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Measuring Systemic Risk in South African Banks

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This paper introduces several market-based measures of systemic risk and examines how they can inform the vulnerability assessment of South African banks from the perspective of both markets and regulators. We make an empirical comparison of three systemic risk measures - ΔCoVaR, Marginal Expected Shortfall (MES), and SRISK in the context of six South African banks, which constitute 92% of the total assets in the South African banking system. We obtain the rankings of these institutions in terms of their contributions to systemic risk across the different measures. In addition, we investigate how the measures may be aggregated to estimate the projected tails of the distribution of GDP growth in order to inform macroprudential policy considerations. This analysis motivates the use of a suite of measures to monitor the level of systemic risk in the financial system, as well as, their implied risk to real economic outcomes in South Africa.

JEL Codes: G01, G32

Keywords: systemic risk, value-at-risk, CoVaR, MES, SRISK, growth-at-risk

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1. Introduction[1]

We often think of systemic risk in the banking system as meaning the likelihood of experiencing joint failures amongst a significant proportion of banks, and enough to have real economy consequences. A systemically important financial institution (SIFI) is one whose distress or failure could crystallize systemic risk. Mitigating systemic risk is considered the premise underlying the need for macroprudential policy intervention where an effective identification of systemically important institutions remains a key focus. Systemic risk measures are designed to overcome the shortcomings of stand-alone measures of an institution's risk, such as value-at-risk (VaR). A survey of the main quantitative measures of systemic risk in the literature can be found in Bisias et al. (2013) and Benoit et al. (2017). The extensive scope of these surveys suggests that there is not yet an agreed upon single approach to the measurement of systemic risk. Nor is there a universally accepted formal definition. Hansen (2013) describes systemic risk as "basically a grab bag of scenarios that are supposed to rationalize intervention in financial markets" by way of macroprudential policy.

The empirical finance literature contains a number of market-based systemic risk measures which differ in terms of the scope of their application. The first group of measures capture the impact on the financial system conditional on an institution being in distress. In this group, the most prominent measures are the Conditional Value-at-Risk (CoVaR) and delta CoVaR (ΔCoVaR) established by Adrian and Brunnermeier (2011). CoVaR is the value-at-risk (VaR) of the financial system conditional on an individual institution being in distress (at its VaR). ΔCoVaR is the change in the VaR of the financial system conditional on an institution being in distress relative to its median state. The next group of measures compute the impact on an institution conditional on the financial system being in distress. The two most prominent approaches are Marginal Expected Shortfall (MES) by Acharya et al. (2010) and SRISK by Brownlees and Engle (2016). The MES of a bank is its short-run equity loss conditional on the banking system taking a loss greater than its VaR. SRISK measures the expected capital shortfall of an institution conditional on a severe decline in market returns.

Both of the above two groups of systemic risk measures focus on the interdependence between a financial institution and the financial system and tend to ignore network interconnectedness among the financial institutions. As described in Hautsch et al. (2015), such approaches cannot detect spillover effects driven by the topology of the risk network and may tend to underestimate the importance of highly interconnected institutions. However, we do not see that as being a significant issue affecting the systemic risk assessment of the South African banking system. The network topology of the South African interbank system suggests a structure that is largely stable. Brink and Georg (2011) show that the interbank system was mostly stable and resilient over the period from March 2005 to June 2010 even in times of distress in international financial markets. However, there are several studies on network analysis which model how inter-linked asset holdings generate and propagate systemic risk (Allen et al., 2010). Haldane and Nelson (2012) discuss how networks can produce non-linearity and unpredictability with the attended extreme events. Hautsch et al. (2015) design a systemic risk beta, which is based on their tail risk interdependence network. Härdle et al. (2016) extend CoVaR to a tail-event driven network that can measure the systemic risk con-

[1] The authors would like to thank Andy Blake, Hugh Campbell, Greg Farrell, Glenn Hoggarth, Videshree Rooplall, and two anonymous referees for their useful comments. The views expressed in the paper are solely those of the authors and should not be taken to represent those of the Bank of England or the South African Reserve Bank.

tribution of a financial institution by taking into account its tail interconnectedness with other relevant financial institutions.

Another strand in the literature relates to measures permitting the computation of the joint probability of default of the banking system. The systemic contingent claims analysis (CCA) proposed by Jobst and Gray (2013), and further augmented by Chatterjee and Jobst (2019), follows this approach. CCA transforms accounting identities into exposures so that changes in the value of assets are directly linked to changes in the market value of equity and expected losses. In this framework, the magnitude of systemic risk is assessed based on the combined default risk of all banks within a banking system during times of stress. This provides a sense of the relative systemic relevance of each bank.

From a regulatory perspective, the ultimate objective of applying these different methods would be to help determine bank-specific capital buffers that reflect systemic risk. Regulators across most jurisdictions now consider it appropriate that a significant proportion of a banking system's going concern equity requirement should be in the form of buffers that can be used to absorb losses under stress, rather than in hard minimum requirements. These buffers serve a macroprudential purpose. By absorbing the impact of stress, they reduce the need for banks to withdraw services, such as credit provision to the real economy. It may be argued, that in doing so, bank capital takes on the attributes of a public good as this has the effect of imposing an externality on bank shareholders (Shin, 2016). A number of regulators now impose an additional buffer of equity for global systemically important banks (GSIBs) depending on their systemic importance. This buffer reduces the probability that these banks will fail, in line with the greater costs of their failure to the global economy. It skews equity in the system towards these banks and raises over-all equity levels in the financial system. In this regard, Haldane (2011) highlights the complexity that arises from model error and states that "model uncertainty, as distinct from risk, is rarely taken into account when interpreting reported capital ratios." Hansen (2013) endorses this view and argues that "even as we add modelling clarity, ... we need to abandon the presumption that we can fully measure systemic risk and go after the conceptually more difficult notion of quantifying systemic uncertainty."

With these caveats in mind, we make an empirical comparison of three measures – ΔCoVaR, MES and SRISK – in the context of major South African banks. By studying their evolution, within a banking system, we want to contribute to a better understanding of their applicability in different types of crises such as the global financial crisis (GFC) of 2007-09, or the exogenous shock from the Covid-19 pandemic.

Taking a macro-financial perspective, some of these measures can be aggregated in an attempt to quantify this systemic uncertainty, through the use of the growth-at-risk (GaR) methodology first introduced by Adrian et al. (2016). In Adrian et al. (2018), the authors suggest that this GaR measure could be used to assess the macroprudential policy stance, and inform policy-makers as to when, and how much, to tighten or loosen their macroprudential policy tools.

The largest 6 South African banks analysed in this paper - Absa, Capitec, First National Bank (FNB), Investec, Nedbank, and Standard Bank - have a diverse set of business models and operate in a broad range of markets in Sub-Saharan Africa. However, the SA banking system is highly concentrated: these 6 banks constitute 92 percent of total SA banking sector assets, with the largest four of these banks (the "Big 4": Absa, FNB, Nedbank, and Standard

Bank) accounting for over 82 percent[2]. These banks are not “global" enough to feature in the Basel Committee on Banking Supervision (BCBS) list of GSIBs[3]. But their distress or failure could have a significant impact on financial stability. Such effects could also impact across borders given the pivotal role South African banks have in the wider South African Development Council (SADC) region. In 2002, the South African banking system underwent a major consolidation after a number of small- and medium-sized banks failed following a liquidity crisis that was partly triggered by the South-East Asian financial crisis of 1997-98. However, even prior to this period - at the end of which Nedbank merged with the 6th largest bank at the time, BoE Bank - the major six banks represented 74.2 percent of total banking assets (SARB (2002)). It should be noted that while all of these banks are listed on the Johannesburg Stock Exchange, Investec is the only bank dual-listed on the London Stock Exchange.

There is currently a very scarce literature on systemic risk measures for South African banks, and the existing papers focus solely on estimating a single measure of systemic risk for the banking sector. Manguzvane et al. (2019) calculate CoVaR for six major banks, and find that FNB, followed by Standard Bank, Absa, and Nedbank, were the largest contributors to systemic risk over the period June 2007 to April 2016. Foggitt et al. (2017) calculate SRISK for the major four banks, along with Investec. They find that Investec is the largest average contributor to systemic risk, despite it being the bank with the smallest market capitalisation, whereas the big banks' contributions appear to be more volatile and increased more dramatically during the global financial crisis.[4] These results support this paper's focus on analysing measures of systemic risk for the 6 largest market-players in the South African banking system.

The contribution of this paper is two-fold. First, whereas the literature on measuring systemic risk in South African banks has typically focussed on a single measure, we develop a more comprehensive assessment across different measures. In the process, we provide additional clarity on the nuances of each measure and their distinguishing characteristics. Second, we have used these measures to demonstrate how higher systemic risk in the banking system implies increased downside risk to GDP growth. In addition, we highlight the importance of having a suite of systemic risk measures in order to assess and inform the stance of macro-prudential policy. To the best of our knowledge, this is the first study that attempts to do so in the context of South African banks.

The rest of the paper is organised as follows. Section 2 provides a description of the three different measures of systemic risk and the common framework used to compare them. Section 3 describes the data used for the estimation of an individual institution's asset returns and those of the financial system. Section 4 describes empirical results for the measurement of systemic risk in South African banks. Section 5 assesses the relationship between these estimated measures of systemic risk and the downside risk to South African GDP growth. Section 6 concludes.

[2] According to the October 2020 BA900 returns of the Bank Supervision Department, South African Reserve Bank.

[3] The BCBS developed a methodology for identifying GSIBs and assessing a higher loss absorbency (HLA) requirement. To accomplish this HLA, Basel III standards require GSIBs to hold more common equity Tier 1 capital (BCBS, 2013).

[4] Foggitt et al. (2017) note, however, that Investec's prominence as a contributor to systemic risk is only emphasized during “quiet” periods in the market. During these periods, the big four banks do not exhibit any SRISK contribution (their estimated SRISK values fall below zero). The period they consider, 2001 to 2013, is dominated by “quiet” markets. During this period, Investec is seen to be the largest contributor to systemic risk on average.

2. Measures of systemic risk

2.1 The CoVaR approach

The Conditional Value-at-Risk (CoVaR) is defined as the Value-at-Risk (VaR) of the financial system conditional on an institution being in financial distress. ΔCoVaR, which captures the marginal contribution to systemic risk, is the difference between the VaR of the financial system conditional on a given institution being in distress, and the VaR of the financial system conditional on this financial institution being in a normal state. As described in Adrian and Brunnermeier (2011), CoVaR focuses on the tail of a distribution and is more extreme than the unconditional VaR, as CoVaR is a VaR that conditions on an adverse event. This condition typically shifts the mean of asset value returns downwards and increases the variance. The distribution of asset values of the financial system depends on the financial health of the individual institutions and their effects on each other. When a financial institution undergoes stress, this will change the distribution of asset values of the system. CoVaR focuses on the co-dependence in the tails of equity returns of an institution and the financial system. By conditioning on another institution's financial distress, CoVaR aims to go beyond idiosyncratic risk and captures possible risk spillovers between financial institutions.

The general framework of CoVaR depends on the conditional distribution of a random variable $R_{s,t}$, representing the equity returns of the entire financial system at time t, given that another financial institution i, represented by a random variable $R_{i,t}$, is in distress.

Before discussing the method used to estimate CoVaR, we describe the steps involved in estimating the market value of asset returns for financial institutions. Bank $i's$ equity returns are evaluated using market equity, $ME_{i,t}$, and book value of assets and equities, $BA_{i,t}$ and $BE_{i,t}$, as follows:

$$R_{i,t} = \frac{ME_{i,t}.LEV_{i,t} - ME_{i,t-1}.LEV_{i,t-1}}{ME_{i,t-1}.LEV_{i,t-1}} = \frac{A_{i,t} - A_{i,t-1}}{A_{i,t-1}} \tag{1}$$

where $LEV_{i,t} = BA_{i,t}/BE_{i,t}$ is the book value of leverage at time t, and $A_{i,t} = ME_{i,t}LEV_{i,t}$ is the total value of assets at time t.

Financial system equity returns, $R_{s,t}$, are evaluated as the weighted average of banks' equity returns, where the weights are the lagged values of institutions' total market capitalisation. Namely,

$$R_{s,t} = \frac{1}{\sum_{i=1}^{n} ME_{i,t-1}} \sum_{i=1}^{n} R_{i,t}ME_{i,t-1} \tag{2}$$

In order to avoid spurious correlations, the conditioning institution is not included in the financial system when evaluating its CoVaR.

Two different definitions of CoVaR appear in the literature using different conditioning events. In the original definition by Adrian and Brunnermeier (2011), CoVaR is defined as the conditional distribution of $R_{s,t}$ given that $R_{i,t} = VaR_t^i$, while in the modified definition of CoVaR, proposed by Girardi and Ergün (2013), the conditioning event is $R_{i,t} \leq VaR_t^i$. In other words, the former definition represents the VaR of the system assuming that institution i is *exactly* at its VaR level, whereas the latter definition of CoVaR represents the same risk metric assuming that institution i is *at most* at its VaR level. In this study we estimate CoVaR using quantile regressions in accordance with the original definition proposed by Adrian and Brunnermeier (2011)[5].

CoVaR is based on the concept of VaR_{α}, which is the maximum loss within the α% - confidence interval. As they use a quantile regression approach, Adrian and Brunnermeier (2011) consider a situation in which the loss is precisely equal to its VaR. ΔCoVaR is defined as the difference between the $\alpha\% - CoVaR$ of the system s conditional on institution i being in financial distress, and the $\alpha\% - CoVaR$ of the system s conditional on institution i being at its benchmark state.

$$\Delta CoVaR_{\alpha}^{s|i} = \left(CoVaR_{\alpha}^{s|R^i = VaR_{\alpha}^i} - CoVaR_{\alpha}^{s|R^i = Median^i} \right) \tag{3}$$

Adrian and Brunnermeier (2011) define institution i's state of financial distress as institution i's asset returns being at their α%-VaR level, and its benchmark state as asset returns being at their median level.

2.2 The Marginal Expected Shortfall approach

The Marginal Expected Shortfall (MES), proposed by Acharya et al. (2010), is defined as the expected loss on a bank's equity conditional on the occurrence of losses in the tail of the financial system's loss distribution. In their original paper, the financial system's returns are proxied by the S&P500 which includes non-financial firms. In this paper, we specify the financial system as comprising our sample of six South African banks. This conforms to the approach taken by Idier et al. (2014) who define MES of a financial institution as its short-run equity loss conditional on the financial system taking a loss greater than its VaR. Our banking system is closer to that considered by Löffler and Raupach (2018), whose MES study is based on the perspective of a smaller country's financial system with a few big players.

Using the same notation as in the CoVaR definition, we denote $R_{i,t}$ as the equity return of the bank and $R_{s,t}$ as the equity return of the financial system. While VaR is simply a quantile (α%) of the loss distribution over a prescribed holding period, expected shortfall is the expected loss knowing that the loss is above VaR.[6]

By definition, the expected shortfall (ES) at the $\alpha\%$ level is the expected return in the worst $\alpha\%$ of cases, but it can be extended to the general case, in which returns exceed a given threshold C. To enable comparison with the ΔCoVaR, we consider a threshold C equal to the conditional VaR of the market return, which is defined as $Pr[R_{st} < VaR_{st}(\alpha)] = \alpha$. We define the ES of the market as the expected loss in the financial system s conditional on this loss being greater than C. Since the banking system is a weighted sum of the performance of the

5 The modified definition of CoVaR proposed by Girardi and Ergün (2013) uses a Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model with the Dynamic Conditional Correlation (DCC) approach of Engle (2002) to obtain joint distributions of the return of the system s and institution i. We use this bivariate DCC-GARCH process for returns to estimate MES, in accordance with Brownlees and Engle (2011).

6 The Expected Shortfall (ES) of a given system is also known as the expected tail loss, or average value-at-risk. It is the expected conditional loss, given that the VaR is exceeded (a worst-case scenario), and will be larger (or more extreme) the more fat-tailed the distribution of returns. The MES is then the contribution of an individual bank to the total estimated shortfall in the system.

participating banks, we have:

$$ES_{s,t}(C) = E_{t-1}(R_{s,t} \mid R_{s,t} < C) = \sum_{i=1}^{N} w_{i,t} E_{t-1}(R_{i,t} \mid R_{s,t} < C) \quad (4)$$

The banking system's return, R_{st}, has been decomposed into each bank's return, R_{it}, that is $R_{st} = \sum_i^n w_{it} R_{it}$, where w_{it} is the weight of bank in the banking system. N is the total number of banks and E_{t-1} is the conditional expectation of all information available in $t-1$:

$$MES_{i,t}(C) = \frac{\partial ES_{s,t}(C)}{\partial w_{i,t}} = E_{t-1}(R_{i,t} \mid R_{s,t} < C) \quad (5)$$

Expected shortfall, $ES_{s,t}$, is an overall measure of systemic risk, being the expected impact on the banking system of a tail shock. The contribution of each bank to the total shortfall, defines the Marginal Expected Shortfall (MES). It is obtained by differentiating $ES_{s,t}$, with respect to the weight of bank i in the banking system. Brownlees and Engle (2011) developed a dynamic version of the MES by applying the asymmetric DCC-GARCH model of Engle and Sheppard (2008) to the measurement of systemic risk. As in the case of CoVaR we estimate MES with a weekly frequency for the panel of 6 SA banks. Appendix B provides details of the modelling framework.

2.3 The SRISK approach

SRISK (Brownlees and Engle, 2016) is a market-based stress test that estimates the expected amount of capital that a bank would need to raise conditional on a crisis affecting the whole financial system. SRISK takes into account a bank's market capitalisation, its prudential capital ratio, and the level of debt given by its total liabilities. The estimation of SRISK involves the following variables: the returns of bank i, $R_{i,t}$, the returns of the financial system, $R_{s,t}$, the book value of debt, $BD_{i,t}$, the market value of equity (market capitalisation), $ME_{i,t}$, and the prudential capital requirement, k_t. These variables are first used to calculate each bank's long-run marginal expected shortfall (LRMES). The LRMES is a measure of an institution's expected cumulative loss of equity over a prolonged period conditional upon a large shock in the financial system. There are two general approaches to calculating the LRMES, both of which require a first step of fitting a DCC-GARCH model. The first is by simulating the trajectories of banking system returns in future periods. We have adopted the second approach based on the following approximation proposed by Acharya et al. (2012):

$$\begin{aligned} LRMES_{i,t} &= E_{t-1}\left(R_{i,t+T} | R_{s,t+T} < C\right) \\ &\approx 1 - exp(-18 \times MES_{i,t}) \end{aligned} \quad (6)$$

where $R_{i,t+T}$ is the cumulative return to a bank's equity, and $R_{s,t+T}$ is the cumulative return on the banking system. Thereafter, SRISK is calculated as:

$$\begin{aligned} SRISK_{i,t} =& max(0, k_t * BD_{i,t} - (1-k_t) * ME_{i,t} * (1 - LRMES_{i,t}) \\ =& max(0, ME_{i,t}[k_t * LEV_{i,t} - (1-k_t) * (1 - LRMES_{i,t} - 1]) \end{aligned} \quad (7)$$

where $LEV_{i,t} = \frac{BD_{i,t} + ME_{i,t}}{ME_{i,t}}$ is the quasi-leverage ratio. This implies that a bank with capital surplus has an SRISK value of zero. SRISK can vary significantly depending on the value of the parameter k, the prudential capital ratio. SRISK will decrease as the value of k decreases. In contrast to Brownlees and Engle (2016), who use a prudential capital ratio of 8 percent[7], this study uses the phased-in minimum capital requirements, as described in Annexure B of Directive 5/2013 issued by the SARB, to capture time-varying capital requirements for South African banks.

Therefore, SRISK adjusts for the amount of regulatory capital that the bank is mandated to hold, and can be decomposed into three main components: (i) the market capitalisation of the bank; (ii) how leveraged the bank is; and (iii) how risky the bank is conditional on a distressed financial system, as proxied by its LRMES. A positive SRISK value thus suggests that the bank, in the event of a systemic market event, would experience financial distress, due to excessive debt, insufficient equity, and/or a deterioration in market-based risk-related measures of the bank. Under the assumption that the prudential capital requirement is time-varying, the decomposition can be represented as follows:

$$\begin{aligned} \Delta SRISK &= \Delta \text{PrudentialRequirements} + \Delta \text{Debt} - \Delta \text{Equity} + \Delta \text{Risk} \\ &= [(D+ME)*dk] + [k*dD] - [(1-k)(1-LRMES)*dME] \\ &+ [(1-k)*ME*dLRMES] \end{aligned} \tag{8}$$

3. Data

The analysis focuses on 6 SA banks - Absa, Capitec, First National Bank (FNB), Investec, Nedbank, and Standard Bank - covering the period 1 March 2002 to 23 October 2020. Following Adrian and Brunnermeier (2011), we work with weekly returns to avoid the non-synchronicity of daily data[8]. We obtain weekly data on Market Capitalisation, Total Liabilities and Leverage from the Bloomberg database and generate returns in the market value of equity. There are 974 weekly returns for each institution in our sample.

In the CoVaR estimation for each bank, a market-value-weighted return index of the remaining banks in the sample is used as a proxy for the financial system. In this way, the resulting system return portfolios can be considered representative of the SA banking system. This enables one to analyse possible spillover effects between a stressed institution and the financial system. Moreover, this approach rules out any spurious correlation that may be induced by banks that are more heavily represented in the proxy for the composition of the financial system. For example, FNB's market capitalisation currently accounts for about 35 percent of the combined market capitalisation of these 6 banks. So, if the corresponding index is used as a proxy for the financial system, systemic risk estimates generated conditional on FNB will be severely affected by the presence and large-scale factor of FNB in the SA banking system's portfolio proxy. The asset (bank) returns data and market (financial system) returns are based on data that have been de-meaned[9].

7 Brownlees and Engle (2016) state that the 8 percent figure is based on the capital ratio maintained by well managed large US financial institutions in normal times. The NYU's VLAB sets it at 5.5 percent for European banks.

8 It may be the case that not all the equity shares of the sample banks trade actively on all days of the week. Including the inactive stock on a particular day would give rise to non-synchronous data. Non-synchronous price data would complicate the estimation of returns in multivariate time series analysis. This complication can be avoided by considering weekly observations.

9 It is generally more efficient to separate estimation of the mean from volatility estimation, and consequently

In the MES estimation, the market-value-weighted return of all the six banks in the sample is used as a proxy for the banking system. This is because MES measures how exposed an individual bank is to tail-shocks emanating from the banking system as a whole.

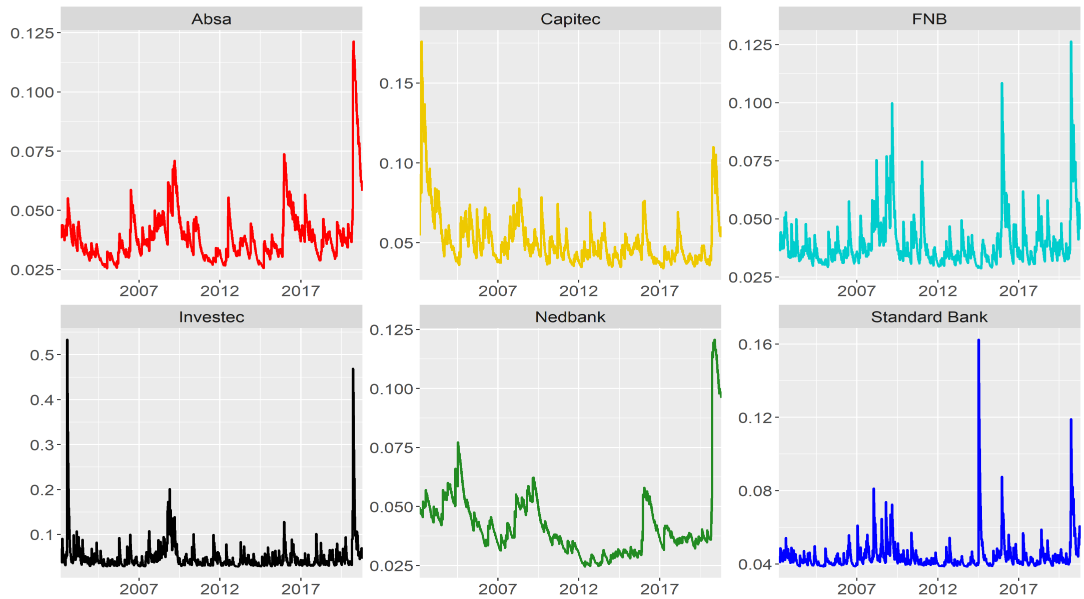
4. Empirical results

In this section we present the empirical results for the conditional volatility of equity returns, CoVaR, MES, and SRISK estimations for each of the six banking institutions.

4.1 Conditional volatility of bank equity returns

The conditional volatility of the time series of bank equity returns is a key input in the estimation of MES. Figure 1 shows the conditional volatility of each institution's equity returns. Conditional volatility (σ_t) is the volatility of a random variable, that is, the equity returns of a bank, $R_{i,t}$, conditioned on some extra information. Asset return volatilities tend to increase with investors' uncertainty about future fundamentals[10].

Figure 1: SA banking sector (Largest listed banks): Conditional volatility of equity returns, 2002-2020



In Figure 1, the initial upsurge in conditional volatility occurred in 2002. Over the period of 2002 up to the first few months of 2003, 22 banks exited the South African banking system. This was an outcome of the contagion subsequent to the imposition of curatorship over Saambou Bank Limited (which was then the fifth largest bank in South Africa) in February 2002. What followed was the takeover of BOE Bank Limited (sixth largest bank in South Africa) by Nedbank, which was a consequence of the strained liquidity environment (Havemann (2018)). Saambou Bank and BOE Bank were viewed by regulators as being systemically important. Both these banks,

the volatility models are based on equity returns that have been *de-meaned* (i.e., the unconditional mean has been subtracted from the returns).

10 In Pastor and Veronesi (2009) volatility increases are caused by a higher "news elasticity" of asset prices when investors' uncertainty about the asset's underlying fundamentals has increased.

as well as some smaller banks, experienced large withdrawals of deposits as confidence in the banking system plummeted, which put downward pressure on banks' share prices.

The second major upsurge in South African banks' conditional volatility occurred during the period of the global financial crises (GFC) in 2007-09. Owing to the relatively more stringent regulatory environment and very limited exposure to overseas wholesale funding markets, the impact of the financial crisis was not as catastrophic for South African banks as it was for banks in the UK and US[11]. Nevertheless, South African banks had to contend with the abrupt drying up of international capital flows that eroded share prices and lead to a depreciation of the rand (Viegi (2008)). While the South African banking system remained relatively stable during the financial crisis, there was a dip in commercial bank's profitability in 2009, amid rising bad debts, curtailment of credit extension, and a progressive decline in domestic demand (SARB (2009)).

In 2014, African Bank experienced liquidity stress and a sharp decline in the share price of its holding company, African Bank Investment Limited (ABIL), generating risk across financial markets. The bank was put under curatorship by the South African Reserve Bank due to its inability to make sufficient provisions for bad debts and engaging in unsustainable lending. However, in the wake of the central bank-led bailout of ABIL, at least 10 SA money market funds "broke the buck"[12], leading to a significant widening of money market spreads. Financial contagion was, however, contained following the imposition of complementary interventions by authorities (Havemann (2018)).

A spike in the conditional volatility of equity returns was experienced by the "Big 4" banks at the end of 2015. This captures the negative economy-wide impacts of the unexpected replacement of South Africa's Minister of Finance, Nhlanhla Nene, by the relatively unknown Desmond van Rooyen by President Jacob Zuma (commonly referred to as, "Nene-gate"). This was, at least partially, mitigated by the appointment of former Minister of Finance, Pravin Gordhan, to the position 4 days later. At the time, this political event exacerbated a negative global sentiment towards emerging markets, resulting in significant capital losses in both the equity and bond markets. According to Hogg (2015), nearly one third of these losses were shared amongst three stocks: FNB, Standard Bank, and Old Mutual. This is reflected in the surge in conditional volatilities for FNB and Standard Bank as depicted in Figure 1.

Over the period of observation, the sharpest rise in the volatility of bank's equity returns was witnessed after the start of the Covid-19 pandemic in March 2020. Conditional volatilities of bank equity returns reached unprecedented levels of between 11-12% during the second quarter of 2020. This was largely driven by the sharp falls in bank equity prices observed at the onset of the pandemic and a deterioration in financial market liquidity[13]. With the support provided by SARB to money markets and interbank markets, bank equity prices went through a gradual process of recovery from June 2020 but remain well below pre-Covid levels. Moreover, South African banks have been exposed to additional aggravating factors.

In South Africa, the government's debt (as a percentage of GDP) has been on an upward

[11] Brink (2009) finds that the direct impact of the GFC on the South African interbank market was modest.

[12] This is a term used to describe a situation where a money market funds' net asset value (NAV) falls below ZAR1. It is considered a signal of financial distress, as money market funds are generally considered to be nearly risk-less investments.

[13] The fall in bank equity prices was observed across most banking systems and was not unique to South Africa. The FTSE 350 Bank Index fell by 40 percent between March-September 2020. The Nasdaq Bank Index fell by 27 percent over the same period.

trajectory for a number of years, culminating in a sovereign debt downgrade by Moody's to sub-investment grade in March 2020. Ratings downgrades have an effect on banks' longer-term funding costs. They may also cause financial stability risks in the banking sector given the large and growing exposure of South African banks to the government.

Following the Covid-19 crisis there has been a surge in capital outflows from South Africa and other emerging market economies (EMEs). Yet, given South African banks' limited dependence on external funding, the overall impact is likely to be modest. Moreover, the South African banking system, at the aggregate level, does not have mismatches between foreign currency assets and liabilities. However, the aggregate level data could obscure FX mismatches facing individual banks. Banks' relative exposure to possible fund outflows or concentration risk to single-name exposures, or particular industries, may exacerbate vulnerabilities.

4.2 ΔCoVaR analysis

The following section estimates, in sequence, the VaR, CoVaR, and ΔCoVaR for each bank.

4.2.1 Estimating CoVaR using quantile regressions

Adrian and Brunnermeier (2011) employ linear quantile regressions regressions to obtain CoVaR estimates. In order to estimate a time-varying conditional CoVaR measure, the authors include systemic state variables that model changes in tail dependence over time, i.e., the time variation in the tails of the joint distribution of $R_{i,t}$ and $R_{s,t}$. In particular, the following quantile regressions are run on weekly data for the institution i and the financial system s. The first step involves predicting the VaR of the asset returns of institution i, $R_{i,t}$, through a linear model of market variables:

$$R_{i,t} = \alpha_i + \gamma_i^{\tau} M_{t-1} + \varepsilon_{i,t} \tag{9}$$

where γ_i^{τ} is the transpose of γ_i, and M_t is a vector of state variables which are able to capture time variation in conditional moments of asset returns. In the context of South Africa, the following state variables can be considered:

(i) The Chicago Board Options Exchange (CBOE) emerging market volatility index (VXEEM[14]) as the implied equity market volatility.

(ii) The weekly change in the 3-month South African Treasury bill (T-bill) rate.

(iii) The change in the slope of the yield curve, measured as the difference between 10-year SA Government bond and 3-month SA T-bill rate.

(iv) The equity market, measured by the weekly FTSE/JSE Top40 Tradeable Index returns.

Figure 2 shows the time series in the lagged state variables. The Covid-19 outbreak resulted in unprecedented volatility in financial markets. VXEEM surged to over 70 on 16th March 2020, surpassing its 2008 record of 55. It has since eased. Likewise, the JSE40 plummeted in March 2020 but has since rebounded. In bond markets, similar fluctuations were observed in the 3-month SA Treasury bill rate, and the slope of the SA yield curve. On 25th March

[14] The CBOE VXEEM is a VIX-style estimate of the expected 30-day volatility of returns on the MSCI Emerging Markets Index (MSCI EEM).

2020 SARB announced measures to ease liquidity strains observed in funding markets that would involve purchasing government securities in the secondary market across the entire yield curve.

Figure 2: South African state variables, 2002-2020

Equation 9 is estimated with the linear quantile regression model developed by Koenker and Bassett Jr (1978) to get the coefficients $(\hat{\alpha}_i, \hat{\gamma}_i)$, with $F^{-1}_{\varepsilon,t}(\tau \mid M_{t-1}) = 0$. The VaR of institution i is then predicted by

$$VaR_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i^{\tau} M_{t-1} \tag{10}$$

Figure 3 shows the VaRs of the banks estimated by equation 10. A bank's VaR is a risk measure representing the loss in asset returns due to the bank being in distress in isolation (see Appendix A for a full description of the VaR measure). The VaR for all 6 banks reached their highest levels during the Covid-19 pandemic. The increases in the VaR have also largely coincided with the surge in volatility of equity returns shown in Figure 1. Figure 4 shows a comparison of VaR across the six banks. However, the question of whether a bank with bigger VaR could mean that it would contribute more marginal risk to the financial system as a whole, is what we investigate in the CoVaR estimation.

In a second step we estimate the financial system return, $R_{s,t}$, as a linear function of the institution's returns, $R_{i,t}$, and market variables, M_t, again using a quantile regression.

$$R_{s,t} = \alpha_{s|i} + \beta_{s|i} R_{i,t} + \gamma_{s|i}^{\tau} M_{t-1} + \varepsilon_{s,t} \tag{11}$$

The coefficient $\beta_{s|i}$ in equation 11 can be interpreted as a standard linear regression coefficient, i.e. it determines the sensitivity of the equity return of the financial system s to changes in equity returns of an institution i when they are at their VaR. In the final step, the CoVaR is

Figure 3: SA banking sector (Largest listed banks): VaR of bank asset returns, 2002-2020

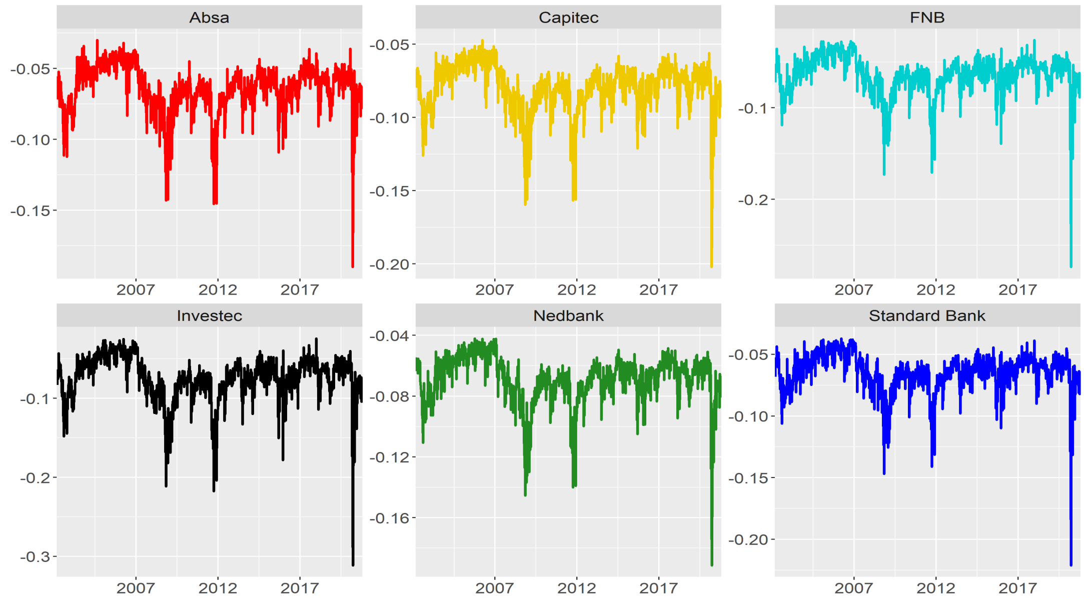
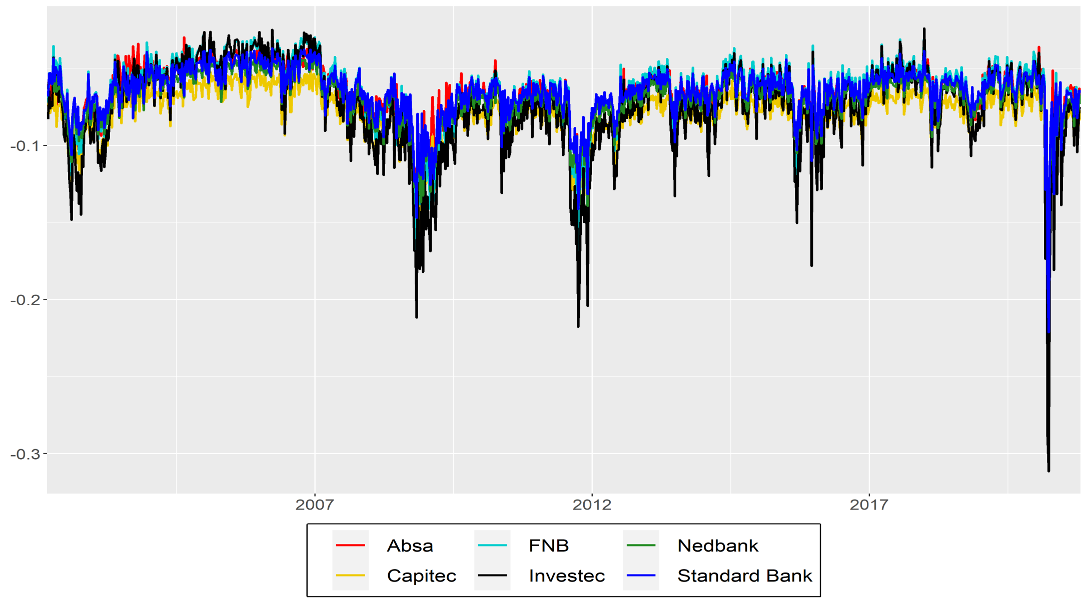


Figure 4: SA banking sector (Largest listed banks): VaR across banks, 2002-2020



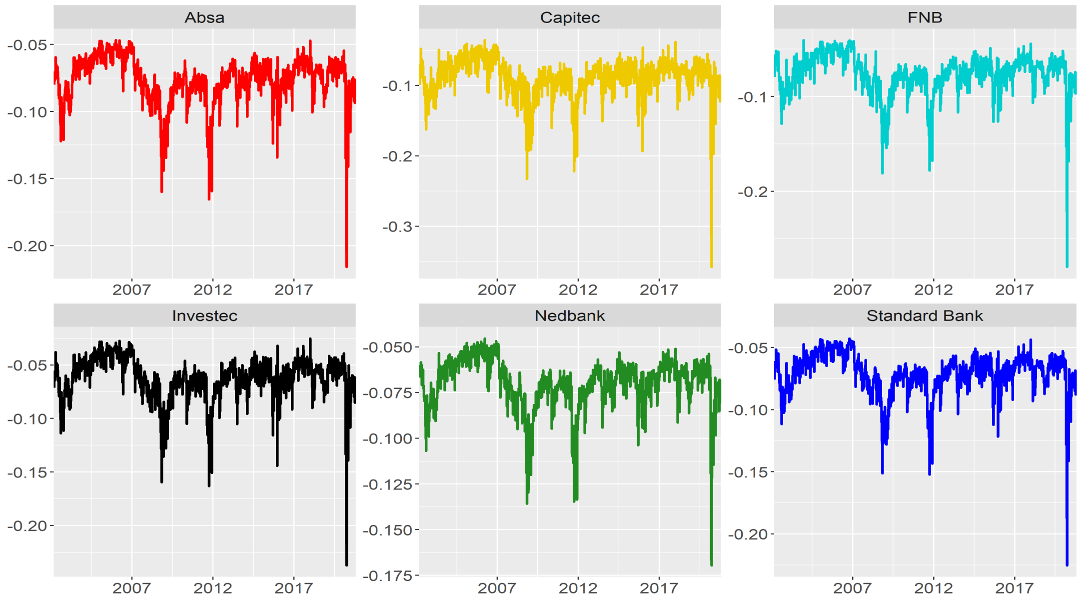
calculated by plugging in the VaR of institution i estimated in equation 10 into equation 12.

$$CoVaR_{s|i,t} = \hat{\alpha}_{s|i} + \hat{\beta}_{s|i} VaR_{i,t} + \hat{\gamma_{s|i}}^{\tau} M_{t-1} \tag{12}$$

Therefore, the risk of the financial system s is calculated on the basis of the VaR of an institution i and the market variables. Here the coefficient $\hat{\beta}_{s|i}$ in equation 12 reflects the degree of interconnectedness between the financial system and the financial institution. The value of the coefficients $(\hat{\alpha}_{s|i}, \hat{\beta}_{s|i}, \hat{\gamma}_{s|i})$ are shown in Table 7 in Appendix A. Figure 5 shows the CoVaR for the six banks. During the Covid-19 pandemic CoVaR levels across all the sample banks reached levels not witnessed in any previous crises.

The ΔCoVaR of bank i is then defined as the difference between the VaR of the financial sys-

Figure 5: SA banking sector (Largest listed banks): CoVaR in individual banks, 2002-2020



tem conditional on this particular bank being in financial distress, and the VaR of the financial system conditional on i being in its median state.

$$\Delta CoVaR_{i,t} = \left(CoVaR_t^{s|R_{i,t}=VaR_{i,t}(\alpha)} - CoVaR_t^{s|R_{i,t}=Median(R_{i,t})} \right) \tag{13}$$

Figure 6: SA banking sector (Largest listed banks): Individual ΔCoVaR estimates, 2002-2020

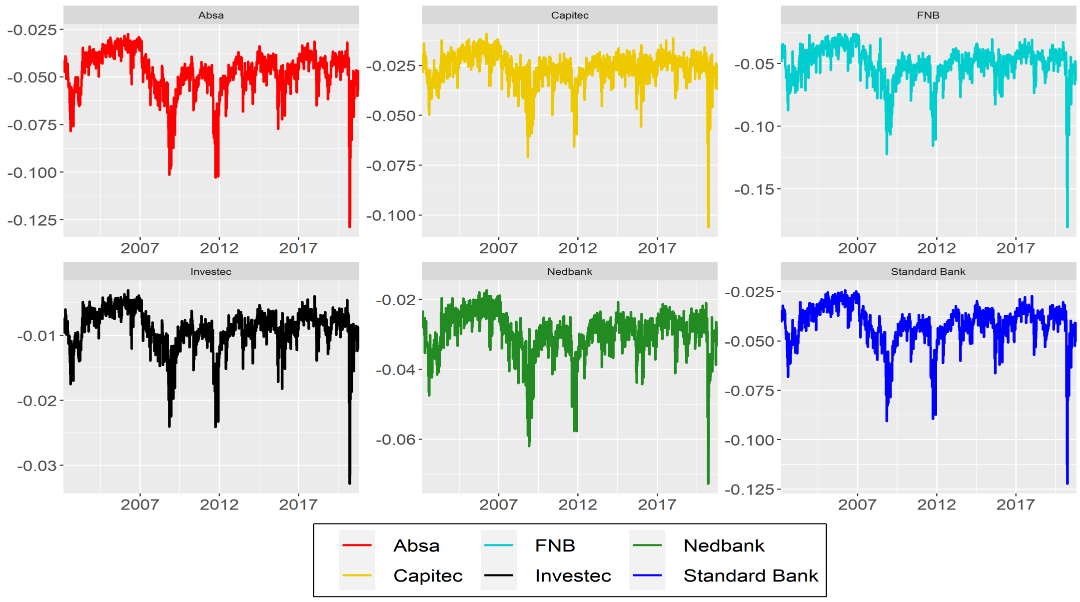


Table 1 shows the VaR of the six banks alongside their respective ΔCoVaRs. The smaller banks, Capitec and Investec, have typically reported the highest VaR in recent years. But the larger banks, FNB followed by Absa and Standard Bank, are the most systemically important on the basis of their respective ΔCoVaRs. It follows that banks with the highest VaR would not necessarily have the highest CoVaR or ΔCoVaR. A plausible explanation is that VaR is not the only risk that gets transmitted to the wider financial system.

Figure 7: SA banking sector (Largest listed banks): ΔCoVaR estimates, 2002-2020

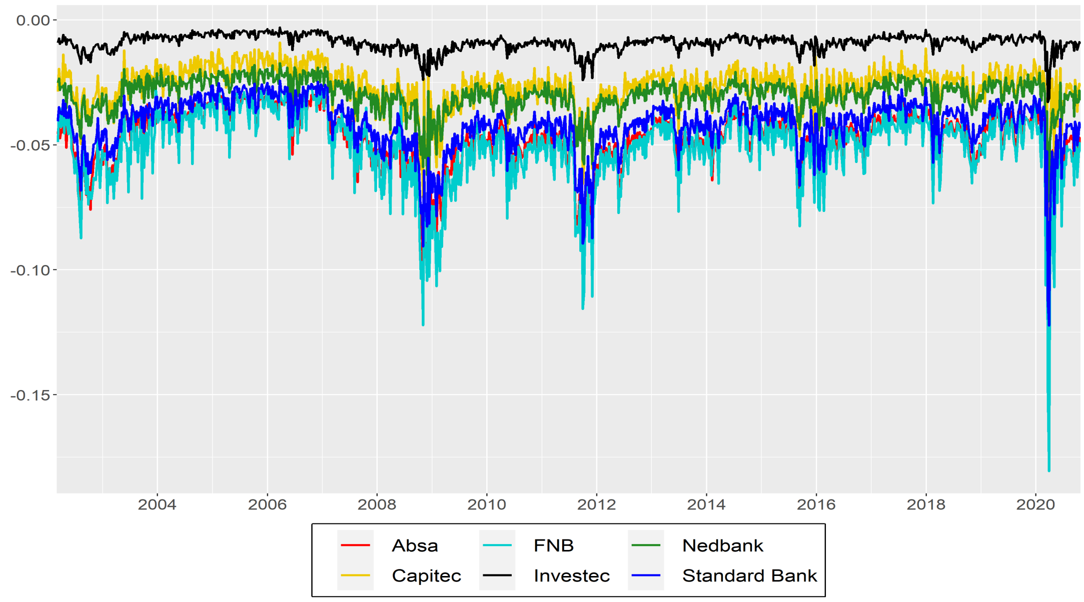


Table 1: SA banking sector (Largest listed banks): Ranking of banks by average VaR and ΔCoVaR estimates, 2002-2020

Institution	Average VaR	Average ΔCoVaR
Absa	-0.0649	-0.0479
Capitec	-0.0799	-0.0266
FNB	-0.0647	-0.0507
Investec	-0.0752	-0.0091
Nedbank	-0.0693	-0.0299
Standard Bank	-0.0652	-0.0418

4.3 MES analysis

We implement the dynamic version of MES proposed by Brownlees and Engle (2011), which is based on the estimation of GARCH volatilities and dynamic conditional correlations of individual bank equity returns with the equity returns of the banking system.

Following Brownlees and Engle (2011) we consider the bivariate DCC-GARCH process for the returns:

$$R_t = H_t^{\frac{1}{2}} v_t \tag{14}$$

where $R_t = \left(R_t^s, R_t^i\right)'$ is the (2 x 1) vector of system and individual bank returns, and the random vector $v_t = \left(\varepsilon_t^s, \xi_t^i\right)'$ is independent and identically distributed, and where $\mathbb{E}(v_t) = 0$

and $\mathbb{E}(v_t v_t^{'}) = I$, a two-by-two identity matrix. H_t is the (2 x 2) conditional covariance matrix:

$$H_t = \begin{pmatrix} \sigma_{s,t}^2 & \rho_{si,t}\sigma_{s,t}\sigma_{i,t} \\ \rho_{is,t}\sigma_{i,t}\sigma_{s,t} & \sigma_{i,t}^2 \end{pmatrix}, \tag{15}$$

where $\sigma_{s,t}$ and $\sigma_{i,t}$ are the conditional standard deviations of the equity returns of the banking system and the individual institution at time t, and $\rho_{si,t}$ is the conditional correlation at time t between R_t^s and R_t^i. $\sigma_{s,t}^2$ and $\sigma_{i,t}^2$ are the conditional volatilities.

The time-varying cross-correlations between a bank and the banking system highlight the systemic dimension of stress. To illustrate this, we compare the conditional covariance matrix for the six banks on 29 March 2019 and 27 March 2020 in Tables 2 and 3.

Table 2: SA banking sector (Largest listed banks): Conditional covariance matrix, 29 March 2019

$$H_{t,\text{ Absa}} = \begin{pmatrix} 0.0015 & 0.0014 \\ 0.0014 & 0.0020 \end{pmatrix} \qquad H_{t,\text{ Capitec}} = \begin{pmatrix} 0.0015 & 0.0010 \\ 0.0010 & 0.0017 \end{pmatrix}$$
$$H_{t,\text{ FNB}} = \begin{pmatrix} 0.0015 & 0.0014 \\ 0.0014 & 0.0017 \end{pmatrix} \qquad H_{t,\text{ Investec}} = \begin{pmatrix} 0.0015 & 0.0019 \\ 0.0019 & 0.0065 \end{pmatrix}$$
$$H_{t,\text{ Nedbank}} = \begin{pmatrix} 0.0015 & 0.0011 \\ 0.0011 & 0.0013 \end{pmatrix} \qquad H_{t,\text{ Standard Bank}} = \begin{pmatrix} 0.0016 & 0.0015 \\ 0.0015 & 0.0021 \end{pmatrix}$$

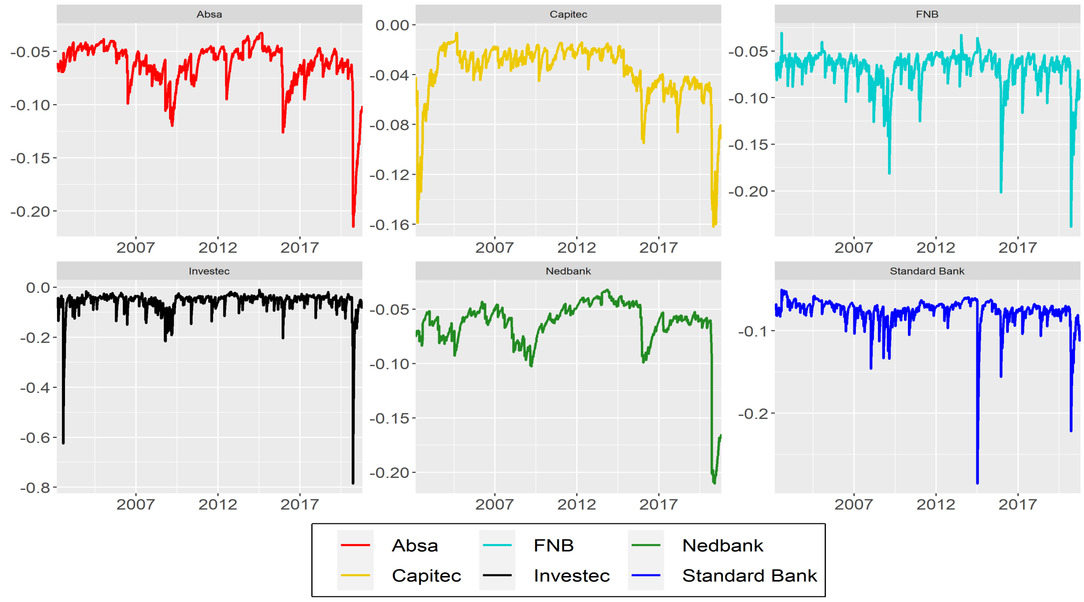
Table 3: SA banking sector (Largest listed banks): Conditional covariance matrix, 27 March 2020

$$H_{t,\text{ Absa}} = \begin{pmatrix} 0.0237 & 0.0158 \\ 0.0158 & 0.0130 \end{pmatrix} \qquad H_{t,\text{ Capitec}} = \begin{pmatrix} 0.0237 & 0.0112 \\ 0.0112 & 0.0091 \end{pmatrix}$$
$$H_{t,\text{ FNB}} = \begin{pmatrix} 0.0237 & 0.0183 \\ 0.0183 & 0.0160 \end{pmatrix} \qquad H_{t,\text{ Investec}} = \begin{pmatrix} 0.0237 & 0.0575 \\ 0.0575 & 0.2200 \end{pmatrix}$$
$$H_{t,\text{ Nedbank}} = \begin{pmatrix} 0.0237 & 0.0151 \\ 0.0151 & 0.0132 \end{pmatrix} \qquad H_{t,\text{ Standard Bank}} = \begin{pmatrix} 0.0307 & 0.0182 \\ 0.0182 & 0.0142 \end{pmatrix}$$

Conditional volatilities of equity returns and correlations between the bank specific returns and banking system returns increase significantly during periods of stress. 29 March 2019 could be described as part of a relatively tranquil period during which the conditional volatility of equity returns (given by the diagonal elements of the H_t matrix) and the correlations (given by the off-diagonal elements) remained low. However, during the Covid-19 pandemic on 27 March 2019 both these volatilities and correlations increased multiple times and reached peak levels.

Figure 8 plots the MES of the 6 banks. Almost all banks witnessed upward movements in 2002, 2008-10, end-2015, and March-June 2020. By far, the most significant upsurge was on account of the fallout from the Covid-19 pandemic. This can be understood from the very definition of MES which is the expected loss in an institution when the financial system suffers an adverse shock. The Covid-19 pandemic is a major shock to the economy but unlike traditional financial crisis, its origin is exogenous to the financial system. The distinguishing feature of all major financial crises is that they gather momentum from the endogenous responses of the financial institutions themselves[15]. MES measures how exposed an individual bank is to tail shocks within the system. In terms of its causality, MES is different from CoVaR. MES measures the impact of distress in the banking system on a bank, rather than the other way round. Figure 9 compares MES across 5 banks, excluding Investec[16].

Figure 8: SA banking sector (Largest listed banks): MES of individual banks, 2002-2020



The difference between systemic risk (danger of the entire financial system collapsing) and systematic risk (the exposure to common market factors) is well documented[17]. Nevertheless, it is usually worth investigating whether higher systematic risk would imply an increase in systemic risk. Benoit et al. (2013) have provided a theoretical proof of the proposition that the MES of a given financial institution i is proportional to its systematic risk as measured by its time-varying beta. The proportionality coefficient is the expected shortfall of the market:

$$MES_{i,t}(\alpha) = \beta_{i,t} ES_{s,t}(\alpha) \tag{16}$$

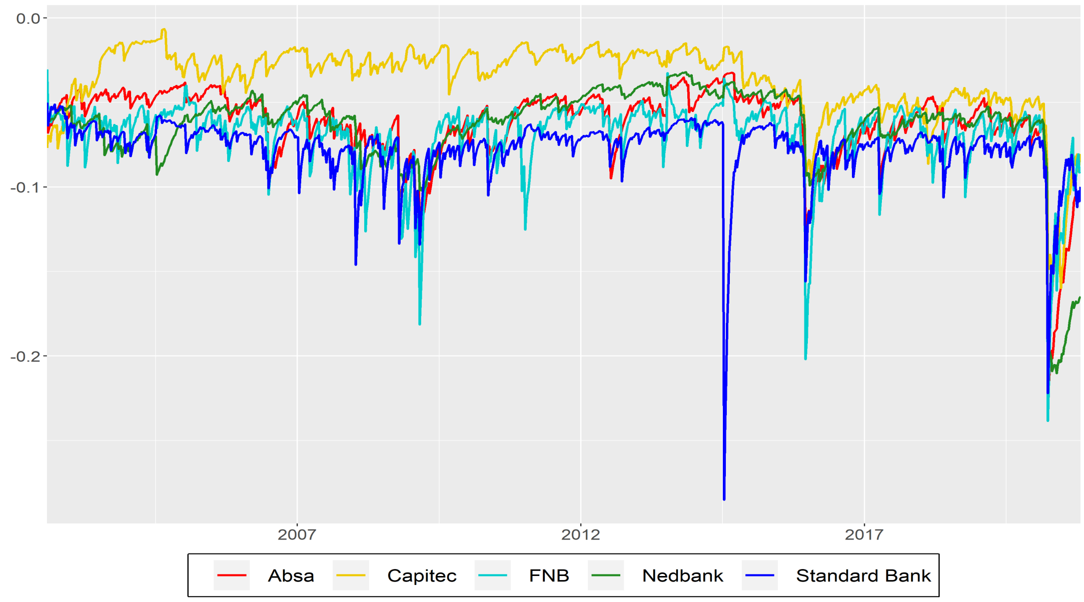
where $\beta_{i,t} = \frac{cov(R_{i,t}, R_{s,t})}{var(R_{s,t})} = \frac{\rho_{i,t}\sigma_{i,t}}{\sigma_{s,t}}$ denotes the time-varying beta of bank i and $ES_{s,t}(\alpha)$ is the expected shortfall of the market. If this were to be the case then ranking banks according to their MES (a measure of systemic risk) should be equivalent to sorting banks according to their market betas (a measure of systematic risk). Since the ES of the banking system is not bank-specific, the greater the sensitivity of the bank's equity return to the banking system's

15 Danielsson and Shin (2003) propose the term "endogenous risk" for risk from shocks that are generated and amplified within the financial system.

16 Investec's market capitalisation has been subjected to sudden downward movements which, although transitory, have been reflected in sharp momentary increments in its systemic risk indicators. Investec's market capitalisation fell sharply for a day in July 2002 resulting in its MES rising to 0.6 in the week of 26 July 2002. Investec market capitalisation recovered within a week. The MES returned to levels below 0.1 by the end of September 2002. There was another sharp fall in Investec's market capitalisation in the week of 27 March 2020 resulting in its MES rising to as much as 0.8. Investec's MES reverted to levels below 0.1 by July 2020. For the other five banks MES never exceeded 0.3 over the entire period of observation. Investec is excluded from the MES comparison chart in order maintain a comparable scale for the other five banks.

17 Benoit et al. (2017) create a framework that demonstrates the distinction between systemic risk and systematic risk.

Figure 9: SA banking sector (Largest listed banks): MES across banks, 2002-2020



equity return, the more systemically risky the bank is. However, for a given bank, the time profile of its systemic risk measured by its MES may be different from the evolution of its systematic risk measured by its conditional beta. Since the banking system ES may not be constant over time, forecasting the systematic risk of a bank may not be sufficient to forecast the future evolution of its contribution to systemic risk.

We examined this proposition in the context of South African banks. Figure 10 plots the relationship between bank betas and MES for each of the six banks. The scatter plot shows that there is positive relationship between MES and bank beta, which implies that the systemic risk rankings of financial institutions based on their MES should broadly conform to the rankings obtained by sorting banks on betas. However, there is noise in the relationship between beta and MES that could be partially explained by the time-varying nature of ES. Figure 11 shows the equity returns of the SA banking system and the time-varying ES corresponding to a VaR confidence level of 95%[18].

However, the time-series average of the MES for each bank and its beta suggests a cross-sectional relationship between the two measures. This is shown in the scatter plot in Figure 12. Each point represents one of the six banks listed in Table 4. The beta corresponds to the average of the time-varying beta, β_{it}, in equation 16. The dashed line in Figure 12 is the OLS regression line with no constant term. The outliers observed in the chart correspond to the two smaller banks – Capitec and Investec – both with relatively low betas.

[18] VaR and ES must be estimated together because the ES estimate depends on the VaR estimate. VaR and ES have been estimated using the non-parametric historical simulation approach. ES of the SA banking system reached its highest ever level in March-April 2020 during the onset of the Covid-19 pandemic and remains elevated. Each bank's contribution to the ES can be measured by its MES.

Figure 10: SA banking sector (Largest listed banks): Relationship between bank beta and MES, 2002-2020

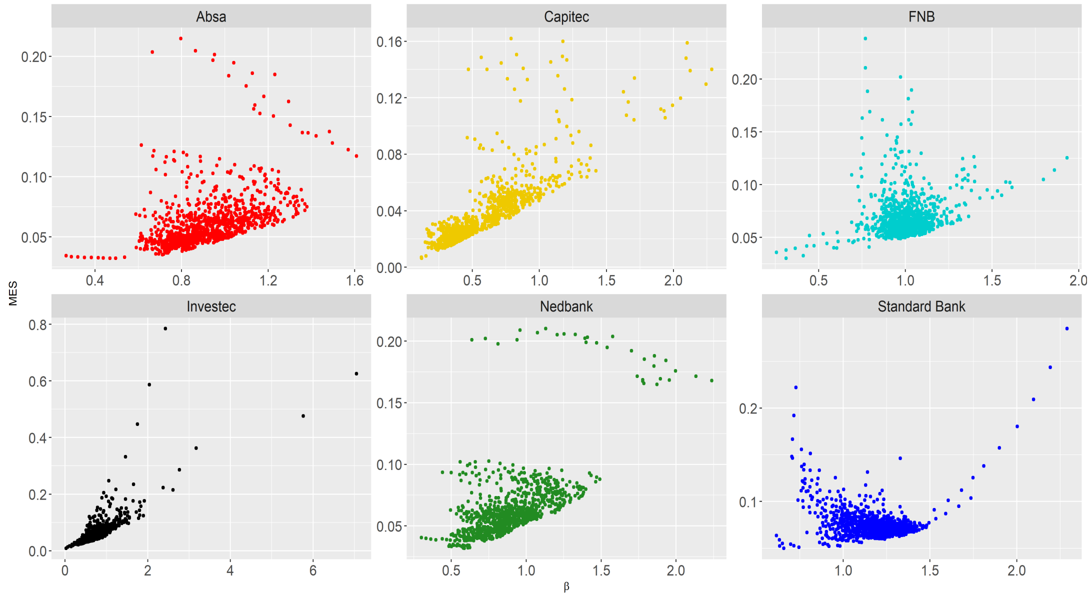


Figure 11: SA banking sector (Largest listed banks): VaR and Expected Shortfall, 2002-2020

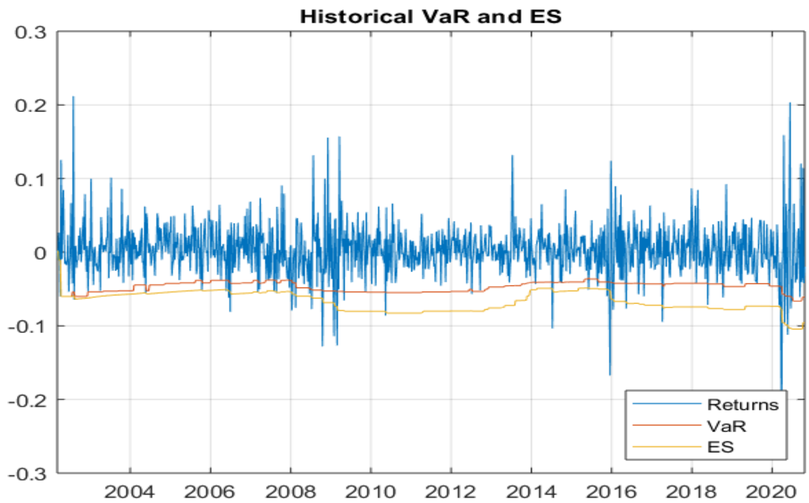
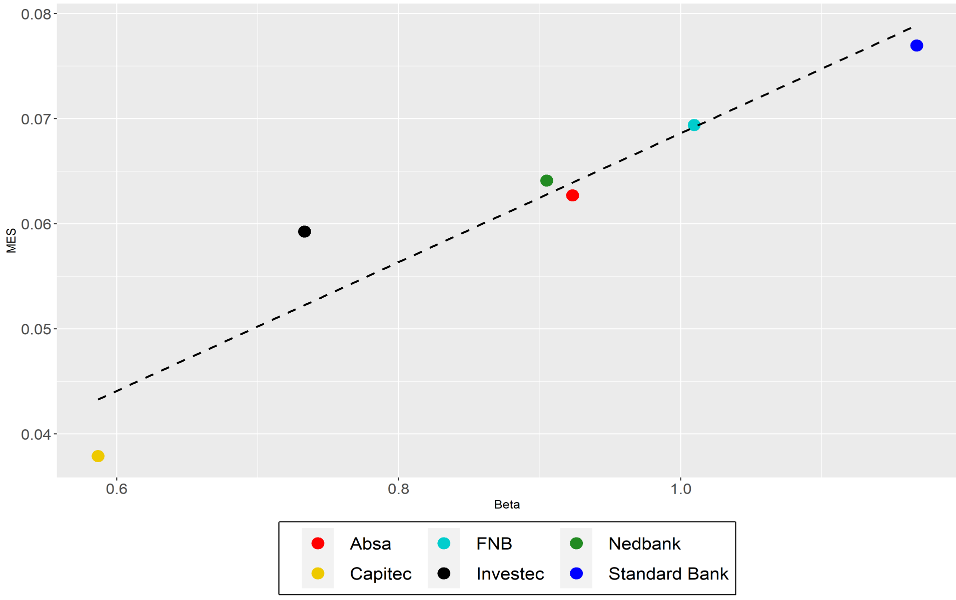


Table 4: SA banking sector (Largest listed banks): MES and firm beta

Institution	Bank β	MES
Absa	0.9233	0.0627
Capitec	0.5866	0.0379
FNB	1.0096	0.0694
Investec	0.7330	0.0592
Nedbank	0.9049	0.0641
Standard Bank	1.1674	0.0770

Figure 12: SA banking sector (Largest listed banks): Relationship between average bank betas and MES, 2002-2020



4.4 SRISK analysis

To arrive at the SRISK measure we first estimate the LRMES using equation 6 in Section 2.3. We then estimate SRISK for an institution i using the following equation proposed by Brownlees and Engle (2016):

$$SRISK_{i,t} = k_t BD_{i,t} - (1 - k_t) ME_{i,t} (1 - LRMES_{i,t}) \tag{17}$$

SRISK measures the capital shortfall a financial institution is expected to experience conditional on a systemic event. $SRISK_{i,t}$ is the capital shortfall of bank i in week t. Figure 13 plots the evolution of SRISK across the six banks over the period of observation. It shows that the level of risk in the banking sector reached a historic all-time high during the Covid-19 crisis.

As described in Brownlees and Engle (2016), the $SRISK_{i,t}$ measure can be used across all the six banks to construct a system wide measure of financial distress:

$$SRISK_t = \sum_{i=1}^{N} SRISK_{i,t} \tag{18}$$

Figure 14 shows the contribution of the six banks to aggregate SRISK. From these figures, it can be seen that Standard Bank – the bank with the largest total liabilities – contributes the most to systemic risk on average, approaching 80% in certain periods. Notably, however, this contribution is more evenly spread across institutions during the crisis periods of 2008-2010, late 2015, and early 2020, highlighting the systemic nature of the risk this metric is attempting to capture. Unlike the previous measures, Investec and Capitec hardly ever contribute to aggregate SRISK, reflecting their significantly smaller balance sheets. SRISK of a bank is an increasing function of the level of debt, as indicated by its total liabilities, and a decreasing function of its market capitalisation.

Figure 13: SA banking sector (Largest listed banks): Estimated SRISK for the South African banking system, 2002-2020

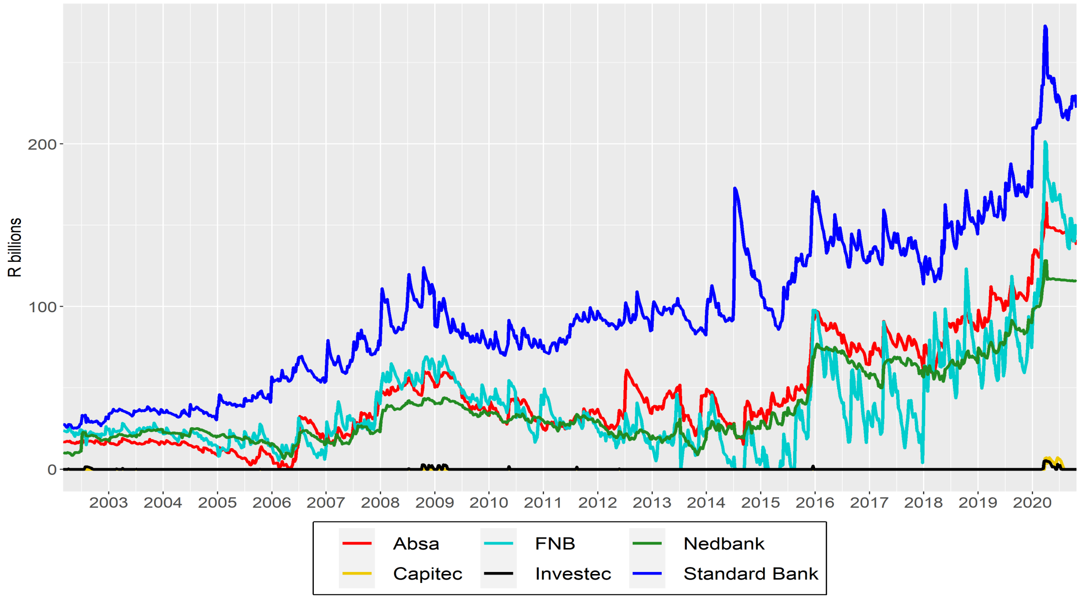
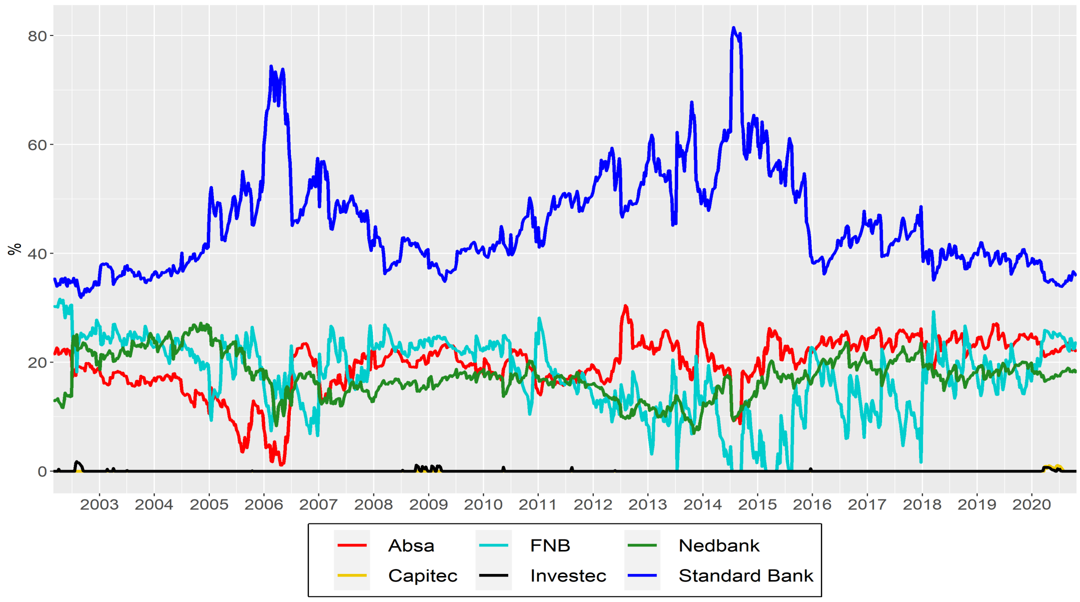


Figure 14: SA banking sector (Largest listed banks): Contribution to aggregate SRISK, 2002-2020



The decomposition of ΔSRISK in Table 5 according to equation 8, shows that, on average, the debt dynamics component contributes the most to increases in SRISK for all of the major banks during the period under consideration. This is much more pronounced for the Big 4 banks, and for Standard Bank in particular. FNB, the bank with the largest market capitalisation, clearly benefits from the average increase in the market value of equity that it has experienced over 2002-2020. However, over this same period Capitec, which has grown to be the biggest bank by number of customers, and has a share price that trades at a significant premium to the other banks in the market, has seen the greatest reduction in SRISK from growing market equity.

Table 5: SA banking sector (Largest listed banks): Average contribution to ΔSRISK, 2002-2020

Institution	Δk	ΔDebt	ΔEquity	ΔRisk
ABSA	7.5310	126.5762	-16.2814	13.7606
Capitec	0.5174	12.9456	-55.1359	36.9254
FNB	6.7165	153.1194	-50.7599	7.7138
Investec	1.0582	2.8786	-5.3115	-3.4907
Nedbank	6.9184	98.0942	-8.5830	12.9863
Standard	13.2830	210.4714	-31.9029	1.9583

4.5 A comparison of systemic risk measures

Table 6 below provides a comparison of the six banks in terms of their contributions to systemic risk based on their respective ΔCoVaR, MES, and SRISK measures.

The different systemic risk measures analysed in this paper have been formulated using very different estimation frameworks. So, it is not surprising that their outcomes differ when it comes to identifying the bank that contributes most to systemic risk in the South African banking system. The ΔCoVaR measure is conditional on a particular bank being in distress and thus varies across the panel of banks. This could impact the ranking of banks based on systemic importance. In this regard, MES facilities the ranking of banks on their systemic importance as the conditioning set is held constant for all banks (the banking system equity returns are below its VaR). The ΔCoVaR measure identifies FNB as the most systemically important bank. This could be partly driven by the fact that FNB has the largest market capitalisation relative to its liabilities, whereby distress for such a bank would have wider ramifications across the banking system. According to the MES, Standard Bank is the most systemically important bank. This could be partly ascribed to its high level of systematic risk as given by its market beta. So, a tail shock to the banking system would impinge more on Standard Bank given its greater interconnectedness with the system as a whole.

It is not possible to observe the actual systemic footprint of a bank. The systemic impact of a bank's failure can only be ascertained once it has defaulted and even then it would be difficult to isolate it from other events. When viewed from this perspective the intuition underlying the ΔCoVaR measure is what happens to the remaining banking system if a bank fails. This definition is similar, in principle, to the loss given default. Thus the systemic risk attributable to a bank is linked to the losses it can cause by some contagion mechanism determined by its default. Moreover, the contribution to systemic risk can be identified as the change in systemic risk when the bank gets into distress rather than when it is added to the system. The MES is a marginal risk measure that is determined by the change in the ES of the system when a bank gains more weight within it. SRISK may be viewed as following an eclectic approach, whereby it is based on the marginal measure of interconnectedness between equity returns, through the MES, but also on nominal values such as market capitalisation and liabilities. However, the measures are in agreement when suggesting that Capitec and Investec are the least systemically important banks in the system over the period 2002-2020. This is likely to change going forward, as Capitec continues to grow into a dominant player in the market.

Table 6: SA banking sector (Largest listed banks): Comparison of banks by average ΔCoVaR, MES, and contribution to SRISK, 2002-2020

Institution	Δ CoVaR	MES	SRISK contribution
Absa	-0.0479	-0.0627	19.5499
Capitec	-0.0266	-0.0379	0.0133
FNB	**-0.0507**	-0.0694	17.5325
Investec	-0.0091	-0.0593	0.0326
Nedbank	-0.0299	-0.0641	17.4438
Standard Bank	-0.0418	**-0.0770**	**45.4279**

An advantage of measures using market information – ΔCoVaR, MES, and SRISK - is that they provide a forward looking and real time view since market prices are quick to reflect the changing expectations of market participants. In contrast, most accounting-based indicators are backward-looking and released with a significant time lag. That being said, market-based measures also have their disadvantages. As a result of random signals and uncertainties in the methods used to estimate them, market-based indicators may provide noisy, and thus potentially misleading, signals.

Löffler and Raupach (2018) find the ΔCoVaR measure responds to idiosyncratic risk[19] in an ambiguous way. When applied in regulation, the use of ΔCoVaR would create incentives for banks to increase idiosyncratic risk in order to lower their estimated systemic risk contribution.

The financial institutions that made the top 5 MES rankings in Acharya et al. (2010), between June 2006 and June 2007, were apart from Bear Stearns, not those bailed out during the GFC. Banulescu and Dumitrescu (2015) state that this is because MES clearly privileges the "Too Connected To Fail" logic rather than the "Too Big To Fail". Most of the shortcomings of MES also apply to SRISK.

Notwithstanding these empirical anomalies, there are clear advantages of estimating market-based measures of systemic risk. Market-based measures of systemic risk can effectively complement prudential reporting as they are not under the direct control of banks.

5. Macro measures of systemic risk and their relationship to Growth-at-Risk

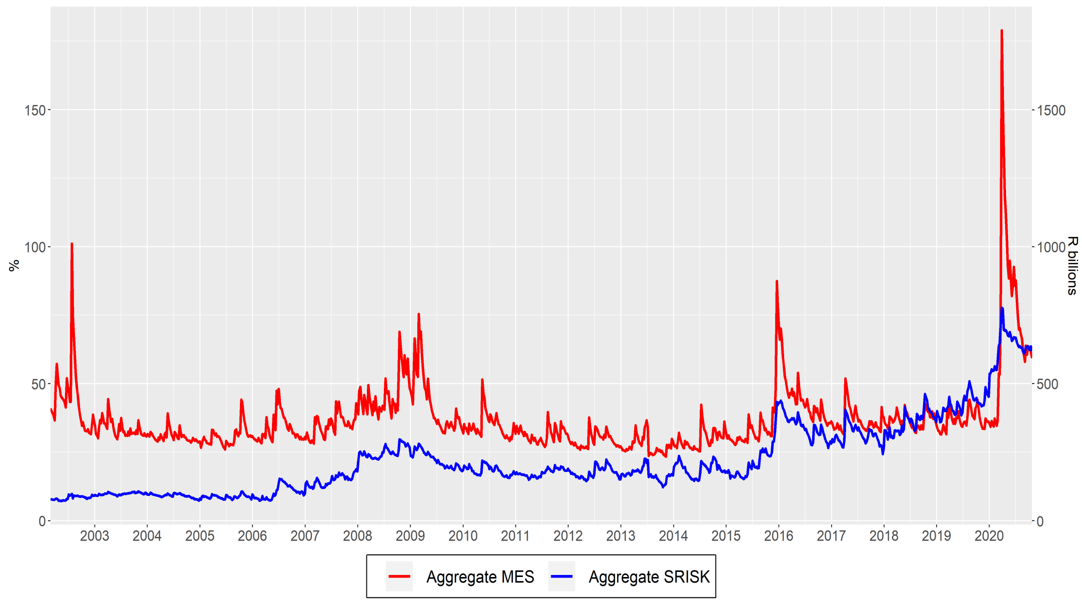
In this section, we assess the relationship between our measures of systemic risk and the projected tails of the distribution of South African GDP growth. In doing so, we view this downside risk to GDP growth as a macroprudential policy issue. We define "growth-at-risk" as the 5^{th} percentile of the GDP growth rate. This implies there is no more than a 1-in-20 chance of experiencing a worse decline in GDP than this. We calculate aggregate MES and SRISK for the South African banking system. CoVaR is a measure of tail interdependence as it assesses the severity of systemic stress conditional on individual institutions being in distress.

[19] Idiosyncratic risk refers to the risk of individual bank failure. Williams and Fenech (2018) argue that systemic risk is analogous to the domino effect of a series of interconnected failures started by the fall of a single domino. The first domino falling (failure of a single bank) is a reflection of idiosyncratic risk while the collapse of the subsequent dominos reflects systemic risk.

It may be regarded as a bottom-up approach in contrast to the more top-down approaches of MES and SRISK. As CoVaR conditions on individual institution distress, it delivers systemic risk outcomes that do not add up to system-wide risk. MES has an additive property[20] and aggregating MES across banks in the system would give a measure of total systemic risk. In our context, aggregate SRISK can be interpreted as the total amount of capital the government would have to provide to bail out the banking system conditional on a financial crisis[21].

The results are plotted in Figure 15, where aggregate MES is measured on the left-hand scale (%) and aggregate SRISK is measured on the right-hand scale (R billions). It shows a broadly common movement across both measures, albeit to varying degrees of intensity. The GFC, "Nene-gate", and the Covid-19 pandemic are clearly highlighted, and the aggregate MES measure captures the mini-banking crisis of 2002.

Figure 15: SA banking sector (Largest listed banks): Macroprudential measures of systemic risk, 2002-2020



5.1 Growth-at-Risk (GaR)

Following the global financial crisis (GFC), the mandates of many central banks around the world, South Africa included, have been expanded from a primary focus on price stability to include financial stability. However, the monitoring and modelling required for these two interlinked, but distinct, objectives can often be delineated by the types of events they are guarding against. Systemic risk is considered to be the risk that a single event could result in

20 Shapley values proposed by Tarashev et al. (2016), as a methodology for systemic risk attribution, also possess this additive property.

21 Tavolaro et al. (2014) discuss some of SRISK's limitations as a supervisory tool. They argue that SRISK is not suitable for aggregating risks in a particular jurisdiction owing to the use of the max operator, $max(0,X)$, in the estimation process. Therefore, adding the SRISK of the institutions of a country does not reflect the SRISK of all the institutions consolidated in a single one. This study argues, however, that it still represents the government's total liability assuming that it would choose to bail-out all of the failing banks. It maintains the assumption that bank liabilities could not be renegotiated or financed via alternative avenues (Brownlees and Engle, 2016), and that the government would not force any institutions with capital surpluses to bail-out those with shortfalls.

system-wide financial instability, with severe impacts on real economy outcomes. Therefore, one aspect of ensuring financial stability is concerned with measuring and monitoring the tail-risks of a given distribution. This is in contrast to the high probability scenarios considered when pursuing price stability (mean/median projections).

As a result of this, recent developments by Adrian et al. (2016), and an extension in Adrian et al. (2018), have considered the full distribution and term structure of future growth, conditional on prevailing economic and financial conditions, with a particular focus on the 5^{th} percentile (or negative, left-tail) of the distribution[22]. They measure prevailing financial conditions using a Financial Conditions Index (FCI), as proposed by Koop and Korobilis (2014). The analysis in this paper is driven by different considerations. Rather than assess broader financial conditions, our aim is to understand the extent to which increased risks to financial stability, as measured via our systemic risk measures, creates downside risks for South African GDP growth, and if so, over what horizon.

In equation 19 we estimate the conditional predicted distribution for GDP growth using a quantile regression. We denote $Growth_t + i$ as the annualised average growth rate for South Africa between t and $t+i$, and $Risk_t$ is a vector of conditioning variables which in our framework represent a measure of systemic risk - MES or SRISK. We then define growth-at-risk (GaR), the VaR of future GDP growth, by

$$Pr(Growth_{t+i} \leq GaR_i(\alpha|\Omega_t) = \alpha \tag{19}$$

where $GaR_i(\alpha|\Omega_t)$ is growth-at-risk for South Africa in i quarters in the future at an α probability. GaR is implicitly defined by the expected growth rate for a given probability alpha (α) between periods t and $t+i$, given Ω_t (the information set available at time t). For a low value of α, GaR will capture the expected growth at the lower end of the GDP growth distribution. That is, there is α percent probability that growth would be lower than GaR. In line with Adrian et al. (2016), GaR has been defined to be the lower 5^{th} percentile of the GDP growth distribution.

As discussed throughout this paper, there is no single measure of systemic risk that could be deemed to be the most effective in all crisis situations. We, therefore, employ both measures – aggregate MES and SRISK – as proxies for current financial conditions, and estimate equation 20 at the 5^{th}, 25^{th}, 50^{th}, 75^{th}, and 95^{th} percentiles[23].

$$Growth_{t+i} = \beta_0 + \beta_1 Risk_t + \beta_2 Growth_t \qquad \text{for i = 1, 2, .., 12} \tag{20}$$

The left-hand side of equation 20, $Growth_{t+i}$, is the average annualised growth rate of real GDP in South Africa over i horizons at time $t+i$ for horizons $i = 1, 2, ..12$ quarters. The variable $Risk_t$ is our systemic risk metric – either aggregate MES or SRISK – measured at time t.

Goodness-of-fit results[24] for these models are presented in Figure 16. The graph plots the goodness-of-fit of the model described by equation 20 by separately using both of the aggre-

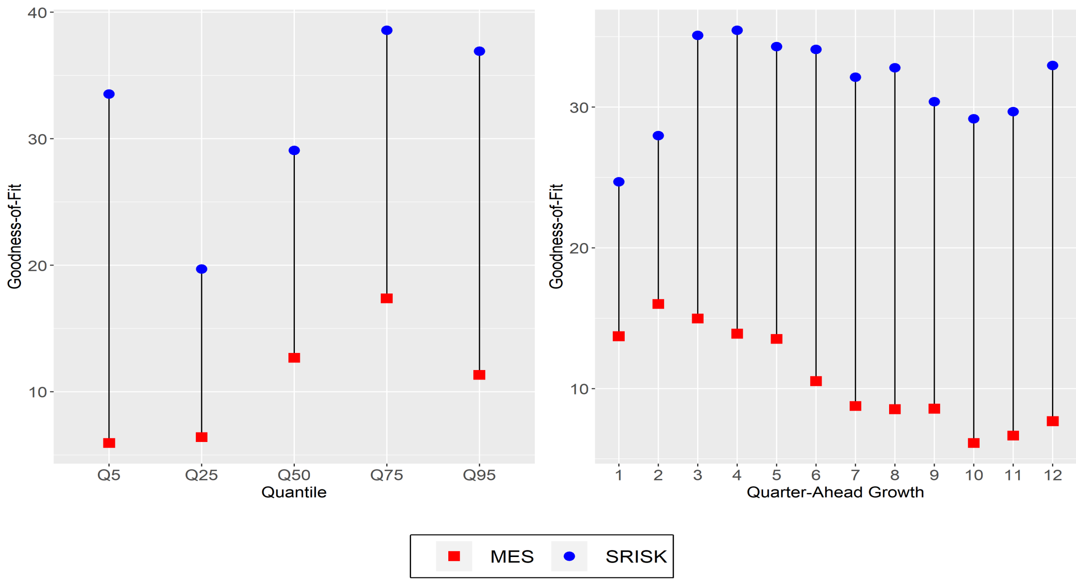
22 The term growth-at-risk was first used by Wang and Yao (1999) who constructed unconditional growth distributions for 84 countries covering the period 1980 to 1998.

23 All results are available from the authors on request.

24 These quantile regression statistics are analogous to an R-squared value in an ordinary least squares regression. They are tabulated in Table 9 in Appendix C.

gate measures of systemic risk as regressors. We can infer that the higher the goodness-of-fit, the better the predictive ability of that particular systemic risk measure. At all quantiles and all time horizons, the models fit best for the SRISK measure, suggesting that SRISK is the best predictor of future economic growth, and more specifically, of future growth-at-risk. However, it should be borne in mind that this performance ranking is a result of the type of systemic risk that has impacted the South African economy historically, and does not imply that SRISK will necessarily be the best measure to use as an early-warning signal for policy makers going forward.

Figure 16: SA banking sector (Largest listed banks): Goodness-of-fit measures, 2002-2020

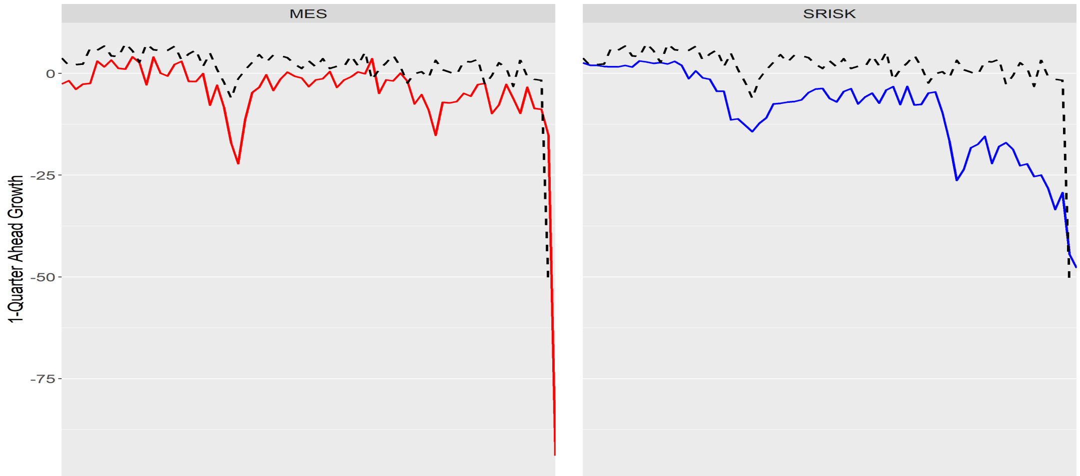


On average, the measures produce better-fitting models at the upper quantiles[25], and in the short- to medium-term (1 to 6-quarter ahead horizon). While it would be preferable for the risk measures to have had the strongest explanatory power at the left-tail of the future growth distribution, at least the goodness-of-fit measures look promising for a policy-relevant time horizon. This is especially true for South Africa, where the recent Financial Sector Regulation Act (FSRA, 2017), explicitly states the need for the South African Reserve Bank to identify and assess the risks to financial stability for (at least) the next 12 months.

The predictions of growth-at-risk and actual 1-quarter ahead growth (shown in black) are depicted in Figure 17, where actual GDP growth is represented by the dotted black line and the remaining lines represent the predicted growth-at-risk associated with each of the three aggregate systemic risk measures. It can be seen that both measures clearly indicated a greater 1-quarter ahead tail-risk leading up to the GFC. However, both measures also issued warnings of significantly increased riskiness in growth following “Nene-gate”, which ultimately did not materialise. This may have something to do with the transitory nature of the shock, as compared to the more sustained stress associated with the GFC or Covid-19. However, it could have led policy makers to tighten their macroprudential policy stance, with a likely cost to economic growth. This emphasizes the importance of having a suite of systemic risk measures in order to assess and inform the stance of macroprudential policy.

25 An analysis of the regression results suggests that this increase in the explanatory power of the model is due to a stronger (statistically significant) relationship between current and future growth.

Figure 17: Predicted 1-quarter ahead growth-at-risk, 2003-2020



Finally, the impact of the Covid-19 pandemic, and the related lock-down restrictions put in place to contain its spread, resulted in the largest quarterly fall in South African GDP since the 1990s in the second quarter of 2020. Given the speed with which the Covid-19 crisis unfolded and the exogenous nature of the shock, neither of the systemic risk measures could have predicted a decline of such magnitude. However, given that both measures have increased to their highest levels during the period of observation, it is not surprising that both models suggest significantly heightened downside risk to South African economic growth going forward. Since systemic risk in the South African financial system remains historically elevated (Figure 15), it implies that downside risk to economic growth will remain heightened. This is despite a strong technical rebound in the third quarter of 2020, attributable to a relaxation in lock-down restrictions and support from policy responses. The SARB, along with other major central banks, recognises that the ability of the economy to recover from the pandemic will depend in part on the availability of credit. The SARB reduced the policy rate several times since the pandemic started supporting borrower's ability to service debts. It also kept low-cost credit flowing to households and businesses.

6. Conclusions

In this study we have made an assessment of systemic risk within the South African banking system by analysing three different market-based measures – ΔCoVaR, Marginal Expected Shortfall (MES) and SRISK. We have taken the South African financial system to comprise its six largest banks – Absa, Capitec, First National Bank (FNB), Nedbank and Standard Bank. All three measures successfully depict the four distinctive episodes of stress: (i) the uncertainty surrounding the consolidation of the South African banking system in 2002; (ii) the impact of the GFC in 2007-09; (iii) the period of heightened uncertainty surrounding what is referred to as "Nene-gate"; (iv) the fallout from Covid-19. The evolution of these systemic risk measures shows that the level of risk in the South African banking system rose to a historic all-time during the onset of the coronavirus pandemic. Covid-19 is a major shock to the economy whose origin is exogenous to the financial system. Unlike the GFC, banking sectors have not been a direct source of stress or an amplification.

The measures of systemic risk analysed in this paper capture different dimensions of systemic risk. ΔCoVaR measures the impact on the financial system conditional on an institution being in distress. MES and SRISK compute the impact on an institution conditional on the financial system being in distress. The ΔCoVaR measure identifies FNB, the bank with the highest market capitalisation, as the most systemically important bank. According to the MES measure, Standard Bank is the most systemically important. Across our sample of six banks we observe a cross-sectional relationship between MES of banks and their market betas – a measure of systematic risk. Standard Bank has a high level of systematic risk as indicated by its market beta. A tail shock on the banking system, such as that imposed by Covid-19, would impinge more on Standard Bank given its greater interconnectedness with the system as a whole. The SRISK measure incorporates some information about balance sheet structure. Standard bank, with the largest total liabilities, contributes the most to SRISK. All three measures find that the two smallest banks in the sample – Capitec and Investec – have the least systemic importance.

We assessed the degree to which heightened systemic risk in the banking system would create downside risk for South Africa's GDP growth. We estimated the expected tail-risk of GDP growth, as the VaR of future GDP growth, conditional on the level of systemic risk. This was over a policy-relevant time horizon of 1- to 6-quarters ahead. Aggregate SRISK (the amount of capital needed by the banking system to survive a financial crisis), proved to be the most robust indicator predicting GDP growth-at-risk. The speed with which the Covid-19 crisis unfolded and the magnitude of the shock suggests heightened downside risk to GDP going forward.

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Appendices

A Value-at-Risk (VaR)

The CoVaR estimation proposed by Adrian and Brunnermeier (2011) use quantile regressions which allows them to focus on the tails of equity returns. The starting point of their analysis is the Value-at-Risk (VaR). Consider a random variable, $R_{i,t}$, that represents the returns of financial institution i at time t ($i = 1,...,N$; t=1,...,T). The VaR of the random variable $R_{i,t}$ is defined as the α-quantile of the return distribution and thus can be formulated in terms of returns in the following way,

$$Pr(R_{i,t} \leq VaR^i_{\alpha,t}) = \alpha. \tag{21}$$

where $R_{i,t}$ is the return of institution i at time t, and $VaR^i_{\alpha,t}$ is the α-quantile of the returns $R_{i,t}$ at time t. This implies that VaR can be written as the upper bound of the integral in the following formulation,

$$\int_{-\infty}^{VaR_{\alpha,t}} pdf_t(R_{i,t})dR_{i,t} = \alpha \tag{22}$$

where $pdf_t(R_t)$ is the probability density function of the returns at time t. Equivalently, (22) can be also be written as

$$VaR^i_{\alpha,t} = F^{-1}_{i,t}(\alpha), \tag{23}$$

where $F^{-1}_{i,t}$ is the generalised inverse distribution function of the return distribution $F_{i,t}$.

For example, Figure 18 shows the VaR for Standard Bank's asset returns, calculated by the historical simulator method of 5% maximum negative asset returns, denoted as VaR 5%.

Figure 18: Distribution of asset returns for Standard Bank, 2002-2020

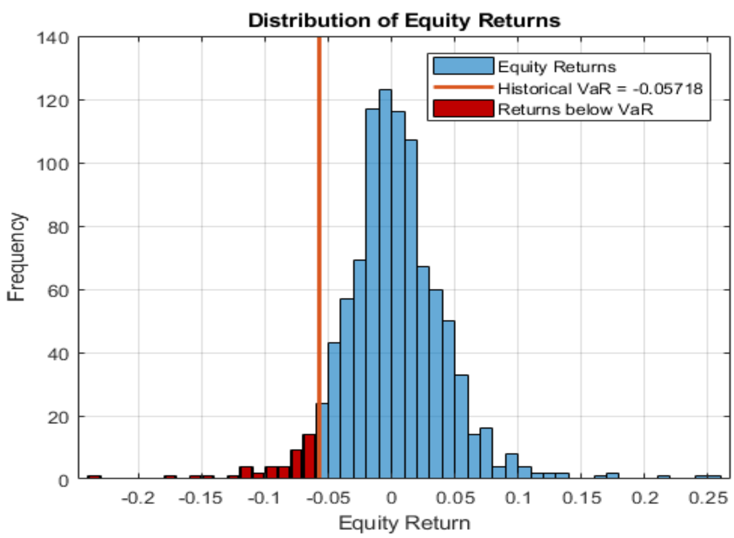

Table 7: Quantile regression parameter estimates

α_s	β_s	$\gamma_{s,1}$	$\gamma_{s,2}$	$\gamma_{s,3}$	$\gamma_{s,4}$
Absa					
-0.0117	0.6274	-0.0010	0.0000	0.0001	-0.0001
Capitec					
-0.0079	0.3035	-0.0023	0.0002	0.0000	-0.0014
First National Bank					
-0.0026	0.6082	-0.0014	0.0000	-0.0001	0.0000
Investec					
-0.0034	0.0597	-0.0022	-0.0002	-0.0002	-0.0011
Nedbank					
-0.0088	0.4138	-0.0014	0.0000	0.0000	0.0011
Standard Bank					
-0.0110	0.5833	-0.0011	-0.0001	0.0000	0.0000

Key:

α_s	Constant term in regression equation
β_s	Coefficient of VaR of the institution
$\gamma_{s,1}$	Coefficient of VXEEM
$\gamma_{s,2}$	Coefficient of weekly change in 3m T-bill rate
$\gamma_{s,3}$	Coefficient of change in slope of the yield curve
$\gamma_{s,4}$	Coefficient of JSE40 log returns

B Estimation method for Marginal Expected Shortfall

Following Brownlees and Engle (2011) we specify a bivariate DCC-GARCH process for returns:

$$R_t = H_t^{1/2} v_t \tag{24}$$

where $R_t = \left(R_t^s, R_t^i\right)'$ is the (2 x 1) vector of system and individual bank returns, and the random vector of the disturbance terms $v_t = \left(\varepsilon_t^s, \xi_t^i\right)'$. In this model, the disturbances $\left(\varepsilon_{s,t}, \xi_{i,t}\right)$ are assumed to be independently and identically distributed over time, and have zero mean and unit variance. But they are not considered to be independent of each other. Episodes of financial stress have a systemic element and tend to affect most financial institutions. H_t is the (2 x 2) variance-covariance matrix. $\sigma_{s,t}$ and $\sigma_{i,t}$ are the volatilities of the financial system and the individual institution at time t, and $\rho_{i,t}$ is the correlation at time t between R_t^s and R_t^i. The H_t matrix denotes the conditional variance-covariance matrix[26]:

$$H_t = \begin{pmatrix} \sigma_{s,t}^2 & \rho_{si}\sigma_{s,t}\sigma_{i,t} \\ \rho_{is,t}\sigma_{i,t}\sigma_{s,t} & \sigma_{i,t}^2 \end{pmatrix} \tag{25}$$

[26] Engle (2002) provides a detailed description of the DCC approach.

Obtaining $H_t^{\frac{1}{2}}$ could be difficult due to the issue of taking the square root of a matrix, whereby the Cholesky decomposition of H_t is used:

$$H_t^{1/2} = \begin{pmatrix} \sigma_{s,t}^2 & 0 \\ \sigma_{i,t}\rho_{i,t} & \sigma_{i,t}\sqrt{1-\rho_{it}^2} \end{pmatrix} \tag{26}$$

To estimate MES, we first model the bivariate process of bank and market returns. Given equation 24, this can be expressed as:

$$R_{s,t} = \sigma_{s,t}\varepsilon_{s,t} \tag{27}$$

$$R_{i,t} = \sigma_{i,t}\varepsilon_{i,t} \tag{28}$$

$$R_{i,t} = \sigma_{i,t}\rho_{i,t}\varepsilon_{s,t} + \sigma_{i,t}\sqrt{1-\rho_{i,t}^2\xi_{i,t}} \tag{29}$$

Therefore, the MES can be expressed more explicitly as a function of correlation and some tail expectations of the standardised innovations distribution

$$\begin{aligned} MES_{i,t-1} &= E_{t-1}(R_{i,t} \mid R_{s,t} < C) \\ &= \sigma_{i,t}E_{t-1}(\varepsilon_{i,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}}) \\ &= \sigma_{i,t}\rho_{i,t}E_{t-1}(\varepsilon_{s,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}}) + \sigma_{i,t}\sqrt{1-\rho_{i,t}^2}E_{t-1}(\xi_{i,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}}) \end{aligned} \tag{30}$$

The estimation of time-varying correlations, stochastic volatilities, and tail expectations follow Brownlees and Engle (2011), and we use the model defined in equations 24 and 25. The estimated parameter values for the six banks are shown in Table 8 below.

Table 8: MES tail expectations parameters

Institution	c	k1	k2
Absa	−0.053	−1.9371	−0.1031
Capitec	−0.053	−1.9371	0.008
FNB	−0.053	−1.9371	−0.1957
Investec	−0.053	−1.9371	−0.2153
Nedbank	−0.053	−1.9371	−0.1815
Standard Bank	−0.053	−1.9371	−0.1197

B1 Volatilities

The conditional volatilities are modelled with an asymmetric GARCH specification described in Scaillet (2005)

$$\sigma_{s,t}^2 = \omega_s + \alpha_s R_{i,t-1}^2 + \gamma_s R_{s,t-1}^2 I_{s,t-1} + \beta_s \sigma_{s.t-1}^2 \tag{31}$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i R_{i,t-1}^2 + \gamma_i R_{i,t-1}^2 I_{i,t-1} + \beta_i \sigma_{i,t-1}^2 \tag{32}$$

where $I_{i,t} = 1_{R_{i,t}<0}$ and $I_{s,t} = 1_{R_{s,t}<0}$ which can capture the leverage effect. It has been demonstrated empirically that volatility in equity returns tends to increase more with negative shocks than positive ones.

B2 Correlation

The time-varying conditional correlations are modelled using the DCC approach introduced by Engle (2002). The variance-covariance matrix is written as follows:

$$H_t = D_t R_t D_t \tag{33}$$

where $R_t = \begin{bmatrix} 1 & \rho_{i,t} \\ \rho_{i,t} & 1 \end{bmatrix}$ is the time-varying correlation matrix of the system and bank returns and $D_t = \begin{bmatrix} \sigma_{i,t} & 0 \\ 0 & \sigma_{s,t} \end{bmatrix}$ is a diagonal matrix for the conditional standard deviations.

The DCC framework introduces what is referred to as a pseudo-correlation Q_t, which is a positive definite matrix such that:

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2} \tag{34}$$

where $diag(Q_t)$ is such that $diag(Q_t)_{i,j} = (Q_t)_{i,j} 1_{i=j}$.

In the DCC framework, Q_t is defined as

$$Q_t = (1-a-b)S + a\eta_{t-1}\eta_{t-1}^{'} + bQ_{t-1} \tag{35}$$

where $\eta_t = (\varepsilon_{i,t}, \varepsilon_{s,t})^{'}$ is the vector of standardised residuals, and a and b are scalars. $S = E[\varepsilon_t \varepsilon_t^{'}]$ is the unconditional correlation of the standardised residuals and is referred to as the intercept matrix. Q_t is a positive definite matrix under certain conditions which are $a > 0$, $b > 0$, $a + b < 0$ and the positive definiteness of S. The matrix S is estimated by

$$\hat{S} = \frac{1}{T} \sum_{t=1}^{T} \eta_t \eta_t^{'} \tag{36}$$

The DCC model is estimated via QML. The steps involved in estimating the dynamic correlation are described in Engle and Sheppard (2008).

B3 Tail Expectations

If we recall equation 30, then the remaining terms to be estimated in order to obtain the MES are the two conditional tail expectations: $E_{t-1}(\varepsilon_{s,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t})})$ and $E_{t-1}(\xi_{i,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}})$. The term $E_{t-1}(\xi_{i,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}})$ captures the tail-spillover effects from the financial system to the bank that is not captured by the correlation. Furthermore, if both marginal distributions of the standardised returns are unknown, then the conditional expectation $E_{t-1}(\varepsilon_{s,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t})})$ is also unknown. As a consequence, both tail expectations must be estimated. In doing so, we follow Brownlees and Engle (2011) and use a non-parametric kernel estimation which is described in Scaillet (2005). In this analysis we consider a threshold C equal to the VaR of the financial system i.e., $C = VaR_{i,t}(\alpha)$. Then, if the standardised innovations, $\varepsilon_{s,t}$ and $\xi_{i,t}$ are i.i.d., the non-parametric estimates of the tail expectations are given by:

$$E_{t-1}(\varepsilon_{s,t} \mid \varepsilon_{s,t} < k) = \frac{\sum_{t=1}^{T} K(\frac{k-\varepsilon_{s,t}}{h})\varepsilon_{s,t}}{\sum_{t=1}^{T} K\left(\frac{k-\varepsilon_{m,t}}{h}\right)} \tag{37}$$

$$E_{t-1}\left(\xi_{i,t} \mid \varepsilon_{s,t} < k\right) = \frac{\sum_{t=1}^{T} K\left(\frac{k-\varepsilon_{s,t}}{h}\right)\xi_{i,t}}{\sum_{t=1}^{T} K(\frac{k-\varepsilon_{s,t}}{h})} \tag{38}$$

where $k = VaR_{i,t}(\alpha)/\sigma_{s,t}$, and

$$K_t(h) = \int_{\infty}^{\frac{t}{h}} k(u)du \tag{39}$$

where $k(u)$ is a kernel function and h a positive bandwidth. Following Scaillet (2005), we fix the bandwidth at $T^{-1/5}$ and choose the standard normal probability function as a kernel function, i.e., $k(u) = \phi(u)$. Using the DCC-GARCH model described above to determine the conditional variance and correlation, we can arrive at a an expression for the MES as follows:

$$MES_{i,t} = \sigma_{i,t}\rho_{i,t}E_{t-1}\left(\varepsilon_{s,t} \mid \varepsilon_{s,t} < k\right) + \sigma_{it}\sqrt{1-\rho^2}E_{t-1}(\xi_{it} \mid \varepsilon_{s,t} < k) \tag{40}$$

C Growth-at-Risk goodness-of-fit results

This section presents the goodness-of-fit results for the growth-at-risk regressions, using aggregate MES and SRISK as measures of financial stability risk. As depicted in Figure 16, we consider goodness-of-fit measures across two dimensions: the regression quantile and the twelve quarter-ahead growth predictions. The bold figures highlight the best fit regressions for each measure. While both aggregate MES and SRISK perform best at the 75th percentile, SRISK outperforms MES at the left-tail. In terms of future growth, aggregate MES performs best for predicting average 2 quarter-ahead growth, whilst aggregate SRISK performs best for average 4 quarter-ahead growth.

Table 9: SA banking sector (Largest listed banks): Goodness-of-fit metrics across quantiles and quarters, 2002-2020

Quantile	MES	SRISK
5	5.94	33.53
25	6.41	19.71
50	12.68	29.08
75	**17.38**	**38.57**
95	11.33	36.91

Quarters ahead	MES	SRISK
1	13.71	24.70
2	**16.01**	27.97
3	14.98	35.09
4	13.90	**35.45**
5	13.53	34.30
6	10.52	34.11
7	8.76	32.12
8	8.54	32.79
9	8.57	30.38
10	6.13	29.16
11	6.66	29.67
12	7.68	32.95