Table 11, where we indeed see a high degree of accuracy in the filtering of the first four state variables, while the filtering of the credit risk factor, X_t^λ, is associated with somewhat greater error.

Table 10: Accuracy of the estimated bond-specific decay parameters in the AFNS-L-C model

Par.	True value	$\sigma_\varepsilon = 1$ basis point Mean	Std. dev.
$\lambda^{L,1}$	**8.5266**	9.3132	0.1442
$\lambda^{L,2}$	**0.0009**	0.0009	0.0000
$\lambda^{L,3}$	**0.0010**	0.0010	0.0000
$\lambda^{L,4}$	**9.4485**	9.4273	0.2133
$\lambda^{L,5}$	**9.0573**	9.3062	0.1699
$\lambda^{L,6}$	**9.4296**	9.2637	0.1643
$\lambda^{L,7}$	**0.0011**	0.0010	0.0001
$\lambda^{L,8}$	**9.6055**	9.2996	0.1783
$\lambda^{L,9}$	**9.9997**	9.9917	0.0217
$\lambda^{L,10}$	**2.1835**	2.5262	1.1913
$\lambda^{L,11}$	**9.9995**	9.8711	0.2312
$\lambda^{L,12}$	**0.0015**	0.0015	0.0000
$\lambda^{L,13}$	**0.0627**	0.0635	0.0039
$\lambda^{L,14}$	**0.0941**	0.0945	0.0040
$\lambda^{L,15}$	**0.1607**	0.1614	0.0065
$\lambda^{L,16}$	**0.4438**	0.4347	0.0171
$\lambda^{L,17}$	**0.0020**	0.0020	0.0001
$\lambda^{L,18}$	**9.9969**	9.7720	0.3166
$\lambda^{L,19}$	**0.0326**	0.0323	0.0022
$\lambda^{L,20}$	**9.9979**	9.6685	0.3691
$\lambda^{L,21}$	**0.0337**	0.0327	0.0031
$\lambda^{L,22}$	**0.0208**	0.0206	0.0016
$\lambda^{L,23}$	**9.9999**	9.2214	1.5146
$\lambda^{L,24}$	**0.0046**	0.0046	0.0001
$\lambda^{L,25}$	**0.1518**	0.1518	0.0048
$\lambda^{L,26}$	**0.1260**	0.1246	0.0055
$\lambda^{L,27}$	**0.2943**	0.2925	0.0102
$\lambda^{L,28}$	**0.0665**	0.0656	0.0031
$\lambda^{L,29}$	**0.4084**	0.4064	0.0171
$\lambda^{L,30}$	**0.1418**	0.1413	0.0046
$\lambda^{L,31}$	**10**	9.0124	1.5345

NOTE: The table shows the mean estimate and the standard deviation (Std. dev.) of the sampling distribution for each of the estimated bond-specific decay parameters in the AFNS-L-C model when using QML in the model estimation based on simulated data with low and high noise, respectively, both with simulated samples of length $T = 290$ and $N = 100$ repetitions.

5.4 Summary

Based on the simulation study, we conclude with great confidence that, for samples with a bond composition structure identical to our South African sample, the AFNS-L-C model can be robustly estimated using QML based on the extended Kalman filter. To what extent these results extend to other bond samples with a different distribution of bonds in terms of coupons and across maturities is an empirical question that we leave for future research. However, we offer a couple of conjectures for this future work. First, for the model to work properly, credit risk must be a material component in the prices under analysis, otherwise the credit risk factor becomes equivalent to a nuisance parameter without a real role to

Table 11: Accuracy of estimated states in the AFNS-L-C model

State variable	$\sigma_\varepsilon = 1$ basis point	
	Mean	MAE
L_t	-17.17	30.38
S_t	3.33	19.77
C_t	1.51	50.96
X_t^L	-1.06	12.99
X_t^λ	25.28	115.61

Note: The table shows the mean errors (mean) and mean absolute errors (MAE) of each estimated state variable in the AFNS-L-C model when using QML in the model estimation with low and high noise, respectively, both with simulated samples of length $T = 290$ and $N = 100$ repetitions. The mean error is obtained by first computing the mean errors in each of the simulated samples across the $T = 290$ observations, and we then report the average of these means across the $N = 100$ simulated samples. Similarly, the MAE are obtained by first computing the MAE in each of the simulated samples across the $T = 290$ observations and then reporting the average of these absolute means across the $N = 100$ simulated samples. The true states are generated from the AFNS-L-C model, as described in section 5.1. All numbers are reported in basis points.

play. It thus seems to make little sense to apply this model to the US Treasury market, for example, and this holds true independent of the number of bonds. It is an empirical question what the required level of credit risk is for the model to work, but the South African data may serve as a guiding example in this regard. Second, provided the model is appropriate for the data, and the first condition is met, a larger and more diverse set of bonds should improve factor identification and the estimation accuracy, all else being equal.

6 Conclusion

In this paper, we introduce a novel dynamic term structure model that jointly identifies liquidity and credit risk premia in panels of prices of bonds issued by the same legal entity. Their separate identification relies on the observations that liquidity risk, defined as the inability of an investor to sell a given bond back to the market at the prevailing prices without having to incur a price discount, is security-specific and unique to each bond, while credit risk is common to all bonds and defined as the inability of the issuer to meet its debt payments.

We then demonstrate the model's applicability by using it to estimate the liquidity and credit risk premia embedded in South African government bond prices. The results show that liquidity and credit risk premia are sizeable in this market and only weakly correlated. Liquidity and credit stresses are thus indeed distinct risks to bond investors. We speculate that this may be tied to the fact that rollover risk is more continuous in sovereign bond markets owing to the large amounts of short-term bills in circulation, but we leave it for future research to explore this further.

A simulation study was used to study and document the validity of the separate identification of liquidity and credit risk. The results overwhelmingly confirm the estimated model's ability to distinguish liquidity risk from credit risk and distinguish both of these components in the bond prices from the frictionless level, slope and curvature factors. This leads us to recommend that the model be used to examine these risks in other emerging bond markets as well as in corporate bond markets in advanced economies, but we also leave that task for future research.

In future work, we aim to expand the presented model with inflation-indexed bonds and decompose breakeven inflation into expectations and risk premium components in the presence of liquidity and credit risk premia. We conjecture that this may improve the identification of the credit risk factor by exploiting the fact that it is common to both bond markets, while liquidity risk is unique to each (as in Beauregard et al. (2024)).

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