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**Commodity Prices and Policy Stabilisation in  
South Africa**

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# Commodity Prices and Policy Stabilisation in South Africa

Byron Botha \*      Eric Schaling †

September 4, 2020

## Abstract

In order to account for the effects of commodity exports on the South African business cycle we use a multivariate extension of the Hodrick Prescott (HP) filter that incorporates commodity prices. We find that ignoring commodity prices results in a monetary policy stance that is more dovish than the one implied by our multivariate measure of the business cycle. This may partly explain why inflation breached the inflation target from 2007Q2 to 2009Q4, and overshoot the upper bound of the target again by mid-2014. In addition we find that incorporating information about commodity prices implies smaller revisions of the estimated output gap. This in turn, enables a more consistent narrative around economic slack and monetary policy over time.

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## 1 Introduction

The output gap is a crucial variable for monetary policy making. It is an indication of excess demand, i.e. if the gap is zero and shocks are absent the economy does not face any inflationary pressure.

In addition, central banks are interested in the gap not only because it can be a (leading) indicator of future inflation, but also because it may want to stabilize the gap at some target level in the context of flexible inflation targeting.

The problem for monetary policy is that the output gap cannot be observed because potential output is unknown. The central bank only observes actual GDP and not its components, namely, the output gap and potential GDP.

Knowing the gap is important as it plays an important role in central bank decision making. Suppose the central bank sets interest rates based on some estimate of the gap. If the true gap is lower than the one used for monetary policy, so that the policy maker overestimates the gap then interest rates will be too high and monetary policy is too tight relative to its correct stance. Since only actual output is observed this means that the central bank underestimates potential output. Conversely, if the policy maker underestimates the gap by definition they are overstating potential output and interest rates will be too low.

There is a long literature regarding the inference of potential output. There are several approaches such as the production function approach, univariate time-series filters, multivariate filters and structural vector autoregression (SVAR) models.

In macroeconomics one of the most widely used statistical approaches is the (univariate) HP filter. This was introduced by Hodrick & Prescott (1997). Multivariate extensions have been introduced by Laxton & Tetlow (1992) and the Bank for International Settlements (BIS) (Borio et al. (2013), Borio et al. (2014), and Alberola et al. (2016)). The BIS approach has been applied to South Africa by Anvari et al. (2014) and Kemp (2015).

In this paper following Alberola et al. (2016) we investigate the role of commodity prices for South Africa. The argument is that booms in commodity prices tend to raise real GDP in the short term. However, it is not clear to what extent such booms raise potential output. A univariate approach such as the HP filter will allocate a part of the boom to potential GDP, thus underestimating the output gap and creating a negative bias in short-term interest rates.

We find that – unlike the official South African Reserve Bank (SARB) output gap – the commodity price augmented gap is in positive territory in late (2010). This may partly explain why inflation was breaching the target by mid (2014). Using the commodity price augmented gap would have helped to tame inflation. These results demonstrate the danger of not responding to the cyclical impact of commodity prices.

The remainder of this paper is organized as follows. We review related literature in Section 2. Section 3 discusses our model. In Section 4 we present the data and descrip-

tive statistics. Empirical results and concluding remarks are the subjects of Sections 5 and 6. Appendix A discusses how filtering and estimation are related in the multivariate filter. In Appendix B we compare the results of the multivariate filter with some well-known univariate filters. Appendix C checks the robustness of the sample and Appendix D investigates the sensitivity of our results with respect to the priors. Finally, Appendix E show the results of unit root tests.

## 2 Review of Related Literature

There are several methods for inferring potential output or the output gap. The most common are the production function approach, univariate time-series filters, multivariate filters and structural vector autoregression (SVAR) models. We briefly discuss each of those below and focus on their application to South Africa.

With regard to univariate time-series filters a popular method is a so-called unobserved components model (UCM). A UCM decomposes a time series into trend, seasonal, cycle and irregular components. Applied to GDP one can thus obtain the cycle component, that is, the output gap.

In macroeconomics one of the most widely used UCMs is the well-known (univariate) HP filter. This was introduced by Hodrick & Prescott (1997). It infers potential output as the solution of an optimization (minimization) problem that trades off deviations of potential output from actual output and the extent to which growth in potential output can vary, that is, the smoothness of the potential output series itself. This can be done in two steps: first, the HP minimization problem is written as a state space model.<sup>1</sup> Secondly, restrictions are imposed on the variances of the equations of the state space model, to reproduce the balance between the elements of the minimization programme, as represented by the (crucial) smoothing parameter  $\lambda$ . Since the filter's inception - following the suggestion of Hodrick & Prescott (1997) - the value of  $\lambda$  for quarterly data has typically been set at 1600.<sup>2</sup> It is important to note that in order to derive the output gap that features in the 2007 version of the SARB's Quarterly Projection Model (QPM) - as outlined by De Jager (2007) - the HP filter with  $\lambda = 1600$  is also used.<sup>3</sup>

Du Toit (2008) argues that this "default value" is inappropriate due to its ad hoc nature and problematic underlying assumptions. Following Pedersen (2001) he uses the

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<sup>1</sup>UCMs have a natural state-space representation. Indeed, Harvey (1985) shows that the HP filter can be cast in state-space form and reproduced with the Kalman filter.

<sup>2</sup>Another problem with the HP filter – being a two-sided symmetric filter – is that it is subject to end-point bias. This means that it converges to the observed values of the underlying series at the beginning and end of the series. Another filter that suffers from an end-point problem is the Baxter and King (BK) filter. In addition to the HP filter the latter has been applied to South Africa by Fedderke & Mengisteab (2016). For a good survey of several time-series filters see Álvarez & Gómez-Loscos (2017).

<sup>3</sup>In the 2017 version of the QPM the HP filter is used in the context of commodity prices. See Botha et al. (2017).

method of optimal filtering to determine the optimal value of the smoothing parameter for South Africa. Then, the optimal smoothing parameter is that value which least distorts the frequency information of the time series. The result depends on both the censoring rule for the duration of the business cycles and the structure of the economy. Based on a sample that runs from 1960 to 2005 - and a business cycle duration of 6 years - he finds that a value of 352 most closely matches the long-run features of the data.

Using the HP filter Ehlers et al. (2013) find South Africa's potential rate of growth equalled 3.5 percent for the period 1998-2008 and 3.4 per cent for the period 1998-2011. Broadly in line with these results are the findings of Fedderke & Mengisteab (2016) For the period 1995-2010 (a sample that includes the first sub-sample of Ehlers et al.) they find that – according to the HP filter – potential output grew by 2.6 percent from 1995-2000, 3.7 percent from 2000-2005 and 3.6 percent from 2005-2010.

The production function approach is squarely based on economic theory and assumes that potential output can be described by a production function which describes the supply side of the economy, where output is determined by available technology and labour and capital.<sup>4</sup> Although it is based on economic theory one still needs to specify a production function (a Cobb- Douglas (CD) or constant elasticity of substitution (CES) type is most often used), and a key challenge is the measurement of the various inputs. For example, even the simplest form (CD) requires removing the cyclical component from both labour and total factor productivity. The latter is sometimes done using the Hodrick-Prescott (HP) filter. Thus, in practice the production function method is not completely separate from a purely statistical univariate time-series filter approach.

Using the production function approach Ehlers et al. (2013) find that South Africa's potential rate of growth equalled 3.9 percent for the period 1998-2008 and 3.6 per cent for the period 1998-2011. Fedderke & Mengisteab (2016) consider the period 1960Q1 to 2015Q2 and find that the production function approach produced results similar to the band-pass filters (Baxter-King and Christiano-Fitzgerald).<sup>5</sup> More specifically for the period 1995-2010 (a sample that includes the first sub-sample of Ehlers et al.) they find that potential output grew by 2.3 percent from 1995-2000, 3.2 percent from 2000-2005 and 3.7 percent from 2005-2010. It is noteworthy that these growth rates all lie below the 3.9 percent found by Ehlers et al.

In univariate filters, only information about a variable itself (in our case output) is used in order to obtain an estimate of the underlying trend (potential output). As a natural extension of a univariate filter is a multivariate filter, wherein other information is

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<sup>4</sup>Fedderke & Mengisteab (2016) show that this method is used by Brazil, Croatia, the IMF, Malaysia, New Zealand, the OECD, the SARB and the US-CBO.

<sup>5</sup>They point out that high-pass filters, such as the HP filter, only allow for stochastic cycles that meet a minimum level frequency and block the lower frequency stochastic cycles. By contrast band-pass filters allow only stochastic cycles within a specified range of frequencies, with any frequency outside the relevant upper and lower bounds being filtered out.



used to sharpen the identification of potential output.<sup>6</sup> A seminal paper is one by Laxton & Tetlow (1992). They generalize the HP filter in the sense that additional variables are considered such as the rate of inflation. More specifically, one can include a (traditional) Phillips Curve that explains inflation by lagged inflation and the output gap.<sup>7</sup> For South Africa such a multivariate HP filter (MVHP) was first used by Ehlers et al. (2013). They include a traditional Phillips Curve, Okun's Law and capacity utilization. Using this approach they find South Africa's potential rate of growth equalled 3.6 percent for the period 1998-2008 and 3.4 per cent for the period 1998-2011. Note that these results are virtually identical to those obtained from the univariate HP filter (see above).

Multivariate HP filters were also used by the Bank for International Settlements (BIS). An important contribution is Borio et al. (2013). They state that identifying potential output with non-inflationary output is too restrictive. They show that including information about the financial cycle such as real credit growth and residential property prices can yield measures of potential output and output gaps that are not only estimated more precisely, but also more robust in real time. In the context of policy applications, such "finance-neutral" output gaps can serve as a complementary guide for monetary policy.

The Borio et al. (2013) approach was applied to South Africa by Anvari et al. (2014). They find that South Africa's potential growth rate has declined from 4.0 per cent in 2007 to 2.5 per cent in 2013. Another application is Kemp (2015). When considering ex post real interest rate, consumer price inflation, real credit growth and residential property prices, his results imply that the growth in potential output declined from an average of 3.6 percent between 2001Q1 and 2007Q3 to 2.6 percent between 2009Q4 and 2014Q1 and that potential growth slowed to just 2 percent in 2014Q1.

Adapting Borio et al. (2013) the BIS (Alberola et al. (2016)) did a similar study focusing on the role of commodity prices and net capital inflows in Latin America. Results indicate that these two factors temporarily boost output and are likely to push up estimates of potential growth in the region to unrealistic levels, thereby resulting in an underestimation of the output gap during the upswing of the commodity cycle. Further they find that commodity prices have been the dominant factor explaining deviations of activity from sustainable levels.

Another way to infer potential output is via a structural VAR (SVAR). This approach was pioneered by Blanchard & Quah (1989) who consider fluctuations in GNP and unemployment.<sup>8</sup> Rennison (2003) used this approach to identify the output gap in Canada.

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<sup>6</sup>In Appendix B we do provide some additional discussion of alternative univariate filters such as the Baxter-King (BK), Butterworth (B) and Christiano-Fitzgerald (CF) filters and show how the associated results differ from those of our MVF filter.

<sup>7</sup>See for example Borio et al. (2014). Doménech & Gómez (2006) estimate a multivariate model including a New-Keynesian Phillips Curve.

<sup>8</sup>Blanchard & Quah (1989) define the two types of disturbances as follows. The first has no long-run effect on either unemployment or output. The second has no long-run effect on unemployment, but may

He uses a bivariate SVAR with the first difference of log real GDP and the inflation rate. The long-run restriction used to identify the structural disturbances (supply shocks) is that demand shocks have no long-run impact on the level of output, whereas both demand and supply shocks can affect the level of prices in the long run (but not the inflation rate). The output gap is then computed as the cumulative response of the level of output to all past transitory shocks (where all supply shocks have been set to zero). A SVAR was applied to South Africa by Aurora & Bhundia (2003). This approach also features in Botha et al. (2018). They point out that the South African economy is regularly hit by transitory supply shocks (such as strikes and drought(s)), that should arguably affect the economy's productive potential rather than the output gap. They therefore update the South African Reserve Bank's current, finance-neutral, estimates of potential growth (in line with Anvari et al. (2014)) to account for these short-lived supply shocks. Using a SVAR they identify supply shocks that should shift potential growth rather than the output gap. They then claim that the resulting output gap estimate is more reflective of demand pressures in the economy.

### 3 Model

#### 3.1 The HP filter

As mentioned earlier the HP filter infers potential output as the solution of an optimization (minimization) problem that trades off deviations of potential output from actual output and the extent to which growth in potential output can vary, that is, the smoothness of the potential output series itself. More specifically, given  $T$  observations on actual output (in logs),  $y_t$ , and an adequately chosen, positive value of  $\lambda$ , there is a trend component that will solve

$$\min_{\{y_t^*\}_{t=1}^T} \left[ \sum_{t=1}^T (y_t - y_t^*)^2 + \lambda \sum_{t=2}^{T-1} (\Delta y_{t+1}^* - \Delta y_t^*)^2 \right] \quad (1)$$

The first term of the equation is the sum of the squared deviations which penalizes the cyclical component. The second term is a multiple of the sum of the squares of the trend component's second differences. This second term penalizes variations in the growth rate of the trend component. The larger the value of  $\lambda$ , the higher is the penalty. This parameter determines the relative variability of the filtered potential output series. When  $\lambda$  becomes very large, potential output approximately follows a linear trend. More specifically, if  $\lambda$  were infinite, no change in the trend's slope would be

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have a long-run effect on output.

allowed, and the trend would be a straight line. Conversely when  $\lambda$  approaches zero potential output mimics actual output.

Barbosa-Filho (2004) points out that according to equation (1),  $\lambda$  is the square of the ratio of the “moderate” change in the cyclical component to the “moderate” change in the trend component. Assuming that  $y_t$  is a quarterly series expressed in logarithmic terms, the most common rule is to define more than 5% as an excessive variation of the cycle component, and more than 0.125% as an excessive variation of the trend component. The resulting smoothing parameter is  $\lambda = [\frac{5}{1/8}]^2 = 1600$ . This is the standard value that is used in the literature which limits the maximum length of the business cycle to approximately 8 years.<sup>9</sup>

For ease of exposition now we set  $T = 7$ . The following first-order conditions are derived by setting the gradient vector of equation (1) equal to zero. We obtain:<sup>10</sup>

$$\begin{aligned} y_1 - y_1^* &= \lambda(y_1^* - 2y_2^* + y_3^*) \\ y_2 - y_2^* &= \lambda(-2y_1^* + 5y_2^* - 4y_3^* + y_4^*) \\ y_t - y_t^* &= \lambda(y_{t-2}^* - 4y_{t-1}^* + 6y_t^* - 4y_{t+1}^* + y_{t+2}^*) \\ y_6 - y_6^* &= \lambda(y_4^* - 4y_5^* + 5y_6^* - 2y_7^*) \\ y_7 - y_7^* &= \lambda(y_5^* - 2y_6^* + y_7^*) \end{aligned}$$

Or more compactly,  $y - y^* = \lambda F y^*$ <sup>11</sup>, which implies that  $y = (\lambda F + I)y^*$ . Thus the HP trend

$$\hat{y}^* = (\lambda F + I)^{-1}y \quad (2)$$

and

$$\widehat{GAP} = y - \hat{y}^* \quad (3)$$

---

<sup>9</sup>As pointed out by Nilsson & Gyomai (2011), one can transform the filter into the frequency domain and understand its effects on various cycles that make up the time series. Then changes to  $\lambda$  determine the shape of the frequency response function of the HP filter and the cut-off frequency. Thus it is possible to align with the aim to filter out economic cycles in a certain frequency range. This is the approach that is followed for South Africa by Du Toit (2008). Also Fedderke & Mengisteab (2017) discuss various filters using the frequency domain.

<sup>10</sup>See for example Reeves et al. (2000) and Barbosa-Filho (2004).

<sup>11</sup>Where  $F = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ -2 & 5 & -4 & 1 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 1 & -4 & 5 & -2 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix}$

Except for the further four endpoints the first-order conditions state that

$$y_t = (\lambda L^2 - 4\lambda L + 6\lambda L^{-1} + \lambda L^{-2} + 1)y_t^* \quad t = 3, 4, 5, \dots$$

Using the notation of Ahumada & Garegnani (1999) and King & Rebelo (1993) we get  $F_{HP}(L)y_t^* = y_t$ . Define  $G_{HP}(L) = F_{HP}(L)^{-1}$ . Then we can write  $y_t^* = G_{HP}(L)y_t$ . This is the form of King & Rebelo (1993). Using the above definition of the output gap we get  $y_t - y_t^* = y_t - G_{HP}(L)y_t = [1 - G_{HP}(L)]y_t = C_{HP}(L)y_t$ .

This means that  $y_t - y_t^*$  is also a moving average of  $y_t$ . In the language of filtering theory  $G_{HP}(L)$  and  $C_{HP}(L)$  are linear filters. Based on the above King & Rebelo (1993) state that the HP cyclical filter  $C_{HP}(L)$  is therefore capable of rendering stationary any integrated process (here log GDP) up to fourth order.

### 3.2 The Multivariate Filter of Borio et al. (2013)

Borio et al. (2013) start with the following state-space form of the HP filter:

$$\Delta y_t^* = \Delta y_{t-1}^* + u_t^\tau \tag{4}$$

$u_t^\tau$  is assumed to be a normally and independently distributed error term with zero mean and variance  $\sigma_\tau^2$ .

$$y_t - y_t^* = u_t^c \tag{5}$$

Here  $u_t^c$  is also assumed to be a normally and independently distributed error term with mean zero and variance  $\sigma_c^2$ .

Borio et al. (2013) state that the parameter  $\lambda_c = \sigma_c^2 / \sigma_\tau^2$  – the so called noise-to-signal ratio – determines the relative variability of the estimated potential output series.<sup>12</sup>

They then extend equation (5) with additional variables and include the lagged output gap. Their specification is

$$y_t - y_t^* = \phi_1(y_{t-1} - y_{t-1}^*) + \gamma x_t + u_t^{c1} \tag{6}$$

Where the  $x_t$  vector of additional economic variables and  $u_t^{c1}$  represents a normally and independently distributed error term with mean zero and variance  $\sigma_{c1}^2$ .

---

<sup>12</sup>Rennison (2003) states that the setting can be interpreted as a prior on the relative variance of supply and demand shocks. He states that in the case one works with artificial data one would expect the HP filter to produce the most accurate estimates of the output gap when the data generating process (DGP) is consistent with this ratio. Alternatively, by varying the ratio of demand-to-supply shocks in the DGP, one can examine the costs of assuming the “wrong” value for  $\lambda$ .

They use the same state transition equation as equation (4) and set the new (or what they call  $\lambda_{ALT}$ ) signal-to-noise ratio of the multivariate filter  $\lambda_{ALT} = \sigma_{c_1}^2 / \sigma_{\tau}^2$  such that

$$\text{VAR}(y_t - y_{(3),t}^*) / \text{VAR}(\Delta^2 y_{(3),t}^*) = \text{VAR}(y_t - y_{(4),t}^*) / \text{VAR}(\Delta^2 y_{(4),t}^*) \quad (7)$$

where  $y_{(3),t}^*$  and  $y_{(4),t}^*$  are the potential output series from equations (5) and (6) respectively.

We provide some intuition for the use of equation (6) which underlies the multivariate filter. Alberola et al. (2016) state that commodity prices and net capital inflows booms temporarily boost output and so are likely to push up estimates of potential growth to unrealistic levels, thereby resulting in an underestimation of the output gaps during the upswing of the commodity cycle.

Now we use equation (6) as the measurement equation where for now we set  $\phi_1 = 0$  and focus on one explanatory variable (commodity prices). Also, for argument's sake we now assume we know the value of  $\gamma$  (filtering only).<sup>13</sup> Then it can be shown that the trend component of output is

$$\hat{y}_{MV}^* = (\lambda F + I)^{-1} (y - \gamma x) \quad (8)$$

Thus higher commodity prices lower potential output and thereby raise the level of the output gap compared to the HP filter.<sup>14</sup> In Appendix A we show how estimation and filtering are interrelated. That is, the output gap depends on parameter estimates and the latter on the output gap.

## 4 Data and descriptive statistics

The sample period chosen for the study is 1972Q1-2019Q1, which represents the span of the commodity price data we had available. The starting point also coincides as closely as possible with the original work of Anvari et al. (2014) that uses the multivariate filter of Borio et al. (2013) on South African data. The chosen sample provides enough data for robust estimation of the model parameters and lends to this study being more comparable to the literature. This is in contrast to the alternative of limiting the data to the period of Inflation Targeting (2000-present) in order to avoid structural breaks. In the appendix we present the results of our benchmark model estimated over this shorter sample as a test of the robustness of the estimates against structural breaks.

To make the rest of the discussion of the data manageable we concentrate our narrative on the period starting in the early 1990s. This period coincides with a monetary

<sup>13</sup>In the full problem potential output depends on actual output, the estimated parameter and commodity prices, whilst the estimate depends on potential output.

<sup>14</sup>More specific, the difference between potential output according to the multivariate and HP filter is  $-(\lambda F + I)^{-1} \gamma x$ .

policy regime beginning with “informal” inflation targeting which moved to a formal framework in the early 2000s and a progressively more freely floating exchange rate policy.

Examination of this period suggests that commodity prices are correlated with growth cycles in SA. The 2000s saw the longest cyclical upswing in SA’s history – ending in 2013 (excluding the 21 month downward phase brought on by the Great Financial Crisis (GFC)).<sup>15</sup> Easy global financing and a more stable macroeconomic environment compared to the often turbulent 1990s, both globally and domestically, promoted relatively robust GDP growth. We posit as Alberola et al. (2016) does in the Latin American case, commodity prices also played a large role. During this time the extended commodity price boom fed into a sustained increase in SA’s terms of trade, a real exchange rate appreciation, and boosted GDP growth. We hypothesise that these changes stimulated bullish investment into the commodities sector and the resulting wealth effects increased demand throughout the economy, thus playing a significant role in the cyclical upswing of the period. This would be consistent with Alberola et al. (2016), who find that “adjusting for commodity prices and net inflows produces level shifts that often lead to substantially higher output gaps at times when these two variables are buoyant”.

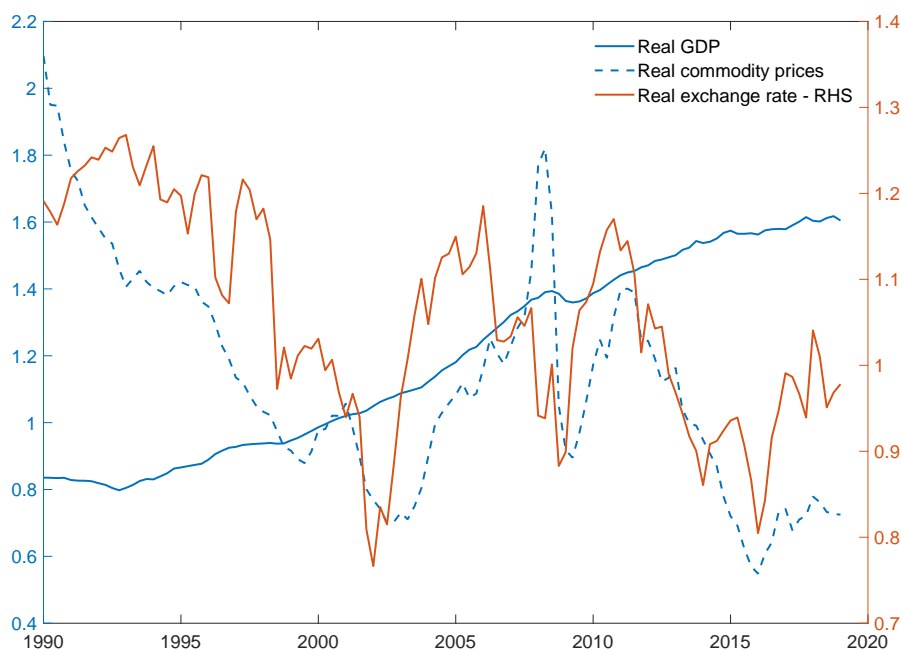
These relationships are shown in Figure 1 while the positive correlation between commodity prices and GDP growth are shown in Figure 2 below. Unlike the Latin American case the post-crisis recovery in SA was not particularly strong as the economy started to suffer from structural constraints in the face of the Eskom crisis.<sup>16</sup> A quick rebound in commodity prices, however, helped the business cycle revert to an upward phase during this period.

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<sup>15</sup>Christensen (2016) mentions – but does not analyse – the effects of commodity prices on potential growth of several African countries. There the focus is on the link between commodity prices and actual GDP growth. Broadly speaking, the results are that the latter varies inversely with the level of commodity prices.

<sup>16</sup>The Eskom crisis refers to capacity constraints at the state-owned power utility that has led to intermittent, managed electricity blackouts since 2010.

**Figure 1: Real GDP, effective exchange rate, and commodity prices**



Commodity prices in USD are export weighted and deflated by the Producer Price Index (PPI). Real gross domestic product (GDP) and the real effective exchange rate are the official measures published by the South African Reserve Bank. Source: SARB.

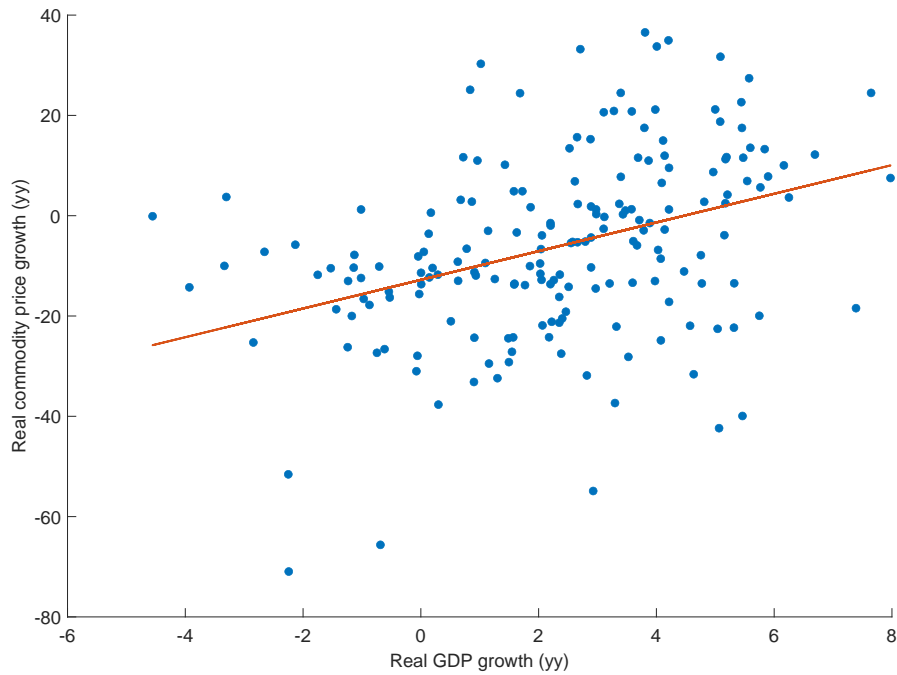
Alberola et al. (2016) further observe that “booms in commodity prices tend to raise real GDP in the short term by increasing the value and production of a key production factor in the economy (natural resources) (...)”.

A key question is to what extent this translates into higher potential GDP. A univariate filter such as HP will attribute part of the higher level of actual GDP – here driven by a boom in commodity prices – to potential GDP. This will result in overestimating potential output and thus in underestimating the output gap in the short-term.<sup>17</sup> If one extends the HP filter by allowing the output gap to depend on commodity prices, and then find a significant positive estimate, we then have good evidence that the output gap was pushed up because of this cyclical factor. After this exercise we will have a better idea of potential growth, which will be revised downwards compared to the HP filter. Of course, if one finds no significant effect then the multivariate filter collapses to the

<sup>17</sup>We further find that estimated potential growth may be underestimated in the period subsequent to the boom.

“plain vanilla” HP filter.

**Figure 2: Correlation of commodity prices and GDP growth**

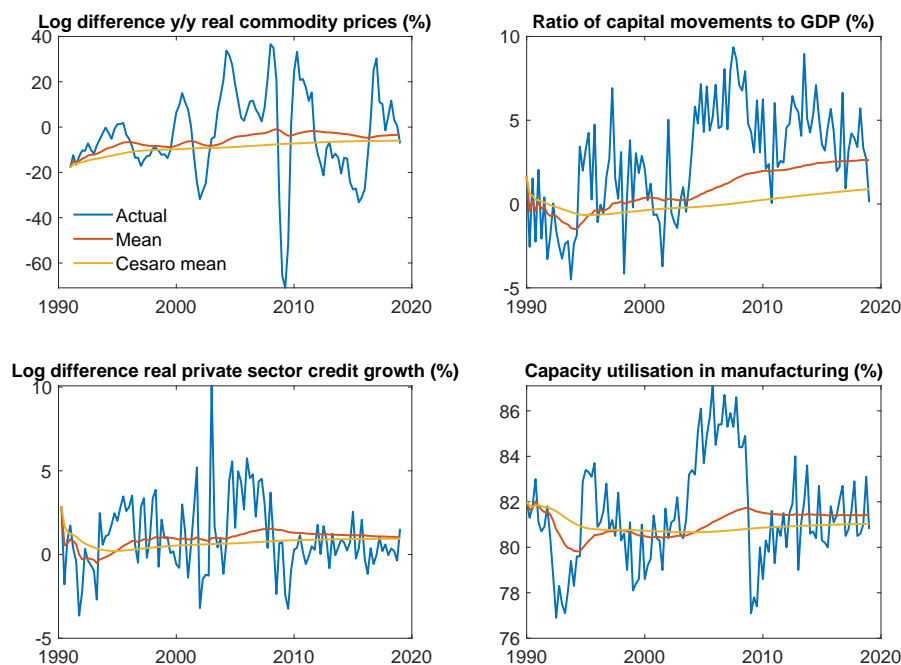


Source: SARB.

We investigate three different variables as being potentially informative for the output gap besides commodity prices. Capital flows as a ratio to GDP, considered by Alberola et al. (2016), as well as real private sector credit extension and capacity utilisation. The latter two variables are used in previous work using the same or similar framework in the South African context, see Anvari et al. (2014) and Botha et al. (2018). They provide useful controls. We excluded variables used in previous work that were shown not to be particularly useful in the multivariate filter framework, including: inflation, real-interest rates, unemployment, and property price growth (Anvari et al. (2014), Borio et al. (2014)).



**Figure 3: Assessing the behaviour of the means**



Source: SARB.

It is important to assess the behavior of the mean of any candidate variable one might want to include in estimating the output gap since any exhibited trend can contaminate the resulting estimated gap. As in Borio et al. (2014) the variables are demeaned using the Cesàro mean. As can be seen from (3) credit growth is fairly stable over the recent history, while commodity prices, capacity utilisation and the ratio of capital flows to GDP exhibit some trend behaviour. The capital flows data shows an especially strong trend over the post-crisis period as can be seen from the divergence of the regular sample mean from the Cesàro mean.<sup>18</sup> This is consistent with the era of quantitative easing in developed countries and the resulting carry-trade that affected capital flows. Some of the model specifications in the following section include the ratio of capital flows to GDP, given the observations of a marked trend in this data the results of those models should be treated with the right amount of scepticism. We present unit root tests for the variables in Appendix E for the reader's reference.

<sup>18</sup>The Cesàro mean is calculated by taking the mean of the sequence of means generated by repeatedly adding a single observation to the calculation. As stated in Borio et al. (2014): "Its key property is that it converges faster to the population mean whenever this mean exists."

## 5 Empirical Results

In this section we discuss the empirical results. We start by writing down the model equation for ease of reference and explaining our choice of priors, which are taken from the literature. Next we present the model estimates, finding that the relationship between commodity price growth and the output gap is statistically significant and stable over multiple specifications. We then look at the resulting output gaps and levels of potential growth resulting from the various specifications. Finally, the section concludes with an analysis of how the policy signal given by the output gap would have been different had a commodity price neutral output gap been employed.

First, the gap equation is shown with  $\phi_1$  the coefficient on the lagged gap and  $\gamma$  the coefficient vector multiplied by the vector of regressors  $x_t$ . The gap equation is followed by the equation for potential output, a unit root process in growth terms.

$$\begin{aligned} y_t - y_t^* &= \phi_1 (y_{t-1} - y_{t-1}^*) + \gamma x_t + u_t^{c1} \\ \Delta y_t^* &= \Delta y_{t-1}^* + u_t^\tau \end{aligned}$$

The priors used for the lagged output gap, real private sector credit extension, and commodity prices are the same as used in the Borio et al. (2014) and Alberola et al. (2016) papers, while the prior for capacity utilisation is set to the same as that employed by Anvari et al. (2014). As in all three papers, the coefficient on the lagged gap is restricted to lie between 0.00 and 0.95.<sup>19</sup>

**Table 1: List of priors**

	Prior distribution	Mean	Standard deviation
Lagged gap, $y_{t-1} - y_{t-1}^*$	Beta	0.70	0.30
Real commodity prices, $x_{i,t}$	Gamma	0.30	0.30
Capital flows to GDP, $x_{i,t}$	Gamma	0.05	0.02
Real private sector credit growth, $x_{i,t}$	Gamma	0.30	0.30
Capacity utilisation in manufacturing, $x_{i,t}$	Gamma	0.05	0.02

Analysing the estimation results shows that commodity prices are (statistically) significant over most specifications. Furthermore, the estimates of the various coefficients are fairly stable over different model specifications. The lagged/dynamic gap term is perhaps the exception, it is consistently less persistent when capacity utilisation is included in the specification and often hits the upper bound of 0.95 when it is not. Model 1, which has no lagged gap, shows no significant role for commodity prices in explaining the output gap. As a result it is near-identical to the univariate HP filter. According to Ollivaud & Turner (2014) among the 19 OECD countries which experienced a

<sup>19</sup>This ensures that the output gap depends positively on itself and is mean-reverting (stationarity condition).

banking crisis over the period 2007-11 the median loss in potential output in 2014 is estimated to be about  $5\frac{1}{2}\%$ , compared with a loss in aggregate potential output across all OECD countries of about  $3\frac{1}{2}\%$ . For this reason we also include a dummy for the global financial crisis (as proxied by the US business cycle) in model 3.

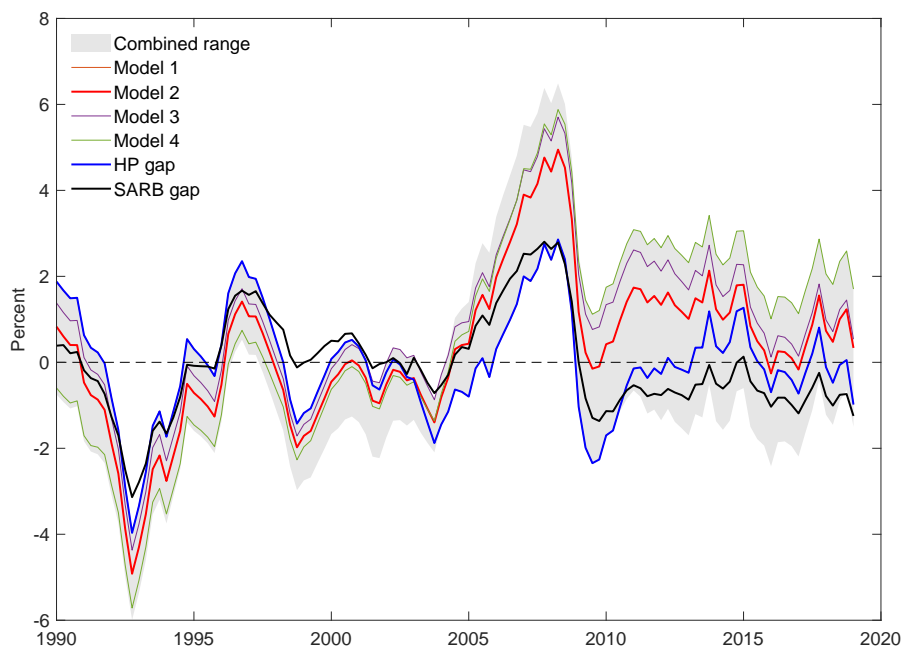
**Table 2: Estimates of model coefficients, posterior mode with t-statistics in parenthesis**

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Dynamic gap		0.950 (27.583)	0.950 (22.343)	0.944 (18.389)	0.854 (20.777)	0.925 (19.662)	0.950 (0.238)	0.871 (21.525)
Commodities	0.000 (0.000)	0.010 (2.652)	0.010 (2.741)	0.010 (2.790)	0.007 (1.935)	0.007 (1.892)	0.008 (2.321)	0.006 (1.594)
Capital flows				0.028 (2.626)	0.026 (2.643)	0.027 (2.629)		
Credit						0.119 (3.936)	0.117 (3.991)	0.100 (3.317)
Capacity utilisation					0.099 (3.806)			0.080 (3.347)
US recession dummy			0.198 (3.192)					

Sample = 1972Q1-2019Q1. Lambda = 1600. t-statistics in parenthesis.

*Model 1* includes commodity prices and is the only specification that excludes the lagged gap term. As can be seen from figure (4) the gap estimated using *Model 1* is barely distinguishable from the vanilla HP filter. *Model 2* is extended with the lagged gap and results in an output gap estimate that comes in somewhere in the middle of the other estimated gaps and is the model we use as a benchmark in the rest of the analysis. The capital flows variable tends to widen the gap substantially post-crisis as a result of the upward trend discussed earlier. The credit variable on the other hand tends to make the gap more negative. Capacity utilisation exacerbates this negative tendency, while the credit variable adds to the magnitude of the upswing during the boom years – as expected. On the whole, including the financial variables makes the gap closer to the official SARB gap (Botha et al. (2018)), which is expected as the modeling choices in that paper also builds off of the work of Anvari et al. (2014) (which estimates the financial-neutral output gap).

**Figure 4: Output gaps, specifications 1-4**



Another generalisation apparent from the results, is that the downswing during the financial crisis was less severe after accounting for commodity prices – as can be seen by looking at the shaded region (which encompasses the range of all the estimated models). The HP filter output gap, which does not account for commodity prices, has the deepest trough of all the estimated output gaps. While all the models estimate a similar magnitude in the decline in the output gap following the crisis, the cyclical build up of output, due to the commodities boom, means that the commodity- price-adjusted output gaps fall from a higher starting point (on average).

**Figure 5: Output gaps, specifications 5-8**

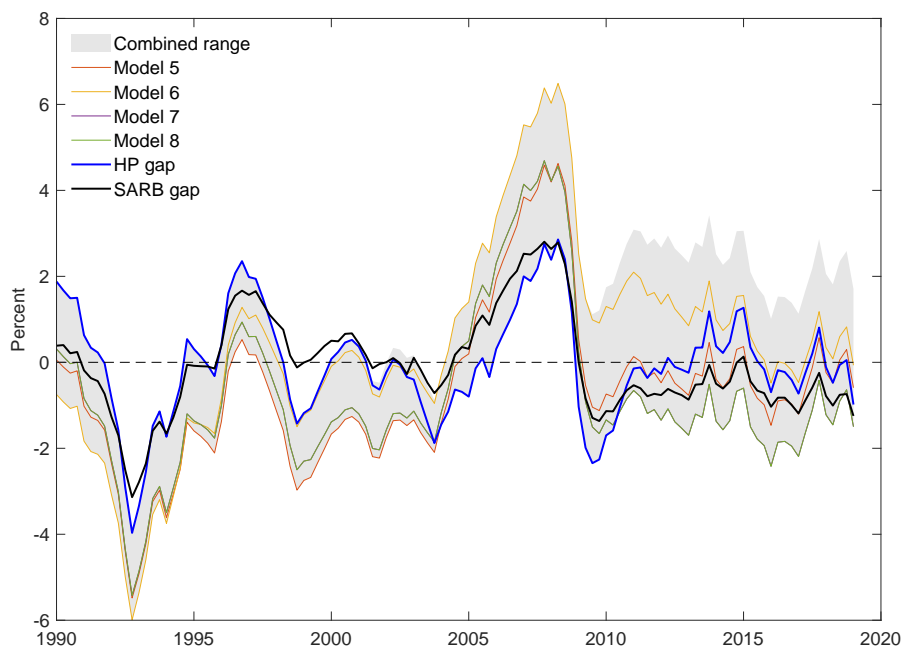
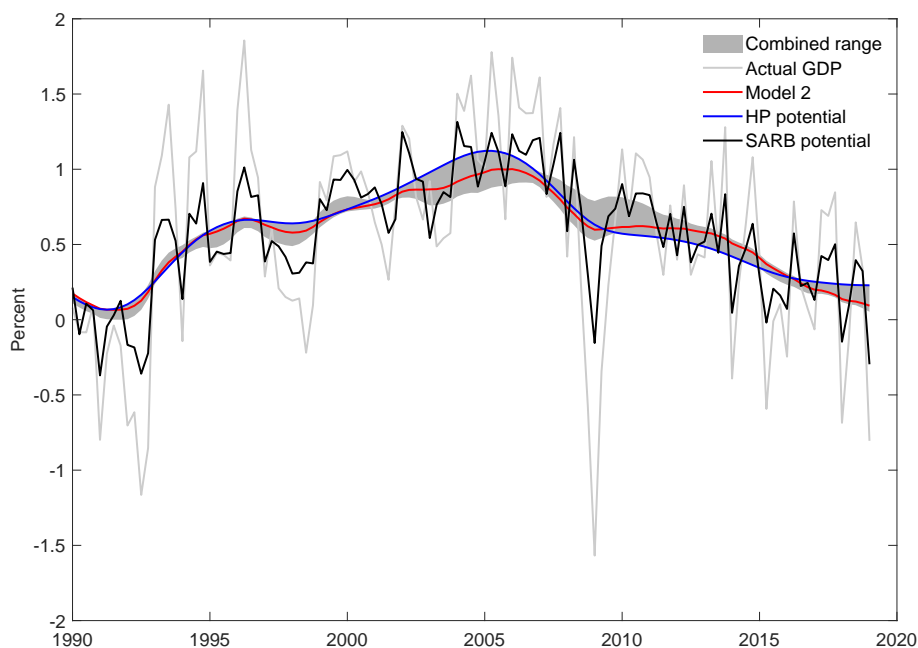


Figure (6) includes actual GDP, the official measure of potential growth estimated by the SARB in black, the HP filter, the combined range of estimated potential growth for all the models, and *Model 2* in red. A striking feature is the difference between the models based off of Borio et al. (2014) and the official measure of potential growth. The latter is a lot more volatile due to the inclusion of short-run supply shock as explained in Botha et al. (2018). Another feature is that the HP filter bounds the models that include commodity prices from above in the lead up to the financial crises and bounds them from below for most of the post-crisis period. That is, the potential growth rate given by the HP filter is at the upper bound of the “combined range” (shaded area) in Figure 6 in the lead up to the financial crisis and at the lower bound from about 2010 to 2015/16.

This shows the pervasive effect of the boom in commodity prices in the run up to the financial crisis. Firstly, interpreting this boom as being largely cyclical requires that one’s view of structural growth during that period be more pessimistic. If the boom in commodity prices feed directly into the output gap, the identity  $y = y^* + GAP$  requires potential growth to be lower over the period. Secondly it requires, all other things equal, that potential growth is relatively stronger post-crisis (but not necessarily during the crisis) in order for the output gap to close. This follows because the gap is stationary

and has a mean of zero which requires that, over time, positive and negative output gaps should average to zero. Our models show that the commodity price adjusted output gap is more positive pre-GFC. It is, therefore, intuitive that (for an observed level of GDP) the rate of potential growth would be higher post-GFC relative to a model that does not control for commodity prices, in order to bring the average of the output gap over the whole period closer to zero.

**Figure 6: Potential GDP growth**



In order to assess the monetary policy implications of accounting for commodity prices in the measured output gap we construct a simple policy rule. The policy rule used, see equation (9), proceeds along the lines originally proposed by Taylor (1993), where  $\pi^{target} = 4.5\%$  is the inflation target (the midpoint of the inflation target band) and  $i_t^*$  is the neutral nominal rate.<sup>20</sup> The nominal neutral rate is filtered from the ob-

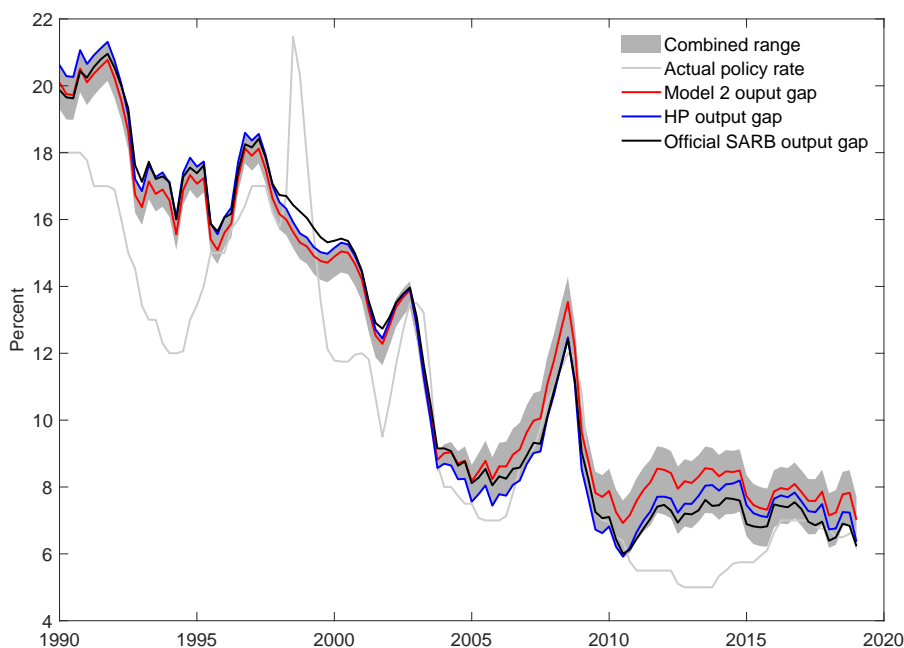
<sup>20</sup>It should be noted that actual monetary policy settings are not based on a mechanical Taylor rule. Additional and/or alternative inputs may be observed. For example: supply or foreign exchange rate shocks, positions on other unobservable variables - such as the neutral rate, and/or other unmodelled factors that affect the effectiveness of monetary policy.

served  $i_t$  and made to converge to 7% from 2008Q1.<sup>21</sup> The 7% reflects a 2.5% mean real interest rate plus the 4.5% inflation target contemplated in the Quarterly Projection Model, Botha et al. (2017).

$$i_t = i_t^* + \frac{1}{2}(\pi_t - \pi^{target}) + \frac{1}{2}(y_t - y_t^*) \quad (9)$$

Figure (7) shows the policy counterfactuals. The grey shaded area represents the range of outcomes based on the output gaps of all the estimated models. Also included is the rule using the official SARB output gap, the HP gap, and the gap estimated using *model 2* – the model augmented with the dynamic gap term and commodity prices. The actual policy rate is shown in light grey for reference.

**Figure 7: Taylor Rule analysis showing the repo rate implied by various output gaps**



*Model 2* suggests that leading up to the crisis the policy rate should have on average, been 91 basis points higher than suggested by the HP filter and 59 basis points higher

<sup>21</sup>Using an HP filter ( $\lambda = 1600$ ), used throughout the literature to extract the economic cycle from macroeconomic time series.

than the official output gap. Similarly policy should have been 60 basis points higher than suggested by the HP filter and 81 basis points higher than the official output gap in the post-crisis period. An important caveat is that the output gaps shown here are calculated ex-post and thus the information set facing the policy maker would, while similar, not have been exactly the same as shown here.

All other things equal, a higher policy rate leading up to the global financial crisis would have helped to tame inflation which breached the upper bound of the inflation target from 2007Q2 to 2009Q4.<sup>22</sup> Unlike the official measure the commodity price augmented gap rebounds into positive territory in late 2010 despite stronger growth in potential output in the post-crisis period. An artefact of the cyclical build-up of the boom period and a moderate recovery in commodity prices. If this result is correct, it could help partly explain why inflation was exceeding the upper limit of the target band again by mid-2014.<sup>23</sup> This result demonstrates the possible danger of not responding to the cyclical impact of commodity price movements in the South African context.

## 5.1 Real-time performance and ex-ante analysis

Figure (8) shows the outputs gaps estimated in real-time, the left panel being estimated using *Model 2* and the right-hand-side panel with the HP filter. The gaps are estimated on vintage GDP data starting with the sample 1972Q1-2000Q1 which is extended period by period, with the full sample being equal to the preceding analysis, 1972Q1-2019Q1.<sup>24</sup>

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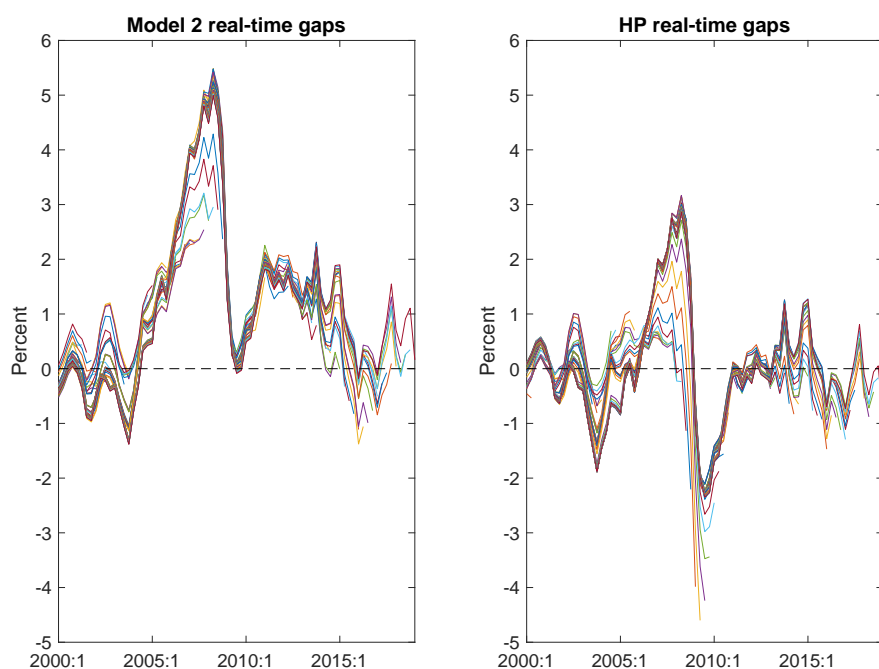
<sup>22</sup>Of course a more formal conjecture would need a specified transmission mechanism - say a Phillips and IS curve - in addition to the monetary policy rule. Here in the Phillips curve, one also needs to be mindful of potential supply shocks.

<sup>23</sup>It is important to note that not all the estimated models show the output gap above zero in the post-crisis period. Specifically those incorporating capacity utilisation.

<sup>24</sup>The model coefficients in the case of *Model 2* are estimated over the full sample.



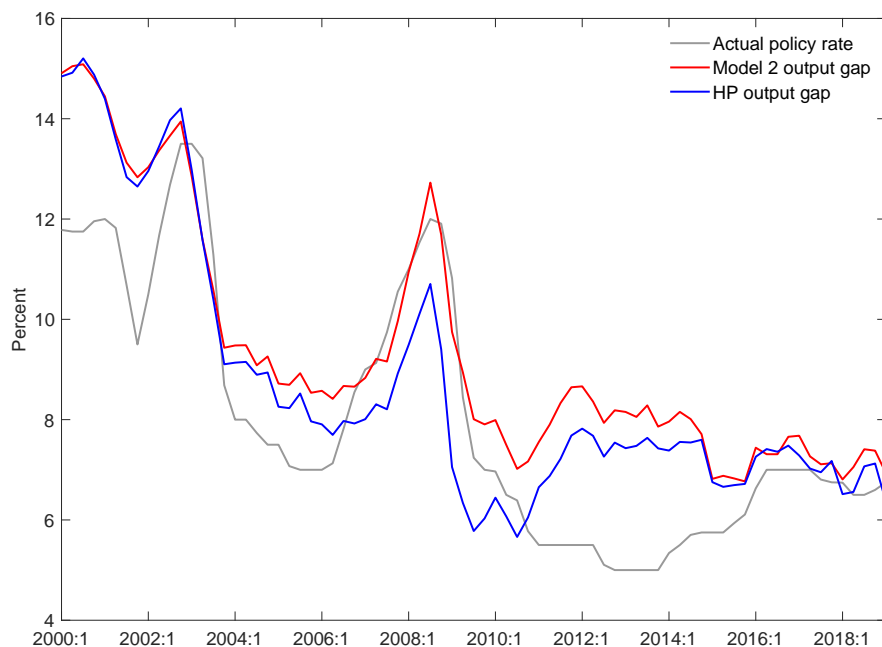
**Figure 8: Real-time output gaps**



Model 2 has a mean absolute revision of 1.04 percentage points after two years while the HP filter gap has a mean absolute revision of 1.12 percentage points, resulting in a ratio of 0.93. This implies that the output gap estimated using the commodity price adjusted output gap is revised, on average, 7% less in magnitude over two years than the HP filter. This result is similar, but less encouraging compared to those produced by Anvari et al. (2014) whose selected model had a mean absolute revision of 0.47, resulting in a ratio of 0.37 relative to the HP filter.

Figure (9) proceeds along the same lines as the Taylor rule analysis in the preceding analysis, but is based on the gaps estimated in figure (8). The actual policy rate is again included in light grey as a reference.

**Figure 9: Taylor Rule analysis, ex-ante**



Using the real-time output gaps we see that Model 2 still suggests a tighter policy rate leading up to the crisis – 99 basis points over the HP filter – and in the post-crisis period – 71 basis points higher than the HP filter. This suggests that the signal to monetary policy coming from extending the output gap with commodity prices are present and significant in real-time.

## 6 Concluding remarks

In this paper we have extended the HP filter to the multivariate case incorporating capital flows, credit, capacity utilization, a US recession dummy and commodity prices.

Focusing on commodity prices we find that this variable has pushed up the output gap compared to the HP filter. This variable is significant across specifications, so our results are robust. For example, the downswing during the global financial crisis was less severe after accounting for commodity prices. Further, unlike the official SARB measure of the output gap the commodity price augmented gap was in positive territory post 2010 (this can be attributed to the cyclical build-up of the boom period and a mod-

erate recovery in commodity prices following the GFC). This may partly explain why inflation was breaching the target by mid-2014.

Of course not taking commodity prices into account results in overestimating potential growth and a monetary policy stance that is too loose. This we illustrate using a simple policy rule along the lines originally proposed by Taylor (1993).

Our benchmark model suggests that leading up to the crisis the policy rate should have on average, been 91 basis points higher than suggested by the HP filter and 59 basis points higher than the official SARB output gap. Similarly, policy should have been 60 basis points higher than suggested by the HP filter and 81 basis points higher than the official SARB output gap in the post-crisis period. We conjecture that all other things equal a higher policy rate leading up to the global financial crisis would have helped to tame inflation which breached the upper bound of the inflation target from 2007Q2 to 2009Q4 and again by mid-2014.

These results demonstrate the danger of not responding to the cyclical impact of commodity prices.

Finally it is desirable to have a measure of the output gap that does not vary wildly with additional information. Our benchmark model (with commodity prices) - has a mean absolute revision error of 1.04 percentage points after two years. The HP filter gap has a mean absolute revision of 1.12 percentage points. Thus, including commodity prices in the output gap specification makes the output gap slightly more robust to revision errors, although, less than what was achieved by Anvari et al. (2014). This is potentially important, as any measure that varies a lot could induce monetary policy mistakes.

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## Appendices

### A Estimation and Filtering Simultaneously

In this appendix we show how estimation and filtering are interrelated. That is, the output gap depends on parameter estimates and the latter on the output gap. Here we estimate and filter using the full sample set-up of the filter.<sup>25</sup> For ease of exposition we show this for the case of one independent variable and no lagged output gap. However, the set-up can be generalized to the case with a lagged output gap and several independent variables.

The problem is

$$\min_{\{y_t^*\}_{t=1}^T \hat{\gamma}} \left[ \sum_{t=1}^T (y_t - y_t^* - \hat{\gamma}x_t)^2 + \lambda \sum_{t=2}^{T-1} (\Delta y_{t+1}^* - \Delta y_t^*)^2 \right] \quad (10)$$

Here we have a stationary series  $\{x_t\}_{t=1}^T$ . The first-order conditions of the multivariate HP filter (conditional on the estimate) are

$$\begin{aligned} y_1 - y_1^* - \hat{\gamma}x_1 &= \lambda(y_1^* - 2y_2^* + y_3^*) \\ y_2 - y_2^* - \hat{\gamma}x_2 &= \lambda(-2y_1^* + 5y_2^* - 4y_3^* + y_4^*) \\ y_t - y_t^* - \hat{\gamma}x_t &= \lambda(y_{t-2}^* - 4y_{t-1}^* + 6y_t^* - 4y_{t+1}^* + y_{t+2}^*), \quad t = \{3, 4, 5, \dots, T-2\} \\ y_{T-1} - y_{T-1}^* - \hat{\gamma}x_{T-1} &= \lambda(y_{T-3}^* - 4y_{T-2}^* + 5y_{T-1}^* - 2y_T^*) \\ y_T - y_T^* - \hat{\gamma}x_T &= \lambda(y_{T-2}^* - 2y_{T-1}^* + y_T^*) \end{aligned}$$

and the OLS normal equation (conditional on the multivariate output gap) is

$$\sum_{t=1}^T ((y_t - y_t^*)x_t) = \hat{\gamma} \sum_{t=1}^T (x_t^2) \quad (11)$$

<sup>25</sup>In the actual estimation in the paper the IRIS toolkit is used that works with MATLAB.

The problem above consists of two interdependent exercises; filtering and estimation.

From the first-order conditions for the multivariate filter it can be easily seen that if the parameter is known we have a simple multivariate extension of the HP filter (no estimation), the additional state variable is the series  $\{x_t\}_{t=1}^T$ .

Conversely if the output gap is known (for example inferred from the univariate HP filter) we can simply run the OLS regression **(11)**.

In the full problem the estimate depends on the multivariate output gap per equation **(11)**, where the latter is filtered conditional on the estimate according to the first order conditions (for the multivariate filter). Thus, estimation and control are interdependent. The system above can be solved as we have  $T + 1$  equations and as many unknowns ( $T$  levels of potential output and one OLS coefficient estimate).

If we find a significant parameter, then there is empirical support for the multivariate filter and potential output does not only depend on actual output (univariate case) but also on the additional state variable  $x$ .

## B Comparison of the Multivariate Filter with Various Alternative Univariate Filters

We now provide some additional discussion of the Baxter-King (*BK*), Butterworth (*B*) and Christiano-Fitzgerald (*CF*) filters and how those results differ from those of our MVF filter.

More specifically, we look the difference between the multivariate filter (*MVF*) - Model 2 including commodity prices and the lagged output gap - and a number of popular univariate band-pass filters (*BK*, *B* and *CF*). We do this by decomposing this difference into (i) the difference between the multivariate HP and univariate HP filters and (ii) the difference between the alternative univariate filter and the univariate HP filter. Thus, we get:

$$(y_t - y_t^*)_{MVF} - (y_t - y_t^*)_i = [(y_t - y_t^*)_{MVF} - (y_t - y_t^*)_{HP}] - [(y_t - y_t^*)_i - (y_t - y_t^*)_{HP}] \quad (12)$$

where  $i = \{BK, B, CF\}$ .

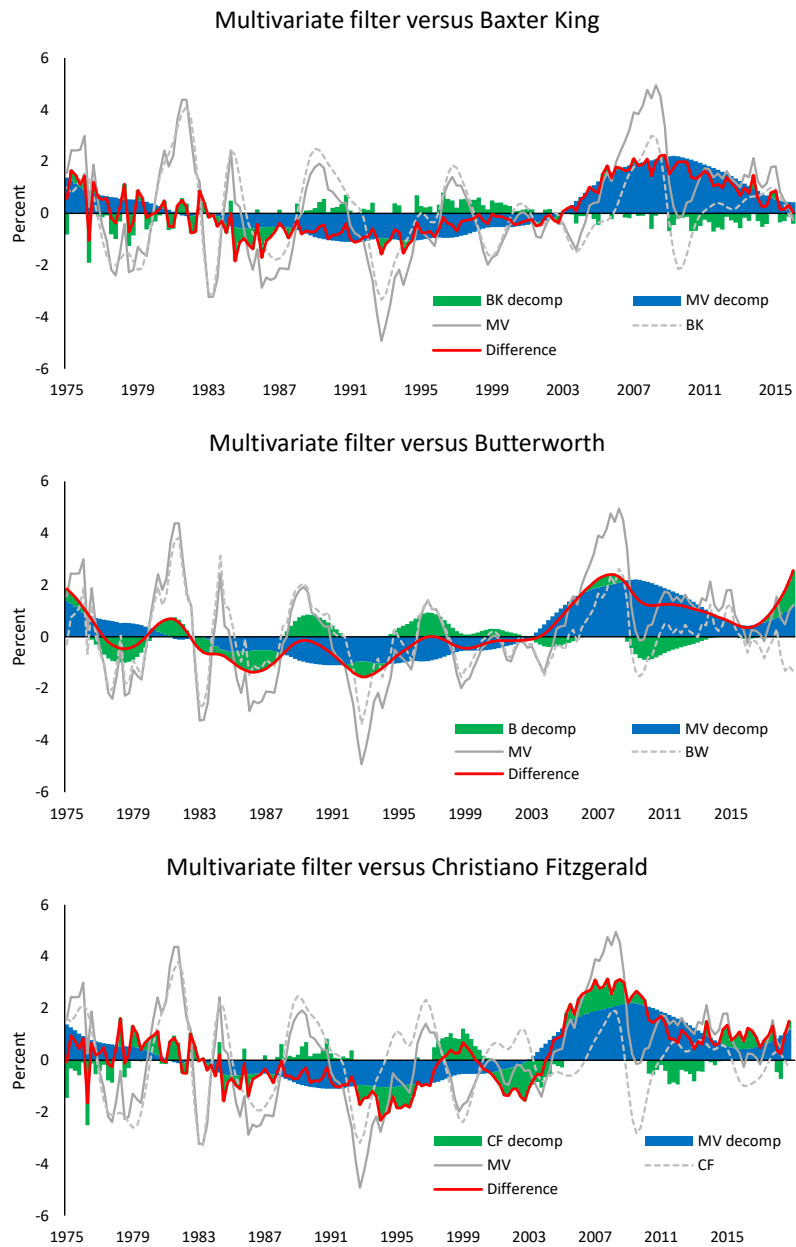
The first term reflects the crux of our analysis (impact of commodity prices and the lagged output gap), whereas the second term investigates the impact of the alternative univariate filtering technique.

In figure **(10)** below, we can see that this simple analysis suggests that the difference in univariate filtering technique is relatively unimportant and what is driving the difference from the baseline HP filter result is the impact of commodity prices. The table that follows lists the parameter specification for the various band-pass filters estimated in R's *m* Filter package. We followed the specification in Fedderke & Mengisteab (2017)

where possible and relied on the package defaults where the literature was not explicit about the parameters used.



**Figure 10: Multivariate filter versus various band-pass filters**



**Table 3: Specification of estimated band-pass filters**

Parameter	Baxter-King	Butterworth	Christiano-Fitzgerald
Type	Fixed	NA	Asymmetric
Lead-length	12	NA	NA
Drift	True	True	True
Period lower	6	NA	6
Period upper	32	NA	32
Order	NA	2	NA
Frequency	NA	10	NA
Root	NA	NA	True
Theta	NA	NA	1

## C Sample robustness

Table (4) presents the benchmark model (Model 2) estimated over the Inflation Targeting era (2000-present) as a simple robustness test against structural breaks. As can be seen from the table all coefficients remain significant, with the coefficient on commodity prices being slightly larger in magnitude.

**Table 4: Shortened-sample estimates of model coefficients, posterior mode with t-statistics in parenthesis**

	Model 2
Dynamic gap	0.946 (22.693)
Commodities	0.012 (3.740)
Capital flows	
Credit	
Capacity utilisation	
US recession dummy	
Sample = 2000Q1-2019Q1. Lambda = 1600. t-statistics in parenthesis.	

## D Sensitivity to priors

By definition all informative priors influence the resulting coefficient estimates. It is, however, a reasonable question to ask how much information content in the estimates is a result of the researcher's priors and how much of the information content is contained within the observed data? In order to give a rough answer to this question we again looked at our benchmark model, Model 2, and re-estimated the model while relaxing our priors. We relaxed the priors by doubling the standard deviation of the prior distributions. The results are reported in table (5).

**Table 5: Estimates of model coefficients with relaxed priors, posterior mode with t-statistics in parenthesis**

	Model 2	Model 2 relaxed priors
Dynamic gap	0.950 (27.583)	0.950 (28.181)
Commodities	0.010 (2.652)	0.009 (2.175)
Capital flows		
Credit		
Capacity utilisation		
US recession dummy		
Sample = 1972Q1-2019Q1. Lambda = 1600. t-statistics in parenthesis.		

The results show that softening the priors makes no difference to the estimate for the dynamic gap, which has very strong support in the data. On the other hand, the coefficient estimate of the contribution of commodity prices declines in magnitude and has less statistical support under the relaxed priors. The coefficient, however, remains statistically significant. Looking at figure (11), we see that the decline in the magnitude of the estimated coefficient has a negligible impact on the estimated output gap.

**Figure 11: Output gaps, specification 2**



## E Unit root test

Table (6) below presents the augmented Dickey-Fuller unit root test applied to the various candidate regressors. Only in the case of capacity utilisation there is not sufficient evidence to reject the null hypothesis that the series contains a unit root at a 5% level of significance.

**Table 6: Augmented Dickey-Fuller unit root test. Null hypothesis: series contains a unit root**

	Test statistic
Commodities	-6.178 (-1.95)
Capital flows	-3.966 (-1.95)
Credit	-5.459 (-1.95)
Capacity utilisation	-0.275 (-1.95)

Sample = 1972Q1-2019Q1. 5% critical value in parenthesis.