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Lauren Kuhn, Franz Ruch and Rudi Steinbach

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South African Reserve Bank

Enquiries

Head: Economic Research and Statistics Department
South African Reserve Bank
P O Box 427
Pretoria 0001

Tel. no.: +27 12 313-3911
0861 12 SARB (0861 12 7272)

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Abstract

The global financial crisis (GFC) saw real interest rates fall to all-time lows as central banks aimed to stimulate economic activity. The effectiveness of such low real rates depends, to a large extent, on the neutral real interest rate — popularly referred to as r-star. Monetary policy is considered expansionary when real interest rates are below r-star, and vice versa. However, the challenge arises from the fact that r-star is unobservable. This paper estimates r-star in the spirit of the popular Laubach-Williams (LW) methodology, but adapts their approach to capture the dynamics of a small open economy. This is achieved by incorporating additional drivers of the neutral rate, such as domestic net savings and investment, South Africa's country risk premium, and the potential growth rate of our trading partners. In addition, foreign linkages like the exchange rate and international commodity prices are included to capture the impact of developments in the rest of the world on South African growth and inflation. The results suggest that South Africa's r-star has fallen less than in advanced economies — from an average of 4.4 per cent from 2000 to 2006 to 1.9 per cent in 2017Q4.

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Corresponding author's e-mail address: lauren.kuhn@resbank.co.za

* *Corresponding author*. Email: lauren.kuhn@resbank.co.za. The views expressed are those of the author(s) and do not necessarily represent those of the South African Reserve Bank or Reserve Bank policy. While every precaution is taken to ensure the accuracy of information, the South African Reserve Bank shall not be liable to any person for inaccurate information or opinions contained herein.

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1 Introduction

The global financial crisis (GFC) and subsequent great recession required extraordinary accommodative policy to stabilize economic activity. This saw real interest rates fall to multi-decade lows as households deleveraged, demand for safe assets rose, and advanced economy investment fell. These factors should also affect estimates of the neutral real interest rate (henceforth r-star) – the rate at which inflation is stable around the central bank's inflation target and output is operating at its potential. The importance of r-star lies in its indication of the extent of monetary policy stimulus. If real interest rates are below the level of r-star, monetary policy is supportive of economic activity, and vice versa. While the output loss following the GFC required real rates to be exceptionally supportive, recent growth momentum in advanced economies, coupled with indications of rising inflation, has seen normalisation of monetary policy to equilibrium levels — in line with their respective r-stars. In South Africa, monetary policy has been gradually normalising in real terms since 2014. To assess the appropriateness of the pace of reduction in policy accommodation, estimates of r-star are crucially important.

We build an open economy version of the Laubach and Williams (2003) model to estimate r-star in South Africa. Laubach and Williams (2003) (henceforth LW) base r-star estimates on two factors: (1) the underlying potential growth rate of an economy, and (2) an unobservable factor z that captures other likely drivers of r-star, such as time preferences of households, within the confines of a closed economy. Similar to Wynne and Zhang (2017), we expand on this formulation by expressing r-star as a function of both domestic and trading partner potential growth. Furthermore, inspired by Pescatori and Turunen (2015), we define likely drivers of the unobserved component z, such as South Africa's savings and investment imbalance, and country risk premium. In addition, the model's description of the processes that determine growth and inflation are enriched by adding the exchange rate channel and allowing for the impact of developments in the rest of world. Unlike LW, we use Bayesian techniques to estimate the model.

The results suggest that r-star has fallen in South Africa since the GFC, although to a lesser extent than estimates for the United States and other advanced economies. The decline in the r-star is mainly the result of slowing domestic potential growth that is estimated to have declined from an average of about 4 per cent in the mid-2000s' to an average of only 0.7 per cent in 2017. R-star is estimated at 1.9 per cent in the final quarter of 2017, this estimate could have been lower if there were no other factors, such as rising domestic country risk keeping it elevated. Given that r-star is unobservable, there is a high uncertainty about its actual value.

The paper proceeds as follows: section 2 provides a brief literature review on the neutral real interest rate; section 3 describes the model; section 4 discusses the estimation approach, including the data used and posterior estimates; section 5 provides the estimation results; section 6 looks at the robustness of the results to certain estimation changes; before section 7 concludes.

2 Literature review

Wicksell (1936) originally conceived the concept of r-star as the *natural* rate of interest that is neutral in respect to prices and "tends neither to raise nor to lower them." Further, he argued that the level of r-star may change over time depending, for example, on factor productivity and availability (Beyer and Wieland, 2017). Similar definitions are used in papers by Laubach and Williams (2003, 2016), Pescatori and Turunen (2015), Orphanides and Williams (2002), among others.

LW defined the natural, or neutral, real interest rate as the real short-term rate that is consistent with stable inflation and output at potential. This is the level of the interest rate that is expected to prevail in normal times, when the economy has recovered from any business cycle fluctuations. The position of the real interest rate relative to r-star then informs the stance of monetary policy. Real interest rates below r-star would be expansionary, while rates above it would be contractionary.

LW uses a semi-structural model that builds on the New Keynesian framework of an intertemporal investment-savings (IS) relationship and a Phillips curve. Together, these two equations describe the dynamics of the output gap as a function of the real interest rate *gap*, and in turn, inflation as a function of the output gap.[1] They apply a Kalman filter to jointly estimate r-star and the level of potential output — ultimately linking r-star to the economy's estimated trend growth rate. Furthermore, the authors use the median-unbiased estimates approach of Stock (1994) and Stock and Watson (1998) on the coefficients of r-star and trend growth variations, and estimate the remaining model parameters using maximum likelihood.

Building on LW's closed economy model, Wynne and Zhang (2017) estimate a two-country framework. Like LW's original model, r-star is linked to the trend growth rate (underlying potential growth rate). However, Wynne and Zhang (2017), extend this r-star calculation to include the underlying potential growth rate of the foreign country, in order to examine the impact of foreign shocks on open economies. In addition, by applying Bayesian estimation, their prior information avoids the pile-up problem that often afflicts maximum likelihood methods.[2] Other work that also builds on LW include Holston et al. (2017) on the US, Canada, and Euro Area; and Beyer and Wieland (2017) for the US, Euro Area, and Germany.

Several alternative frameworks have been used to estimate r-star, including the Taylor rule (Taylor and Wieland, 2016); consumption-based methods (Hamilton et al., 2016 and Fuentes et al., 2007); the yield curve to determine market participants' beliefs of future short-rates (Bomfim, 2001); and statistical methods such as averaging and filters (see, for example, Hodrick and Prescott, 1997; and Cogley, 2002 for applicable filter options).

More theoretically founded models such as the dynamic stochastic general equilibrium (DSGE) are also used to estimate r-star, as opposed to the popular semi-structural approach of LW. However, r-star estimates from these models are found to show more variability than is commonly attributed to equilibrium real interest rates (Pescatori and Turunen, 2015). Additionally, given that a semi-structural model in the spirit of LW only imposes mild cross-equation restrictions, and is able to exploit information from other variables; the model is less likely to be miss-specified — especially in the presence of near

[1] The real interest rate gap is the difference between the real interest rate and r-star.

[2] In models where series are decomposed into trends and cycles, the variance of a trend's growth rate may be non-zero, but small in comparison to the overall variance of the trend. In these situations, the pile-up problem refers to the tendency of maximum likelihood techniques to estimate the standard deviation of the trend growth rate as being zero, when in fact it is not. However, through proper allocation of priors, Bayesian estimation can avoid this so-called pile-up problem of maximum likelihood highlighted in Stock (1994). An alternative approach would be to use the methodology of Holston et al. (2017) by imposing the Stock and Watson (1998)'s median unbiased estimator of the ratio of the relevant nuisance parameters when estimating the remaining model parameters through maximum likelihood.

non-stationarity in observed real rates (Barsky et al., 2014; Cúrdia et al., 2015; Pescatori and Turunen, 2015).

An alternative, but complementary definition of r-star that reflects the interest rate dynamics of a small open economy, is the uncovered interest parity condition, as it captures global factors via exchange rate movements and changing risk premia. Bernhardsen and Gerdrup (2007) model the risk-adjusted uncovered interest rate parity whereby the domestic real interest rate is driven by its foreign counterpart, the expectation of the change in the real exchange rate and risk premiums. They find that interest rate parity provides a reasonable explanation for why domestic interest rates may be influenced by the foreign interest rates, at least to avoid exchange rate depreciation. This is also true for global savings and investment behaviour, whereby the global r-star may influence the r-star in a small open economy. However, depending on the small open economy's idiosyncrasies, the relationship between the global and domestic neutral rate may not be easy to identify.

3 The model

We use a modified version of LW's semi-structural model to estimate r-star, by adjusting their model in two dimensions. First, since South Africa is a small open economy reliant on commodity exports, we include open economy aspects such as the exchange rate, commodity prices, and foreign demand pressures. To introduce these channels we modify the IS curve as:

$$\tilde{y}_t = \alpha_y \tilde{y}_{t-1} + \alpha_r (r_t - r_t^*) + \alpha_{y^f} \tilde{y}_t^f + \alpha_q (q_t - q_t^*) + \alpha_c com_t + \varepsilon_t^{\tilde{y}} \tag{1}$$

where the output gap, $\tilde{y}$, is the difference between real GDP and its potential (i.e. $\tilde{y}_t = y_t - y_t^*$), $r_t - r_t^*$ is the gap between the real interest rate and its neutral rate, $\tilde{y}^f$ is the foreign output gap, $q_t - q_t^*$ is the gap between the real exchange rate and its equilibrium, and com_t is the real price of commodity exports.[3,4]

Reminiscent of the neo-classical growth model, LW's key motivating equation bases the relationship between the real interest rate, underlying potential growth rate and behavioural parameters on basic household intertemporal utility maximisation:

$$r = \sigma g_c + \theta \tag{2}$$

where r is the real interest rate, g_c is the per capita growth in consumption, σ is the inverse intertemporal elasticity of substitution in consumption, and θ is the rate of time preference. LW simplify the theoretical relationship in Equation (2) by linking the neutral real interest rate, r_t^*, to the underlying potential growth rate of an economy g_t, as well as z_t, which captures all other determinants of r_t^*:

$$r_t^* = c g_t + z_t \tag{3}$$

[3]Unlike LW, we do not use more than one autoregressive coefficient in the estimation step as there is not a significant difference in dynamics and it is more closely linked to the theoretical foundations of intertemporal optimisation.

[4]Since the model is not closed by a policy rule, the real interest rate is modelled as a random walk. We assume com_t and q_t follow AR(1) processes.

Wynne and Zhang (2017) extends LW's r-star formulation to show that the neutral rate for an open economy setting would evolve as:

$$r_t^* = c_1 g_t + c_2 g_t^f + z_t \tag{4}$$

where g_t and g_t^f are the underlying growth rates of potential output for the domestic and foreign economy, respectively. Together, c_1 and c_2 represent the inverse elasticity of substitution found in Equation (2).[5]

Due to its unobservable nature, z, or the 'headwinds' to r-star, has been the subject of criticism.[6] For example, z can either be a random walk or autoregressive, and choices about its statistical properties are often subjective and may have a measurable impact on the eventual estimate of r-star.

In order to address some of this criticism, we adjust the model in a second dimension by introducing potential drivers of z, although in a somewhat ad hoc way. We choose to focus on specific drivers from two other theories of interest rates: loanable funds and uncovered interest parity. Loanable funds theory shows the need to include shifts in the supply (savings) and demand (investment) of loanable funds as drivers of interest rates. Uncovered interest parity highlights the importance of country risk. Similar to Pescatori and Turunen (2015) and Blanchard et al. (2014), z is modified such that:

$$z_t = \gamma_z z_{t-1} + \gamma_p \Delta p_t - (\gamma_s \Delta s_t - \gamma_i \Delta i_t) + \varepsilon_t^z \tag{5}$$

where p_t is the country risk premium (proxied by the EMBI+ strip spread for South Africa), and the domestic current account balance is proxied by net savings (s_t) and net investment (i_t) to GDP. The country risk premium and current account proxies are modelled as autoregressive functions with a persistence term ρ_j with $j \in p, s, i$. The autoregressive term (γ_z) is set to 1 in order for z to mimic a random walk, but with time-varying drift.

Unlike in Pescatori and Turunen (2015), we only use domestic factors to drive outcomes in z. All changes to r-star that occur, as a consequence of global headwinds, are assumed to be reflected either directly through foreign potential growth or indirectly through its impact on the country risk premium and the current account via savings and investment. The advantage of this approach is that we can link changes in the domestic savings-investment schedule directly to changes in r-star, without having to make claims about the indirect impact of the demand for safe assets or the size of global savings. The impact of these factors, in particular, can also be ambiguous. As an example, consider the fall in global r-star as a result of rising demand for safe assets. From an uncovered interest rate parity perspective, the falling global r-star should decrease the domestic neutral as well, suggesting the sign on the demand for safe assets proxy should be negative. However, shifts in risk perceptions that accompany flight to safety may lead to significantly higher country risk premiums as capital outflows occur.

The Phillips curve is modified such to reflect the inflation process in a small open economy:

$$\pi_t = (1-\beta_\pi)\pi_t^e + \beta_\pi \pi_{t-1} + \beta_y \tilde{y}_t + \beta_q (q_t - q_t^*) + \beta_o \pi_t^o + \varepsilon_t^\pi \tag{6}$$

[5] If σ is the inverse elasticity of substitution, and γ is the degree of openness of the home economy, then it can be shown that $c_1 = \sigma - \gamma(\sigma - 1)$ and $c_2 = \gamma(\sigma - 1)$ (see Clarida et al., 2002).

[6] See, for example, Pescatori and Turunen (2015) and Beyer and Wieland (2017).

where π is core inflation, π_t^e is inflation expectations, $\tilde{y}_t$ is the output gap, $\tilde{q}_t$ is the real exchange rate gap, and π_t^o is rand-denominated oil price inflation. Unlike LW, but consistent with the South African Reserve Bank's (SARB) main forecasting model (see Botha, de Jager, Ruch, and Steinbach, 2017), we use the expectations-augmented Phillips curve.[7] As the model is semi-structural and not closed with a policy rule, we observe inflation expectations from the Bureau of Economic Research (BER) survey of inflation expectations (labelled π_e). Although we model core inflation, we use two-year ahead headline inflation expectations, which should largely exclude relative price shock related to food and energy; and be a good proxy of core inflation.

Potential output and underlying potential growth are modelled as:

$$y_t^* = y_{t-1}^* + g_t/4 + \varepsilon_t^{y^*} \tag{7}$$

and

$$g_t \quad = \quad g_{t-1} + \varepsilon_t^g \tag{8}$$

where the (log) level of potential output (y^*) evolves by the quarter-on-quarter annualised underlying potential growth rate, g. In turn, g is a random walk. These equations allow for idiosyncratic factors such as droughts and strikes to affect the level of potential output, creating for temporary deviations from the underlying growth process, g. This approach to modelling potential output dates back to Clark (1987), who applied it to US data within a highly parsimonious model structure.

In the model, the rest of world is proxied by the United States, and the model structure and prior calibration are based on LW. In contrast, coefficients on autoregressive functions of lags greater than one are summed and used as a single coefficient. Since the SARB has an in-house global economy model, based on Carabenciov et al. (2013), we use data from this model instead of LW. See the data section for details on the data used and its source.

4 Estimation

We implement a Bayesian estimation procedure instead of the maximum likelihood approach in LW. While Bayesian inference could be subject to misleading results, if not correctly specified, it allows us to impose prior views and take account of the uncertainties related to the parameters. Through analysis of historical data, as well as many well-studied coefficients (including the slope of the Phillips curve; the persistence in inflation; and the impact of the foreign output gap on its domestic counterpart), the use of prior information yields more plausible results, especially for the model's unobserved variables.

Bayesian estimation re-weights the likelihoods of the parameters of interest using prior information, and maximises these re-weighted likelihoods as follows:

$$P(\Omega|X^T) = \frac{P(\Omega)L(X^T|\Omega)}{P(X^T)} \tag{9}$$

where the posterior distribution of the unknown parameters conditional on the observed data over time

[7]See Friedman (1968) for a Phillip's curve that incorporates inflation expectations.

T, $P(\Omega|X^T)$, is proportional to the product of the prior distribution, $P(\Omega)$, and the likelihood function of the observed data over time T conditional on the unknown parameters, $L(X^T|\Omega)$. $P(\cdot)$ is the probability density function which can take on any possible distribution.

We set out the specific prior used for each parameter in Table 2, describing its prior mean, probability distribution and standard error. The posterior distribution is then estimated through the integration of the likelihood function, with the help of the Kalman filter, over the parameter space using priors as weights. However, as pointed out by Rabanal and Rubio-Ramírez (2005), an analytical solution of the above problem is only available in a restrictive set of examples and therefore we require Markov chain Monte Carlo methods such as the Metropolis-Hastings algorithm, in order to solve for the unknown posterior distributions. The specific type of algorithm used in this estimation is a rejection sampling algorithm. A jump distribution is used to sample specific values of the unknown parameters, where the posterior value is either accepted or rejected based on acceptance probabilities to monitor the frequency of acceptance in the Metropolis step (Gelman et al., 1996). An initial burn-in phase is also taken to ensure that the posterior distribution is formed from the stable section of the Markov chain.

The priors on z, as well as r-star, are based on Ordinary Least Squares (OLS) estimates. Given that z is unobserved we generate OLS estimates for the parameters of the assumed underlying observed components of z to an HP-filtered version of SA's real interest rate.

4.1 Data

Both foreign and domestic data are used in the estimation step (described in Table 1). We observe foreign trend potential growth (g^f), the output gap ($\tilde{y^f}$), r-star (r^*_f), and real interest rate for the US economy from the global projection model (GPM) of Carabenciov et al. (2013). Other foreign variables include US core personal consumption expenditure (PCE) inflation (core PCE inflation, π^f), Brent crude oil prices, and real commodity prices (deflated by US core PCE inflation) that is weighted by SA exports.

Domestically, real GDP and consumer inflation excluding food and energy (as a measure of core inflation), are obtained from Statistics South Africa (Stats SA). We also use inflation expectations from the Bureau of Economic Research (BER) survey of inflation expectations. Although we model core inflation, we assume that the available headline inflation expectations of two-years ahead should exclude relative prices, hence approximate core inflation.[8] We observe real interest rates, in accordance with the Fischer equation, as the difference between the nominal repurchase rate and the one-year ahead inflation forecast from the South African Reserve Bank's core econometric model.[9] The country risk premium is proxied by the EMBI+ spread for South Africa.[10] The current account is proxied by net savings less net investment as a percentage of SA's GDP. We deduct consumption of fixed capital at replacement value from gross savings and gross fixed capital formation, to calculate net savings and investment to GDP. We use the HP-filtered outcomes to capture the trends in both country risk premium and the savings-

[8]Given that the series is survey based, we assume that the candidates surveyed do not consider relative price shocks to food and energy for their expectations of inflation two years ahead.

[9]As the SARB's policy reaction function, using a Taylor type rule, was based originally in the SARB's core econometric model projections of headline inflation.

[10]To deal with the volatility in the spread, we applied a HP filter which (in line with the actual data) suggests that country risk has remained elevated post-GFC. This is corroborated using the Bloomberg index of country risk for South Africa which remains significantly elevated compared with a pre-crisis view.

investment balance.

4.2 Posterior estimates

The model is estimated using a Metropolis-Hastings algorithm with 100 000 iterations on data from 2000Q1 to 2017Q4. The priors and posteriors are provided in Table 2. Priors for some of the parameters, such as the slope of the Phillips curve are based on work by Anvari et al. (2014) and Kabundi et al. (2016). The prior of 0.05 on c_2, the coefficient on the contribution of the foreign economy's potential growth rate to r-star, is broadly consistent with an elasticity of substitution of 0.85 and degree of openness around 0.3. Other priors that are specific to the model used here are based on Pescatori and Turunen (2015) and LW. The priors are assumed to follow standard probability distributions. We also estimate foreign economy coefficients (see Appendix C for details).

For the standard deviation of the IS equation's residuals, $\varepsilon_{\tilde{y}}$, the prior is based on that of an HP-filter output gap. Following Borio, Disyatat, and Juselius (2013), we scale the prior on the standard deviation of the trend growth residual, ε_g, such that the signal-to-noise ratio of the Kalman filter is similar to that of the HP-filter. Importantly, this signal-to-noise ratio is not imposed on the posterior.

Most posterior values are in line with plausible expectations. In the core inflation Phillips curve, the persistence of inflation is aligned to its prior at 0.65. The slope of the Phillips curve (β_y) has a posterior of 0.2 implying that a one per cent change in the output gap raises core inflation by 0.2 per cent (marginal elasticity). A 10 per cent undervaluation of the real effective exchange rate tends to increase inflation by 0.22 per cent. In the IS curve equation, the output gap has a persistence of 0.55. A one percentage point increase in the foreign output gap raises the SA output gap by 0.21 per cent, while a 10 per cent undervaluation of the currency raises the output gap by 0.08 per cent.

5 Results

Using a modified version of the LW model to generate r-star estimates for the South African economy, indicates that the neutral real interest rate has fallen since 2006 in line with global neutral rates and worsening potential growth. South Africa similarly suffered from a widening output gap and falling trend potential during the GFC, as was the experience of many economies at the time. But other factors also affect r-star. The unobserved z component now has a more clear interpretation of these factors by reflecting the country risk premium and South Africa's savings-investment imbalance.

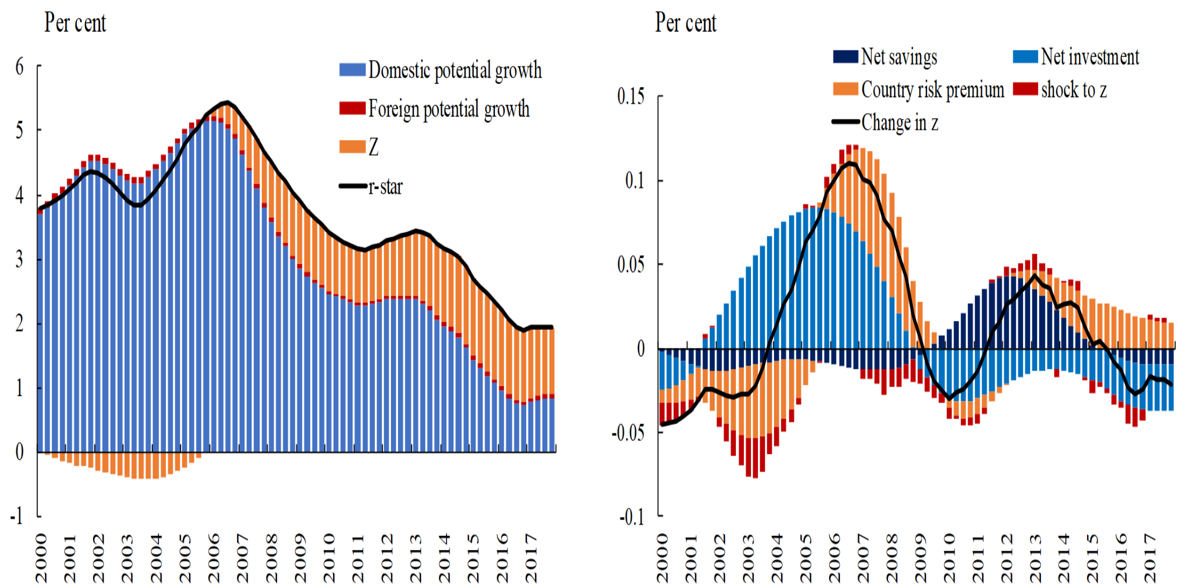
5.1 Neutral real interest rate

Figure 1a plots the decomposition of r-star into underlying domestic (g_t) and foreign potential output growth (g_t^f), as well as the unobserved z component. We then plot z into its main drivers in Figure 1b — graphed as the change in z given the unit root process. Domestic potential growth is the most important driver of r-star, explaining on average three-quarters of the outcome. z is the second biggest driver explaining most of the rest, with foreign potential only contributing marginally to r-star.

Potential growth in South Africa rose sharply in the early- to mid-2000s, reaching a peak of 4.6 per cent in 2006Q1. This was driven by robust investment and credit growth, rising house prices, and a favourable global backdrop with rising commodity prices and external demand. The neutral interest rate reflected the high potential growth rates but was partially offset by other factors in z, including a generally low and falling country risk premium and rising savings, which put downward pressure on the domestic r-star. As a consequence, r-star is estimated to have averaged 4.4 per cent from 2000 to 2006, rising through most of this period from 3.8 per cent in the first quarter of 2000 to 5.4 per cent in 2006Q4.

Figure 1: Neutral real interest rate

(a) Decomposition of r-star **(b) Decomposition of the change in z (first difference)**



The GFC led to a significant slowdown in global demand, ended the housing market boom in both South Africa as well as other advanced and emerging markets, and later in the next decade, the fall of commodity prices. From 2007 onwards, potential growth in South Africa is estimated to have slowed from 4.6 per cent to an average below 1 per cent in 2017. A number of factors could have contributed to this decline. These include a significant slowdown in investment and hence the accumulation of capital stock since 2009; falling domestic and international total factor productivity; worsening terms of trade due to falling export commodity prices; binding domestic constraints in electricity; and falling global potential growth particularly due to the Chinese economy's shift away from exports and investment. The r-star fell along with the post-GFC slowdown in South Africa's potential growth, however, other factors have contributed to keeping its level elevated. The unobserved component z has raised r-star by an average 1.0 percentage point since 2009.

To better understand the drivers of the unobserved component z, Figure 1b decomposes z recursively into its main drivers, including the country risk premium, and South Africa's savings-investment schedule. The figure is shown as the first difference given the unit root assumption made for z.

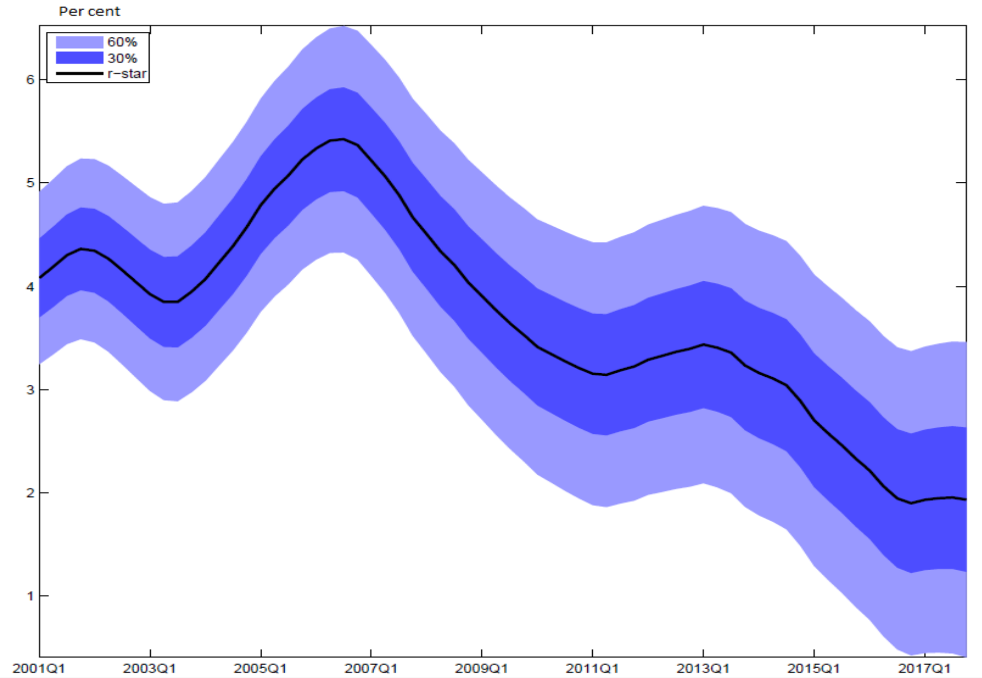
In the years preceding the GFC, South African GDP growth exceeded 5 per cent. Figure 1b illustrates how the need for investment over this period required a higher r-star. In the context of the theory of loanable funds, higher investment induces a rightward shift of the investment curve. However, as the increase in domestic savings at that time was not enough to fully address the need for more investment, it only marginally offset the rise in investment and resulted in a rising z. The interplay between strong investment growth with only marginal gains in savings growth underpinned a significant widening of the current account deficit.

Following the GFC, investment slowed dramatically, implying a leftward shift of the domestic investment curve and a lowering of the neutral real interest rate. There was also a significant deterioration in domestic savings as the government started to implement countercyclical fiscal policy. Household savings also declined, averaging 0.2 per cent of GDP between 2000 and 2008, before deteriorating to -0.7 per cent post-GFC. On the net, lower savings requires r-star to be relatively higher in order to attract capital that would meet South Africa's investment needs. Since 2016, however, the additional slowdown in investment has meant that the savings-investment balance has been putting downward pressure on r-star.

These two distinct periods in the savings-investment balance are also reflected in the impact of country risk premium on z. Prior to the great recession, country risk premiums in South Africa were comparatively low — reflecting strong growth outcomes and fiscal surpluses. Declining country risk premium meant that r-star was also comparatively low over this period. From 2008 onwards, however, the marginal contribution from country risk rose, following the deterioration in economic growth and fiscal balances, and compounded by rising policy and political uncertainty. The EMBI+ spread over this period rose from an average of 1.6 per cent from 2000 to 2008, to an average of 2.2 per cent post GFC (2009-2017Q4).

Figure 2 plots uncertainty around the r-star estimate over the sample. As the estimated standard deviation of r-star is 1.4 per cent, the implied bounds of a 60 per cent confidence interval have a range between 0.5 and 3.25 per cent in 2017Q4. The relatively large standard deviation of r-star is partly a consequence of the choice of a random walk for z. The alternative choice is to make z a mean-reverting process which ensures its variance will not be time dependent. When hit by the same set of shocks, a random walk process will always have a larger variance, compared with, an autoregressive mean-reverting process when $t > 1$.

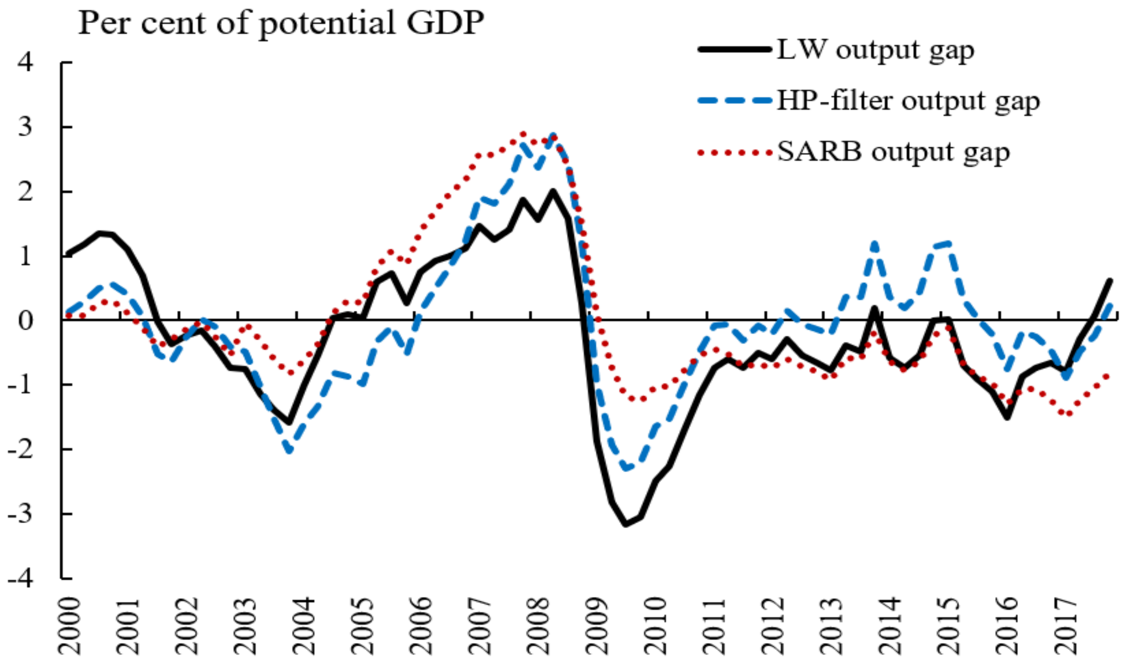
Figure 2: Uncertainty around r-star



5.2 Output gap

Estimating the neutral real interest rate using the LW methodology also generates a model-consistent output gap via the IS curve of Equation (1). Figure 3 compares this output gap (labelled LW) to the preferred estimate of the SARB (see Botha et al., 2018), as well as a simple HP-filter based estimate. The model's output gap is well correlated with other estimates of the gap. During the boom years of 2005 to 2008, both the LW and SARB output gaps are positive for longer than the HP-filter based gap. However, the LW-gap's peak is lower than that of the other methods. One likely driver of this may be the role of finance-neutral potential in the Bank's methodology (see Anvari et al., 2014, and Botha et al., 2018). During the great recession, the model suggests a significantly larger output gap compared with the other methods. Despite this, all methods suggest that the gap is closed by 2014. The model-consistent output gap shows that the gap shifted towards 0 and in fact became positive in the final quarter of 2017. This differs from the -0.8 per cent gap using the Bank's method. This difference is a function of a lower potential growth estimate; 0.7 in 2017Q4 while the SARB's estimate of potential is at 1.5 per cent 2017Q4.

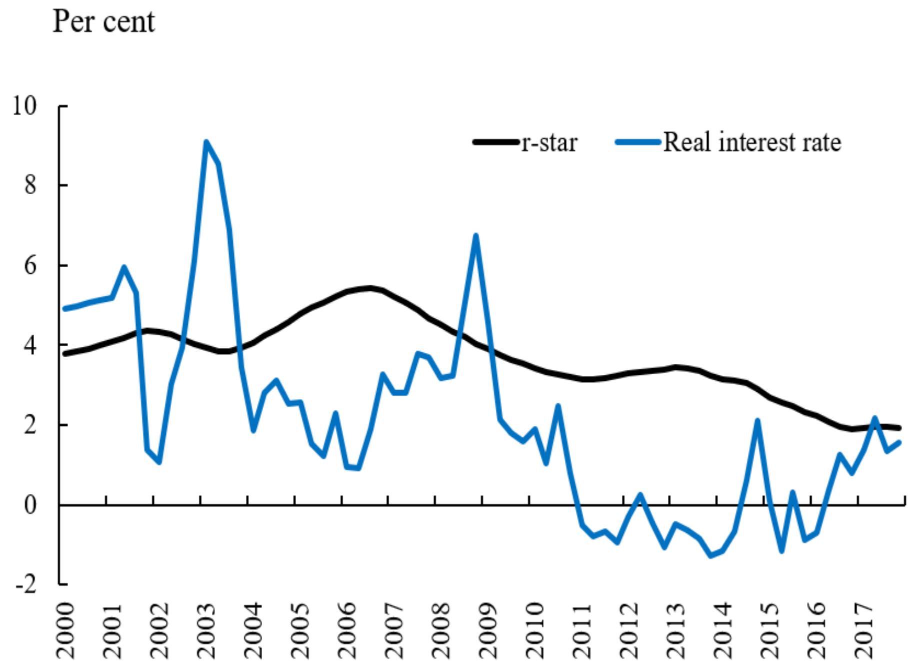
Figure 3: Output gap comparisons



5.3 Policy implications

When comparing the estimate of r-star to the real interest rate, we are able to identify over which periods monetary policy in South Africa has been either expansionary or contractionary. Figure 4 indicates that the first period of tightening of monetary policy occurred in the wake of the rand depreciation at the end of 2001. This rand weakness led to a bout of inflation very early in South Africa's inflation-targeting regime, to which policy responded. Subsequently, the rand appreciated and inflation fell, allowing policy to be eased. The estimates suggest that from 2004 to 2008, monetary policy was broadly expansionary, as the real interest rate remained below the neutral rate. Over this period, the South African economy was growing robustly, while the rand was mostly benefiting from a positive external environment due to the commodity price boom. Only for a brief period in 2009, policy tightened to the extent that it exceeded r-star. From 2010 onwards, the global financial crisis required accommodative monetary policy and our estimate suggests that over much of 2011 to 2014 policy was at its most accommodative since inflation targeting was introduced. The tightening cycle that commenced in early 2014 moved the real rate towards a neutral position by early 2017.

Figure 4: r-star relative to real interest rate



6 Sensitivity analysis

In this section we discuss some of the properties of the model, and its estimation, to provide an assessment of the sensitivity to our estimates. We look at the sensitivity of the results of z and r-star to the variance of the residual of z ($\varepsilon_{\tilde{z}}$).

6.1 Sensitivity to changes to the variance of z

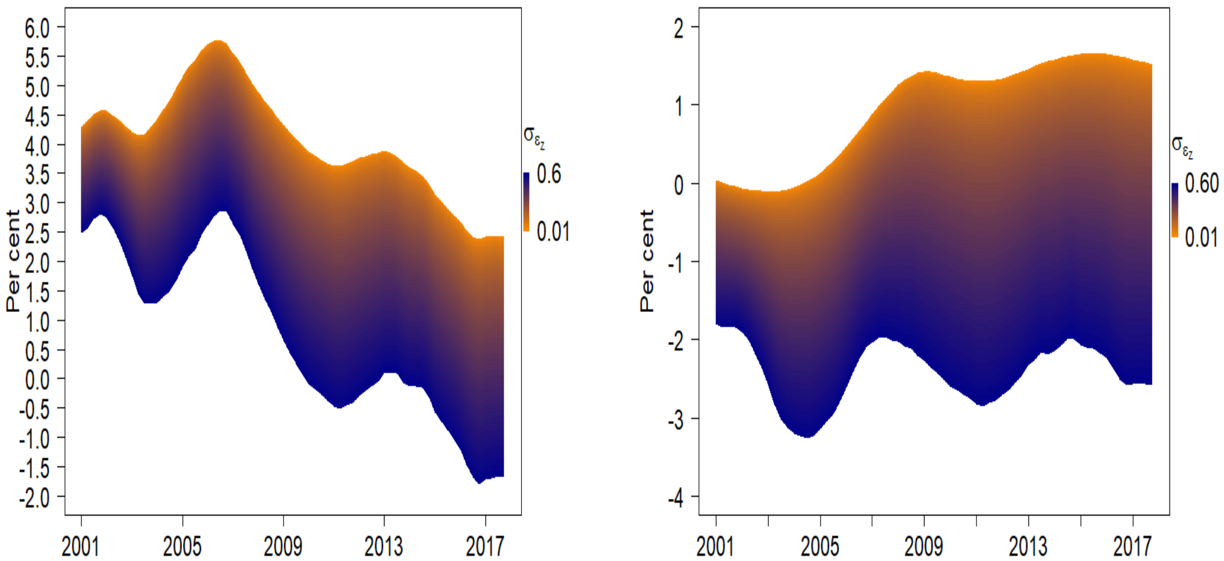
The LW approach is often criticised for the fact that a significant proportion of r-star is explained by the unobserved component z and that the estimated r-star is sensitive to z*'s* error variance. Pescatori and Turunen (2015) highlights three shortcomings with LW's approach including the fact that insufficient information is used to determine z[11]. We have addressed part of this criticism by providing a structure to z with drivers that are observable. However, it remains important to determine the sensitivity of the z process to changes in its error variance.

Figure 5a and 5b display the sensitivity of r-star and z to changes in the variance of ε_z. We plot heat maps which show the standard deviation of z between 0.01 and 0.6 in increments of 0.01. The estimate of r-star (in 5a) deviates by 4.2 percentage points between the smallest and largest standard deviation. On average, over the entire sample the difference between the largest and smallest standard deviation outcome is 3.4 percentage points. The deviations in r-star are entirely the result of changes to the shape and size of the unobserved component (figure 5b).

[11]The other two criticisms include that the output gap estimate bears no resemblance to estimates currently used for the US output gap and that there was no formal accounting for the zero-lower bound.

Figure 5: Sensitivity of r-star and z to changes to the variance of z

(a) Neutral real interest rate **(b)** z



7 Conclusion

The neutral real interest rate is an important benchmark for monetary policy. Policymakers can use estimates of r-star to assess whether the current policy stance supports or constrains growth which, in turn, either fuels or moderates inflation. In this paper, we adapt the commonly used Laubach and Williams (2003) methodology for estimating neutral real interest rates — or r-star — to the small-open economy characteristics of South Africa. In essence, Laubach and Williams (2003) model r-star as a function of the trend growth rate in an economy, and some unobservable factor. The dynamics of this unobservable factor can have far reaching consequences for the estimate of r-star. We follow recent research by identifying some of the likely drivers of this unobservable factor, thereby making its outcome easier to interpret. These factors include South Africa's country risk premium, a measure of SA's current account through net savings and investment; and we allow the foreign sector's underlying trend rate to influence the domestic economy's r-star in an open economy framework. In addition, we further expand the Laubach and Williams (2003) model to account for the impacts that the exchange rate, international commodity prices and developments in the global economy have on South African growth and inflation.

Our estimate of r-star rises in the build-up to the global financial crisis, and peaks at 5.4 per cent by the end of 2006. While this rise mostly reflects rising domestic and foreign potential growth, a comparatively low country risk premium over this build-up period served to moderate the level of r-star. In the aftermath of the global financial crisis, South Africa's potential growth fell dramatically, and pulled r-star down with it. However, this post-crisis period was also characterized by falling domestic savings, and a rising domestic risk premium. These factors have raised the estimate of r-star above what would be suggested by the fall in potential growth. As a result, the South African r-star is estimated at around 1.9 per cent by the end of 2017.

Appendices

A Data

Table 1: Data

	Variable	Transformation	Source
Domestic	Real GDP	Log level (seasonally adjusted and annualised)	StatsSA
	Core consumer inflation	Log first difference (seasonally adjusted)	StatsSA
	2-year ahead Inflation expectations	Log first difference (seasonally adjusted)	BER
	Real exchange rate gap	Per cent deviation from equilibrium	de Jager (2012)
	Real interest rate	Repo rate less one-year ahead inflation forecast from core model	SARB
	Country risk premium	SA EMBI+ spread (HP-filter), first difference	Bloomberg
	Net savings	Gross savings less consumption of fixed capital at replacement value per cent of GDP (HP filter), first difference	SARB
	Net fixed capital formation	Gross fixed capital formation less consumption of fixed capital at replacement value per cent of GDP(HP filter), first difference	SARB
Foreign	Output gap	US Real GDP as a per cent of potential GDP	Global Projection Model
	Underlying potential growth (g)	Percent	Global Projection Model
	Neutral real interest rate	Percent	Global Projection Model
	Core PCE inflation	Log first difference (seasonally adjusted)	FRED
	Brent crude oil	Log first difference	Bloomberg
	Real commodity price gap	Export weighted, Per cent deviation from HP-filter trend, deflated using US core PCE	Bloomberg and own calculations
	Effective federal funds rate	Percent	FRED

B Domestic economy prior and posterior estimates

Table 2: Prior and posterior estimates

Parameters	Priors			Posteriors	
	Distribution	Mean	Standard deviation	Median	5^{th} & 95^{th} percentiles
α_y	beta	0.5	0.1	0.546	[0.528 ; 0.594]
α_r	gamma	0.05	0.02	0.027	[0.024 ; 0.034]
α_{y^f}	gamma	0.25	0.1	0.205	[0.195 ; 0.213]
α_q	gamma	0.02	0.01	0.008	[0.007 ; 0.009]
α_c	gamma	0.05	0.02	0.023	[0.021 ; 0.024]
c_1	normal	1	0.20	1.120	[0.826 ; 1.509]
c_2	gamma	0.05	0.02	0.035	[0.032 ; 0.039]
β_π	beta	0.7	0.1	0.652	[0.491 ; 0.806]
β_y	gamma	0.25	0.05	0.198	[0.184 ; 0.225]
β_q	gamma	0.1	0.05	0.022	[0.020 ; 0.025]
β_o	gamma	0.02	0.02	0.319*	[0.188 ; 0.556]
γ_s	gamma	0.87	0.3	0.319	[0.297 ; 0.334]
γ_i	beta	0.33	0.05	0.299	[0.264 ; 0.364]
γ_p	beta	0.33	0.05	0.913	[0.495 ; 1.405]
ρ_p	beta	0.78	0.05	0.490	[0.311 ; 0.640]
ρ_c	beta	0.5	0.1	0.802	[0.795 ; 0.810]
ρ_o	beta	0.5	0.1	0.314	[0.221 ; 0.410]
$\rho_{E\pi}$	beta	0.5	0.1	0.827	[0.817 ; 0.838]
ρ_s	beta	0.5	0.1	0.477	[0.321 ; 0.637]
ρ_i	beta	0.5	0.1	0.487	[0.341 ; 0.634]
$\varepsilon_{\tilde{y}}$	inv.gamma	0.84	0.2	0.251	[0.243 ; 0.257]
$\varepsilon_{\tilde{\pi}}$	inv.gamma	1.39	0.2	0.745	[0.655 ; 0.855]
$\varepsilon_{\tilde{z}}$	inv.gamma	0.17	0.02	0.184	[0.168 ; 0.189]
ε_g	inv.gamma	0.21	0.05	0.167	[0.156 ; 0.179]
ε_r	inv.gamma	0.95	0.2	0.465	[0.452 ; 0.487]
ε_q	inv.gamma	5.16	1	2.470	[2.345 ; 2.572]
$\varepsilon_{\bar{y}}$	inv.gamma	0.21	0.05	0.132	[0.126 ; 0.137]
ε_o	inv.gamma	1	0.2	18.47	[16.17 ; 21.39]

C Foreign economy prior and posterior estimates

Table 3 shows the priors and posteriors for the estimated parameters.

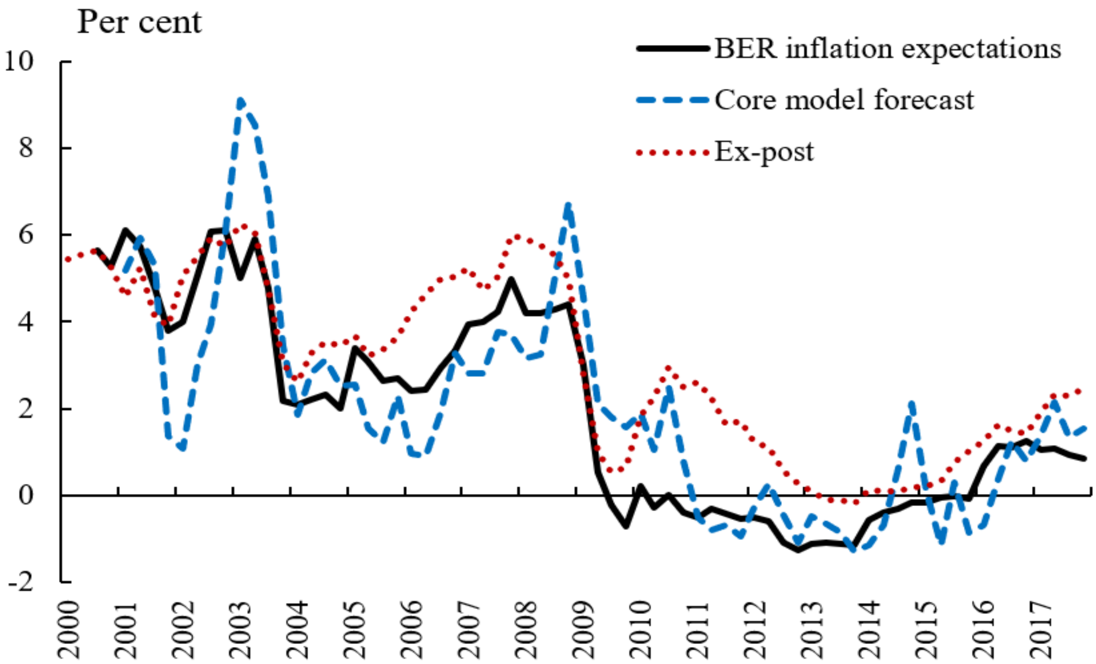
Table 3: Prior and posterior estimates

Parameters		Priors		Posteriors	
	Distribution	Mean	Standard deviation	Median	5^{th} & 95^{th} percentiles
α_{y^f}	beta	0.5	0.1	0.950	[0.947 ; 0.952]
α_{r^f}	gamma	0.05	0.02	0.067	[0.065 ; 0.070]
c^f	normal	1	0.20	0.976	[0.708 ; 1.253]
β_{π^f}	beta	0.7	0.1	0.976	[0.960 ; 0.989]
β_{y^f}	gamma	0.25	0.05	0.092	[0.051 ; 0.119]
β_{o^f}	gamma	0.02	0.02	0.006	[0.006 ; 0.007]
$\varepsilon_{\tilde{y}^f}$	inv.gamma	0.84	0.2	0.220	[0.218 ; 0.223]
$\varepsilon_{\tilde{\pi}^f}$	inv.gamma	1.39	0.2	0.282	[0.268 ; 0.306]
$\varepsilon_{\tilde{z}^f}$	inv.gamma	0.17	0.02	0.081	[0.078 ; 0.082]
ε_{r^f}	inv.gamma	0.21	0.05	0.137	[0.133 ; 0.141]
ε_{g^f}	inv.gamma	0.95	0.2	0.021	[0.021 ; 0.022]
$\varepsilon_{\bar{y}^f}$	inv.gamma	0.21	0.05	0.374	[0.368 ; 0.393]
ε_{o^f}	inv.gamma	1	0.2	20.58	[20.36 ; 20.90]

D Comparisons of real interest rates

We observe real interest rates in the model, using the one-year ahead inflation forecast from the SARB's core econometric model. This is done to ensure that the real interest rate aligns to the forecasts announced by the SARB's Monetary Policy Committee and the implicit real rate that it implies. Figure 6 compares the real interest rate we use in the model (labelled 'core model forecast') to other estimates, including inflation expectations from the BER's inflation expectations survey, using current inflation (labelled 'Ex-post'). All measures generally follow the same path and co-move well, with correlations above 0.75.

Figure 6: Real interest rate: Inflation expectation comparisons



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