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Estimating a Time-Varying Phillips Curve for South Africa*

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Abstract

In this paper we estimate a Phillips curve for South Africa. The slope of the Phillips curve, the inflation persistence, the natural rate of unemployment and the central bank's inflation target band are time-varying. We find that the slope of the Phillips curve has flattened since the mid 2000s - particularly after the Great Recession - which is in line with the findings in most advanced countries. Our results indicate that inflation persistence increased from 1994 to 2001, remained constant from 2001 to 2008, and eventually decreased around 2008. This pattern is different from that of advanced countries where expectations became better anchored relatively early in the inflation targeting (IT) regime and stayed there. Finally, we suggest that the increased stability of inflation expectations after 2008 – which coincides with the Great Financial crisis - may be a result of "good luck" not just a good policy framework.

JEL Classification Numbers: C51, E52, E58.

Keywords: Monetary Policy, Inflation Targeting, Inflation Expectations, Time-varying parameters.

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1 Introduction

The positive relationship between inflation and economic activity which was dubbed the "Phillips" curve by Samuelson and Solow (1960) following the celebrated contribution by Phillips (1958), remains one of the cornerstones of modern macroeconomics. It features prominently in rational expectations guise in New Keynesian models such as Gali et al. (2012) and Gali (2015a).

On the 50th anniversary of Phillips' paper Gordon (2011) provides a roadmap of its evolution since 1975. According to Gordon (2011), in the post - 1975 era, the Phillips Curve has evolved into two strands: a 'left fork' and a 'right fork'. The left fork involves an inflation model where expectations are backward-looking and prices sticky. Demand shocks, inflation inertia, and supply shocks explain the dynamics in inflation. [1]

A recent contribution by Blanchard et al. (2015) - clearly along the left fork of the road - investigates the relationship between output, unemployment and inflation over the course of about 50 years for 23 advanced economies. They estimate for each country a benchmark relationship between inflation, long term inflation expectations, lagged inflation and the unemployment gap. Their specification allows for the natural rate of unemployment as well as the coefficients to change over time. Focusing on the Great Financial crisis, they find that inflation targeting stabilised inflation for the two decades preceding the crisis, and shifted expectations toward the target of the central bank. In addition they find that the slope of the Phillips curve has flattened over time - with much of the decrease taking place from the mid 1970s to the mid 1990s - and the slope being insignificant for most countries today. Even as inflation expectations have become more forward-looking and anchored, the tradeoff disappeared.

In this paper we investigate this for South Africa where price stickiness is normally thought to be high. Accordingly we build a 'left fork of the road' Phillips curve model in unemployment - inflation space along the lines of Chan, Koop and Potter (2015) (CKP). We allow the slope of the Phillips curve, the central bank's perceived inflation target, the natural rate of unemployment and inflation persistence to be time-varying and estimate the model with Bayesian methods using a Markov Chain Monte Carlo algorithm. The

[1] Demand shocks are proxied by the unemployment gap, whereas supply shocks include variables such as changes in the relative prices of food, energy and imports. And the inflation inertia is represented by the formation of expectations and persistence due to fixed-duration in wage and price contracts. The New Keynesian model follows the right fork in which forward-looking expectations play a central role in determining the Phillips curve. See Kydland and Prescott (1977), Sargent(1982).

inflation persistence measures the degree to which expectations are adaptive or rational. As in CKP, we constraint some of our state variables to be consistent with economic theory or with the structure of the South African economy. For example, the South African Reserve Bank's (SARB) official inflation targeting band is 3 - 6%. Therefore, we constrain the central bank's perceived target to fall within this band. Unlike CKP we explicitly tie the inflation persistence parameter to private agents' inflation expectations formation process and link trend inflation with the central bank's perceived inflation target.

This paper estimate a Phillips curve for South Africa in a time-varying parameter framework and is able to answer several questions in the spirit of Blanchard et al. (2015). Our results show that in South Africa inflation persistence and the slope of the Phillips Curve are time-varying. Specifically, the inflation persistence parameter has decreased since the Great Recession in 2008, as expectations appear to become more forward-looking. This is consistent with Blanchard et al. (2015) view that inflation persistence has decreased since the 1980s and that the adoption of inflation targeting (IT) in most advanced economies has led to less persistent inflation. Our view is that the fall in inflation persistence may result from increased policy credibility post 2008, but could also result from global decline in energy and food prices. As for the slope parameter, results show that there is a slight decrease since 2008. The fall in the slope suggests that changes in the unemployment gap have relatively small effects on inflation, although a fall in the gap appears to push inflation up more than a decline reduces it. This is consistent with the view that inflation is only a partially demand-driven phenomenon in South Africa.

The remainder of the paper is organized as follows. Section 2 introduces the unobserved components model. In Section 3 we discuss the data, while our empirical results are given in Section 4. Section 5 concludes.

2 The Unobserved Components Model

The model we use is a standard unemployment-based Phillips curve (see for example Ball and Mazunder (2011), Matheson and Stavrev (2013) and Ball (2015)). More specifically, we assume that the economy is characterized by the following Phillips curve:[2]

$$\pi_t = \pi_t^e - \alpha_t(u_t - u_t^n) + \varepsilon_t \tag{1}$$

[2]Ball and Mazumder (2011, p. 346) define core inflation as the part of inflation explained not by supply shocks, but rather by expected inflation and economic activity. Thus in equation (1) core inflation can be written as $\pi_t^c = \pi_t^e - \alpha_t (u_t - u_t^n)$.

where π_t is the inflation rate between time $t-1$ and t, π_t^e is expected inflation, u_t is the time t unemployment rate, α_t is the slope of the Phillips curve which is allowed to be time-varying and ε_t is a residual capturing other factors such as supply (cost-push) shocks. Here u_t^n is the natural rate of unemployment that prevails when inflation is equal to expected inflation ($\pi_t = \pi_t^e$) and when shocks are absent ($\varepsilon_t = 0$). [3]

Note that in equation (1) the variable u_t^n is unobserved. Moreover, we need to make an assumption about how inflation expectations are formed in (1) to fully describe the dynamics of inflation. Kabundi and Schaling (2013) model inflation expectations in South Africa as a weighted average of lagged inflation and the South African Reserve Bank's (SARB) implicit inflation target.[4] We follow the same logic to specify the inflation expectations process. However, we take into account the fact that the inflation expectations formation process may vary over time. We allow the weight on lagged inflation to be time-varying. Moreover, the SARB does not have an explicit point target but instead has a target band of 3% - 6%. Thus, we assume that the SARB's implicit inflation target point is unobserved and is time-varying. In this case, the inflation expectation process is given by:

$$\pi_t^e = \rho_t \pi_{t-1} + (1-\rho_t)\pi_t^* \tag{2}$$

where π_t^* is a proxy for the SARB's inflation target point and is assumed to be time-varying. ρ_t is a time-varying parameter that captures the weight agents put on past inflation. As mentioned before when $\rho_t = 0$, inflation expectations are completely anchored to the central bank's inflation target ($\pi_t^e = \pi_t^*$). When $\rho_t = 1$, inflation expectations are unanchored and fully backward-looking.[5]

From equations (1) and (2) it can be seen that a rise in inflation caused by a supply shock raises expected inflation, which in turn raises future inflation. More specifically, a supply shock that raises the time t inflation rate pushes up inflation expectations for period $t+1$ which then raises inflation in that period. The latter then feeds in inflation expectations for period $t+2$ and so on.

[3] For u_t^n to be well identified we will impose $\alpha_t > 0 \; \forall$ t.

[4] More specifically, in Kabundi and Schaling (2013), inflation expectations are modelled as: $\pi_t^e = \rho\pi_{t-1} + (1-\rho)\pi^*$ where the weight parameter ρ and the inflation target $\pi^*$are constant over time.

[5] See Kabundi *et al* (2015) for further discussion on the link between ρ and central bank credibility. Using our notation Matheson and Stavrev (2013) estimate $\pi_t^e = \rho_t \pi_{t-1}^4 + (1-\rho_t)\overline{\pi_t}$ where $\overline{\pi_t}$ is long-run inflation expectations (sourced from the Federal Reserve Board), $\pi_{t-1}^4$is year-over-year headline CPI inflation (lagged one quarter). Thus, they anchor inflation expectations to *long-run* inflation expectations, not to the central bank's inflation target. However, to the extent that the former are influenced by the Fed's (implicit) inflation target our specifications is not that dissimilar.

We assume that π_t^* (unobserved) follows the random walk process[6]

$$\pi_t^* = \pi_{t-1}^* + \varepsilon_t^* \tag{3}$$

Combining equations (1), (2) and (3), the reduced form of the inflation process can be written as:

$$\pi_t - \pi_t^* = \rho_t(\pi_{t-1} - \pi_{t-1}^*) - \alpha_t(u_t - u_t^n) + \nu_t \tag{4}$$

where $\nu_t = \rho_t \varepsilon_t^* + \varepsilon_t$

Equation (4) is a Phillips curve in terms of the cyclical component of inflation (if we assume that the central bank target π_t^* is a good proxy for trend (or core) inflation). Basically, the dynamics of the cyclical component of inflation, $\pi_t - \pi_t^*$, are explained by built-in persistence due here to agents expectations formation process; excess demand, $u_t - u_t^n$; and other supply shocks ν_t. This formulation of the Phillips curve based on cyclical inflation has been adopted in previous work including Chan, Koop and Potter (2015), Stock and Watson (2007) and Stella and Stock (2013). The difference with our specification is that we explicitly tie the persistence parameter ρ_t to economic agents' inflation expectations instead of just an ad hoc statistical parameter to be estimated.[7]

Note that when $(1 - \rho_t L)(\pi_t - \pi_t^*) = 0$ and $\nu_t = 0$ we have $u_t = u_t^n$ (where L is the lag operator). Thus, the expression for u_t^n as the natural rate of unemployment is slightly different from the traditional definition of the non-accelerating inflation rate of unemployment (NAIRU) where one only needs $\pi_t = \pi_{t-1}$. Our specification does not explicitly include a supply shock variable as an explanatory factor in the Phillips curve and thus differs somewhat from the triangle specification of Gordon (1998).[8] However, we allow the residual ν_t to follow a stochastic volatility process to correct for potential heteroscedasticity introduced by the omission of supply side variables.

We assume that the parameters ρ_t and α_t follow random walk processes

$$\rho_t = \rho_{t-1} + \varepsilon_t^\rho \tag{5}$$

$$\alpha_t = \alpha_{t-1} + \varepsilon_t^\alpha \tag{6}$$

where ε_t^ρ and ε_t^α are error terms whose processes will be made clear shortly.

Since the natural rate of unemployment, u_t^n, is unobserved, the cyclical unemployment $u_t - u_t^n$ is unobserved as well. Following Chan, Koop and Potter (2015), we specify an AR(2) process for cyclical unemployment as follows

[6] Note that the random walk process is adopted to account slow change in the inflation target.

[7] For a recent specification related to our work see Blanchard, Cerutti and Summers (2015).

[8] For an analysis about optimal monetary policy in a triangle model see Bullard and Schaling (2001).

$$u_t - u_t^n = \varphi_1(u_{t-1} - u_{t-1}^n) + \varphi_2(u_{t-2} - u_{t-2}^n) + \varepsilon_t^u \quad (7)$$

where φ_1 and φ_2, are constant parameters and ε_t^u is an identically independently distributed (i.i.d.) error term with mean 0 and variance σ_u^2.[9] Since, we want to use inflation and unemployment data to estimate the model, we also specify a process for the natural rate of unemployment as a random walk process:[10]

$$u_t^n = u_{t-1}^n + \varepsilon_t^n \quad (8)$$

where ε_t^n is the error term to be defined below. Note that this specification of the natural rate discards the possibility of the hysteresis hypothesis of unemployment where the natural rate of unemployment depends on past unemployment rates.[11]

Although supply shocks such as cost-push pressure by unions or 'oil sheiks' are not explicitly modelled, we stress that our Phillips curve model is close to the 'left fork of the road' (Gordon (2011)) or econometrically sound main stream triangle approach (as opposed to the 'right fork' or rational expectations approach). This can be seen by combining (4) and (7). This yields

$$\pi_t = \rho_t L \pi_t - \alpha_t \left(\varphi_1 L + \phi_2 L^2\right)(u_t - u_t^n) + (1 - \rho_t L)\pi_t^* - \alpha_t \varepsilon_t^u + \nu_t \quad (9)$$

This expression is similar to equation (13) of Gordon (2011).

The system of equations (4), (3), (5), (6), (7) and (8) can be viewed as a bivariate unobserved components model of inflation and unemployment where the parameters ρ_t and α_t are time-varying.

Note that without further restrictions on the state variables u_t^n, π_t^* , ρ_t, α_t and with the standard distributional assumptions on the error terms, this system can be estimated using a Kalman filter algorithm. However, this would mean that these variables are in principle unbounded when the variances of the error terms are significantly large. We depart from this assumption for the following reasons. First, the SARB has a clearly specified inflation target range between 3% and 6%. Even if a target point is not clearly specified, it would not be reasonable to assume that the SARB's unobserved target point π_t^* can take any value. Therefore we have to impose a restriction of the type $3\% \leq \pi_t^* \leq 6\%$ at any time t. Second, here the inflation persistence parameter ρ_t in (4) captures the extent to which inflation expectations are anchored.

[9] In a preliminary analysis, we find that, using a HP filter, the cyclical unemployment follows a AR(2) process and there is no evidence of time variation of the coefficients φ_1 and φ_2. Matheson and Stavrev (2013) assume that the persistence of the unemployment gap is constant.

[10] Equation (8) allows for a slow variation in the NAIRU from one quarter to another.

[11] For a recent analysis of unemployment hysteresis see Gali (2015b).

A preliminary analysis using a HP filter shows that the cyclical component of unemployment follows a stationary AR(2) process. This implies that the parameters φ_1 and φ_2 in equation (7) must satisfy: $\varphi_1 + \varphi_2 < 1$, $\varphi_1 - \varphi_2 < 1$ and $-1 < \varphi_2 < 1$. Moreover, the natural rate of unemployment is a steady-state concept and should be bounded. Finally, the Phillips curve slope parameter α_t should be positive on theoretical grounds.

In general the random walk trend variables in this model are bounded from below and above as

$$a_x \leq x_t \leq b_x \tag{10}$$

where $x_t = u_t^n$, π_t^* , ρ_t, α_t and a_x and b_x are constant real numbers. Using the random walk property of these variables, the inequalities in equation (10) imply that:

$$a_x - x_{t-1} \leq \varepsilon_t^x \leq b_x - x_{t-1} \tag{11}$$

where $\varepsilon_t^x = \varepsilon_t^n, \varepsilon_t^*, \varepsilon_t^\rho$ and ε_t^α.

Thus, for the boundedness restrictions of the random walk processes to be satisfied in (3), (5), (6), and (8), we can simply bound the error terms below and above by time-varying bounds. Chan, Koop and Potter (2015) use a similar framework to study inflation and unemployment trends for the U.S. economy. Matheson and Starvrev (2013) and Blanchard *et al* (2015) estimate a similar Phillips curve with time-varying coefficients for twenty OECD countries where the slope coefficient and the inflation persistence parameter are constrained. As in Chan, Koop and Potter (2015), we assume that the error terms $\varepsilon_t^x = \varepsilon_t^n, \varepsilon_t^*, \varepsilon_t^\rho$ follow a truncated normal distribution, that is, we have

$$\varepsilon_t^x \rightsquigarrow TN(a_x - x_{t-1}, b_x - x_{t-1}, 0, \sigma_x^2)$$

where $TN(\theta, \beta, \mu, \sigma^2)$ is the normal distribution of mean μ and variance σ^2 and truncated within $[\theta \quad \beta]$.

Empirical evidence suggests that inflation volatility has decreased in recent years in most inflation targeting countries (See for example Mishkin (2007), Svenson (2010) and Seedwell, Albert and Fulbert (2012)). To account for this fact, we specify a stochastic process for the conditional volatility of the disturbance term ν_t in the Phillips curve as

$$\nu_t \rightsquigarrow N(0, h_t^2)$$

$$\log(h_t) = \log(h_{t-1}) + \varepsilon_t^h$$

where ε_t^h is an i.i.d. process with mean 0 and variance σ_h^2 and uncorrelated with ν_t. We also assume that each error term is uncorrelated with the others.

We estimate several parameters of the model by a Bayesian method. Let the time invariant parameters be summarized in the following vector
$\psi = (\varphi_1, \varphi_2, \sigma_u^2, \sigma_h^2, \sigma_{u^n}^2, \sigma_{\pi^*}^2, \sigma_\rho^2, \sigma_\alpha^2, a_{u^n}, a_{\pi^*}, a_\rho, a_\alpha, b_{u^n}, b_{\pi^*}, b_\rho, b_\alpha)'$.

Note that the bounds parameters can be fixed or estimated. During the estimation, we will fix the bound parameters $a_\rho, b_\rho,\ a_\alpha,\ b_\alpha$ and estimate the inflation target and the natural of unemployment bounds parameters a_{π^*}, a_{u^n}, and b_{π^*}, b_{u^n} respectively. So, the parameter vector to be estimated becomes $\psi = (\varphi_1, \varphi_2, \sigma_u^2, \sigma_h^2, \sigma_{u^n}^2, \sigma_{\pi^*}^2, \sigma_\rho^2, \sigma_\alpha^2, a_{u^n}, a_{\pi^*}, b_{\pi^*}, b_{u^n})'$.

With these "boundedness" restrictions, the model is highly nonlinear and standard Kalman Filter algorithms do not apply. We follow the algorithm proposed by Chan, Koop and Potter (2015) to estimate the model. The algorithm is a Bayesian estimation method using a Markov Chain Monte Carlo (MCMC) drawing procedure which takes into account the restrictions imposed on the state variables.

Before we give a brief summary of the algorithm, some notation needs to be introduced. For each variable z, we denote by $z = (z_1, z_2, .., z_T)'$ the sample vector of z where T is the sample size and $y = (\pi', u')'$. After specifying the priors for the parameters vector ψ and the initial state variables, the drawing procedure is a six step MCMC algorithm and can be summarized as follows

1. $p(\pi^*|y, u^n, \rho, \alpha, h, \psi)$
2. $p(u^n|y, \pi^*, \rho, \alpha, h, \psi)$
3. $p(\rho|y, \pi^*, u^n, \alpha, h, \psi)$
4. $p(\alpha|y, \pi^*, u^n, \rho, h, \psi)$
5. $p(h|y, \pi^*, u^n, \rho, \alpha, \psi)$
6. $p(\psi|y, \pi^*, u^n, \rho, \alpha, h)$

For the specification of the priors we assume that all the standard deviations follow inverse gamma distributions (IG), the bound parameters $a_{\pi^*}, b_{\pi^*}, a_{u^n}, b_{u^n}$ follow uniform distributions and the AR(2) parameters φ_1, φ_2 follow normal distributions. The initial state variables u_0^n, π_0^* , ρ_0, α_0 follow truncated normal distributions (TN). In general, we use relatively non-informative priors in the analysis. To save space we do not present here all the details of the algorithm and refer the reader to Chan, Koop and Potter (2015) for a detailed description of the drawing procedure of each of the six steps above.

3 Data Analysis

The analysis uses quarterly unemployment rate data obtained from the South African Reserve Bank (SARB) covering the period 1994Q1 to 2014Q1. It is important to note that labour market data in South Africa are somewhat unreliable. Therefore, the choice of the sample size is determined by the availability of the unemployment data. The SARB constructs the unemployment time series by linking different labour surveys conducted by Statistics South Africa (Stats SA) from 1994 to most recent. In addition, the weights used in the different surveys are different. The first sample, from 1994 to 1999, is based on the annual October household survey. From 2000 to 2007 both the frequency and the approach used changed. The new sample is biannual and it is based on the Labour Force Survey (LFS). Finally from 2008 to present Stats SA publishes quarterly series of unemployment based on the Quarterly Labour Force Survey (QLFS). The QLFS is a household-based survey on the labour market activities of population aged 15-64 years. It uses the strict definition of unemployment, that is, it does include discouraged work-seekers.

We emphasise that this is the only available unemployment series for South Africa that goes as far back as 1994. The current analysis necessitates an adequate number of observations in order to obtain statistically meaningful results. Moreover, starting in 1994 enables the analysis of the evolution of the Phillips curve before and during the inflation targeting (IT) regime, which started in 2000. Viegi (2015) avoids altogether using the unemployment series in the estimation of his Phillips curve and instead substitutes it with the more reliable employment series. However, the unemployment rate is an essential variable especially for policymaking. It is crucial for the monetary policy authority to have an idea of the nature of relationship between inflation and unemployment. In addition, the estimation of the non-accelerating inflation rate of unemployment (NAIRU) is important for the conduct of monetary policy.

For inflation we use the year-on-year headline inflation data obtained from the South African Reserve Bank. However, we are aware that from 2000 to 2008 the SARB was targeting CPIX-inflation instead of the headline CPI.[12] In 2008 the SARB reverted back to targeting headline CPI inflation. We use headline CPI inflation to avoid statistical issues such as structural breaks that are inherent when two different series are used.[13]

[12] CPIX refers to headline CPI excluding mortgage costs.

[13] The results are qualitatively the same when we use both the headline CPI inflation and the CPIX inflation.

Figure 1: Unemployment and Inflation

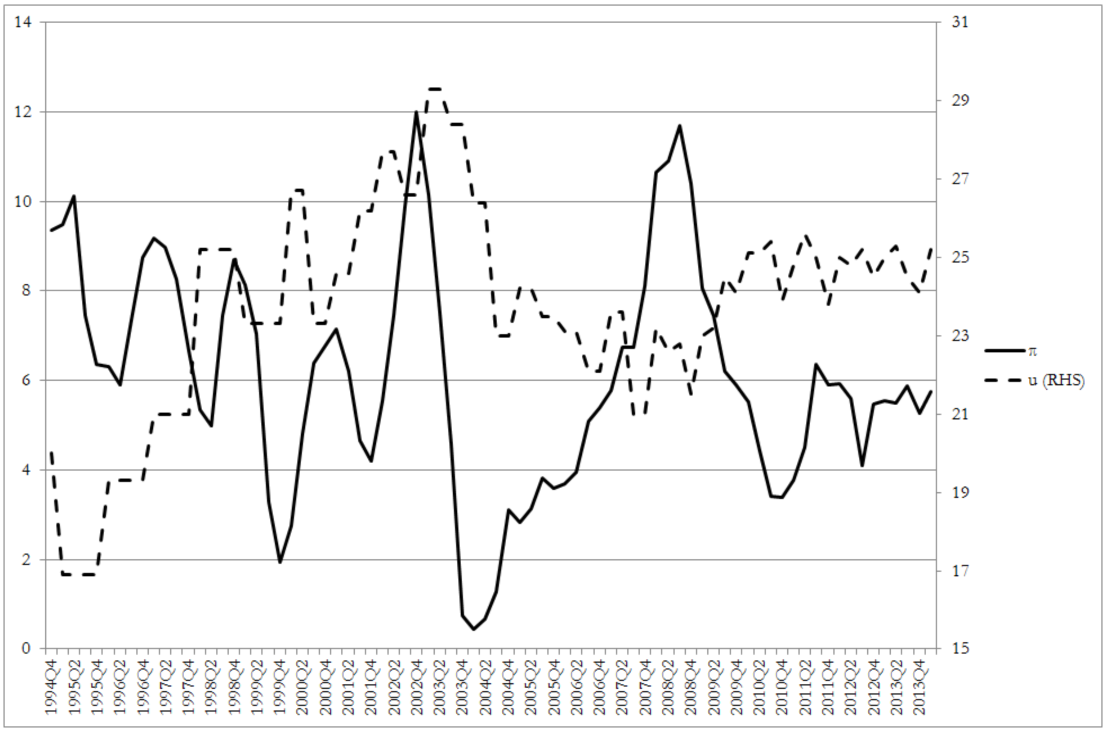


Figure 1 depicts the unemployment and inflation rates from 1994Q4 to 2014Q1. It is clear from the figure that unemployment displays a discrete pattern from 1994 to 2007. The series becomes continuous from 2008 until the end of the sample. The graphical representation points to evidence of a Phillips curve in the beginning of the sample, but the curve flattened in early part of 2000 until the later part of 2003 when the negative relationship recurred again all the way to the end of the sample. The surge in inflation in 2004 was caused by many factors, among others, an increase in demand, the rise in petrol price and food prices. Unemployment decreased and reached a minimum of 21% in the fourth quarter of 2007, in the meantime inflation peaked at 11.69% in 2008Q3. The trend reversed when the impact of the financial crisis in the US spilled over to South Africa, driving the economy into recession, which subsequently pushed unemployment to about 25% and at the same time inflation plummeted to 3.39% in 2010Q4 before stabilising around the 6% level. The analysis based on the graphical representation of these two variables points to the changing nature of the Phillips curve in South Africa.

4 Empirical Results

This section presents the results obtained by estimating equation (4). We first discuss the extracted trends which represent on the one hand the implicit inflation target and the estimated target band, and on the other hand the trend unemployment rate or

NAIRU. Secondly, we discuss the relationship between the inflation cycle and the unemployment cycle. The empirical section concludes with the analysis of the estimated parameters, among others, inflation persistence, the slope of the Phillips curve, and lastly the conditional volatility of inflation.

4.1 Trend Inflation and Trend Unemployment

The estimated trends of inflation and unemployment from the state-space model follow a bounded random walk process. The initial values are set in a way that is consistent with the two series. We set trend inflation between 3% and 7%, and trend unemployment 15% and 30%. Figure 2 represents inflation together with the implicit inflation target and the estimates target bands. Interestingly the estimated band is consistent with the official target between 3.25% and 6.41%. The estimated inflation target (π^*) is relatively constant with a mean of 5.12%, and maximum and minimum values of 5.25% and 5.03%, respectively. These results are consistent with an implicit target of financial analysts of 5.51%, estimated by Kabundi et al. (2015), using a different sample, from 2000 to 2013. Unlike these authors, the approach used in this paper allows the estimation of the target band. It is not surprising that the mean of the target is in line with the mid-point of the estimated band, in this case 5%.

Figure 2: Inflation, Implicit Inflation Target and Implicit Target Bands

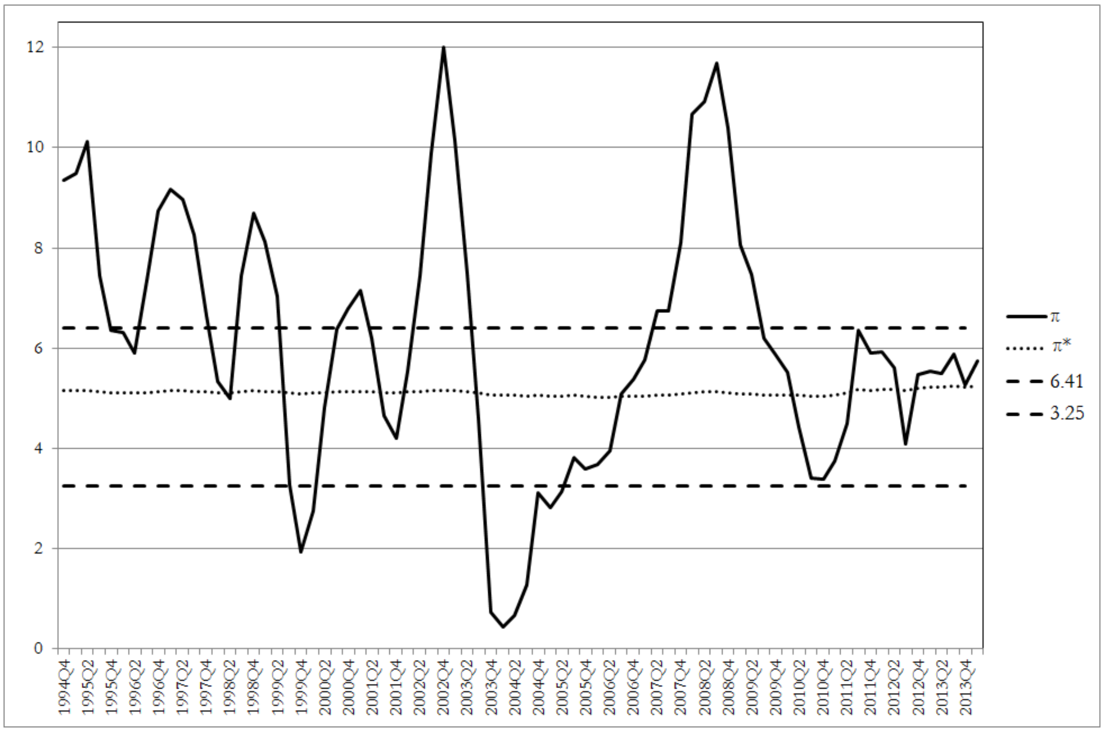
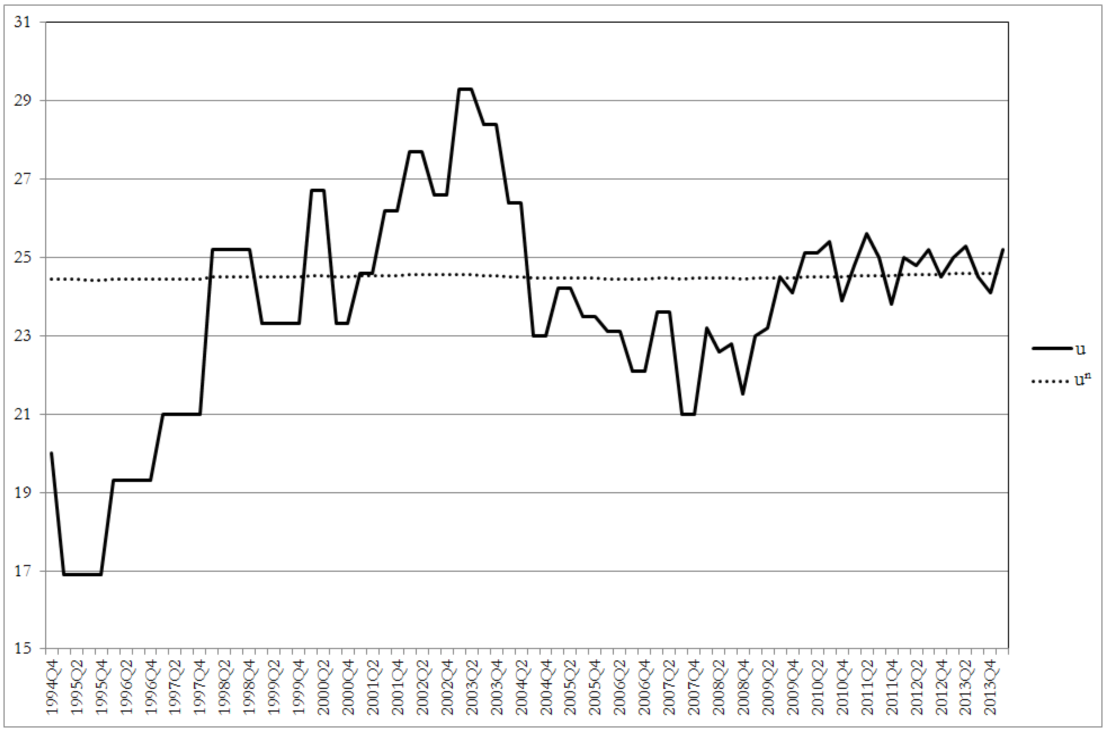


Figure 3: Unemployment and NAIRU



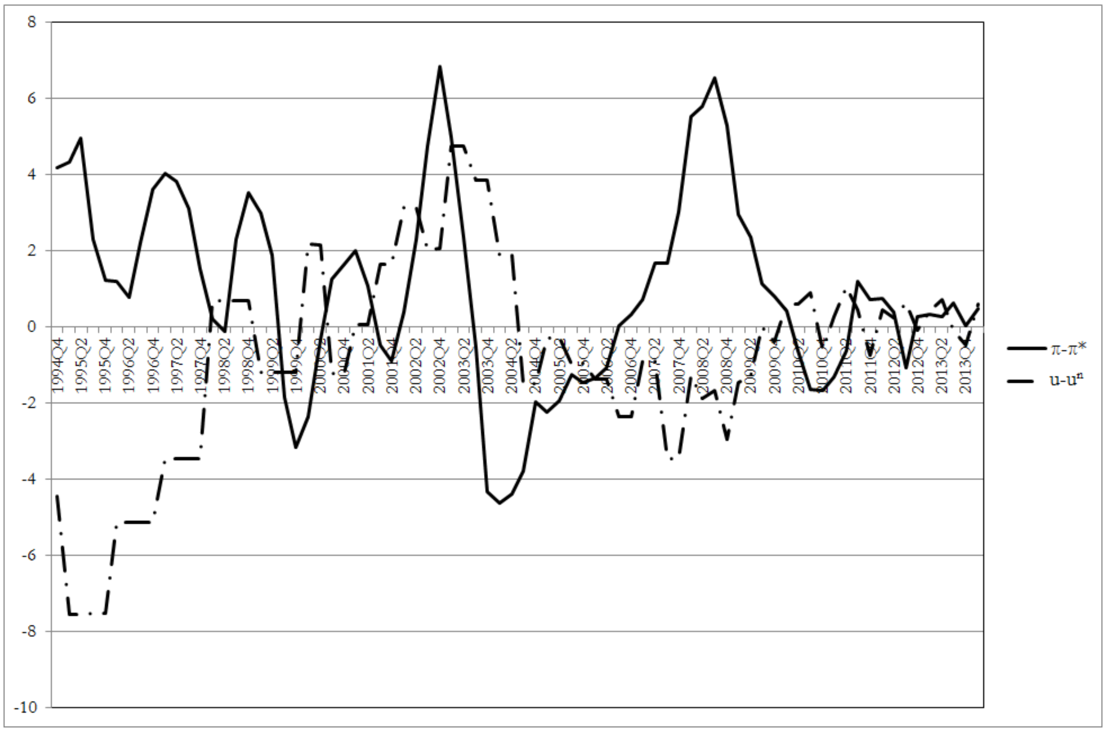
Similarly, the estimated unemployment trend (u^n) or NAIRU in figure 3 is relatively flat throughout the entire sample period, around 24.5%. Previous work suggest that the NAIRU for South Africa is about 25%.[14] Figure A.1 in the appendix shows that the NAIRU estimated using the Phillips curve (PC) is slightly different from the estimation obtained from an atheoretical approach, in this case an AR(2) process (AR). The PC estimates are consistently higher than the AR ones and the difference widens from 2008Q4 onward. Figure 3 shows that unemployment stabilises somewhat around the NAIRU from 1998Q2 to 2001Q1. From 2001Q2 the unemployment rate rises markedly way above the NAIRU and stays high for approximately three years. During this period the economy was weak with a negative output gap. And from 2004Q3 until the recent financial crisis the economy was at an expansionary stage and hence it created more jobs. Consequently unemployment decreased below the NAIRU for about five years. The financial crisis pushed the South African economy into recession in 2009 which at same juncture exerted pressure on unemployment which rose back to the level of the NAIRU and it has remained stable ever since.

[14] See for example Viegi (2015).

4.2 Cyclical Components

In order to get some perspective on the Phillips curve in South Africa, in Figure 4, we show the relationship between the cyclical components of inflation (inflation gap) and unemployment (unemployment gap) inferred from the model. According to equation (4), the Phillips curve relationship involves the cyclical components of inflation and unemployment and not necessarily the levels as shown in Figure 1. From Figure 4 we can see the changing nature of the relationship between the two cycles for the period 1994-2014. There is evidence of a negative relationship between the two variables at the beginning of the sample, that is, from 1994Q4 to 1998Q1. The relationship becomes ambiguous from 1998Q2 to 2001Q3, which in turn is followed by a positive relationship between the two cyclical components. This result suggests that during this period of 1998Q2 - 2001Q3, the South African economy experienced stagflation whereby high cyclical inflation was explained by other factors than cyclical unemployment (see equation (4)). Note that, the domestic currency (Rand) depreciated significantly in 1998Q2 - 2001Q3. That may explain a rise in inflation while at the same time the economy was subdued. The negative relationship between the two variables re-emerges in 2004 and remains in existence although weakens somewhat towards the end the sample.

Figure 4: Cyclical Components



Importantly, the inflation gap seems more persistent than the unemployment gap. In addition, the inflation gap follows a long cycle with a large amplitude from 2001 to 2008.

Even if the 2001-2002 cycle displays a large amplitude of 6.85%, it is short and lasts only a year. However, the 2002-2008 cycle is both long and with a large amplitude, reaching a minimum of -4.63%. This cycle lasts four years and peaks in 2008Q3 at 6.55%. From this finding it seems that the management of the inflation cycle was somewhat ineffective in reducing either its length and/or its amplitude.

Figure 5 depicts the relationship between the estimated unemployment gap $(u - u^n)$ and the output gap $(y - y^n)$ proposed by Anvari, Ehlers, and Steinbach (2014). Note that the two variables are negatively correlated, especially since 2000, with a correlation coefficient of -0.72. Generally, an increase in the ouput gap corresponds to a decline in the unemployment gap. This close relationship between the two series is clearer from Table 1, when we estimate the Phillips curve in equation (4) with constant parameters from 2000 to 2014. The results show high persistence of inflation with ρ of 0.89 and 0.85 for the unemployment gap regression and the output regression, respectively. Finally, the Phillips curve is flat in both cases with slopes of 0.22 and 0.25. The next section provides in-depth analysis of the dynamic nature of the Phillips curve, but also the persistence of the inflation cycle.

Figure 5: Unemployment Gap and Ouput Gap

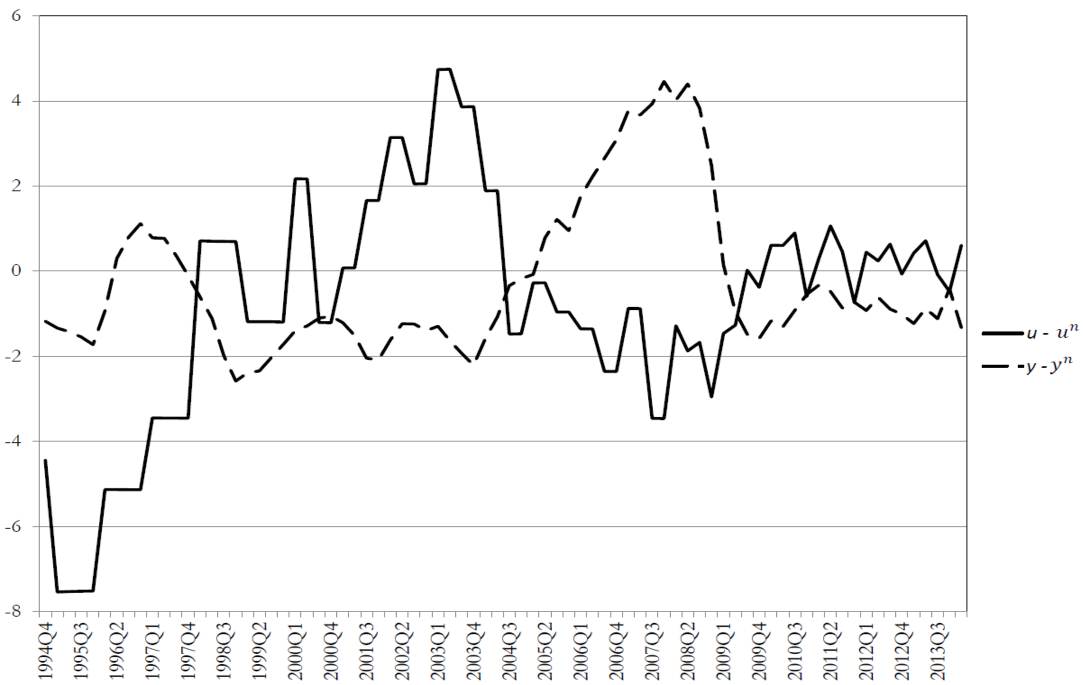


Table 1: Phillips Curve with the Unemployment Gap and the Ouput Gap

	$u - u^n$	$y - y^n$
ρ	0.89***	0.85***
α	-0.22**	0.25***
$\bar{R}^2$	0.79	0.80

*** significant at 1%, **significant at 5%,

4.3 Parameter Estimates

In this part, we present the results of the parameters estimates. As discussed in the model section, we fixed the bound parameters during the estimation except the bound parameters of inflation target a_{π^*}, b_{π^*}, and the natural rate of unemployment bounds a_{u^n}, b_{u^n} which have been estimated.

We set $a_\rho = 0, b_\rho = 1$ in line with our interpretation of ρ as the degree of inflation expectations anchoring to the SARB inflation target. The slope of the Phillips curve bounds are set as $a_\alpha = 0$, $b_\alpha = 1$. We draw 250,000 observations from the posterior distributions and discard the first 100,000 observations before computing the statistics.

The posterior estimates indicate that the inflation target bounds are slightly higher than the official SARB inflation target bounds. The estimate of the posterior median of the lower bound of the inflation target is $a_{\pi^*} = 3.25$ whereas the posterior median of the upper bound is estimated to be $b_{\pi^*} = 6.47$. The posterior median of the natural rate of unemployment lower and upper bounds are $a_{u^n} = 17.50$, $b_{u^n} = 27.31$ respectively. The unemployment gap is very persistent with estimated posterior means of the AR(2) coefficients $\varphi_1 = 0.86$ and $\varphi_2 = 0.007$. We present in appendix (Table A.1) some descriptive statistics of the posterior distributions of the estimated constant parameters.

In a particular version of the model, we conduct the estimation without imposing the boundedness restrictions on the unobserved state variables and coefficients (u_t^n, π_t^* , ρ_t, α_t). However, then in some periods the estimated values of ρ_t and α_t lead to model instability. This highlights the importance of the restrictions imposed on ρ_t and α_t in this time-varying framework. Chan, Koop and Potter (2015) find similar results for the U.S. economy. They find that bounding the random walk processes (of the states) as in our case, yields better out-of-sample forecasts of inflation and unemployment compared to the unbounded case because of its consistency with underlying economics.

In Figure 6 we present the smoothed estimates of ρ_t and α_t whereby the filtering is based on the full sample information. These two parameters are critical for the dynamics of inflation in the Phillips curve specified in equation (4). The parameter ρ_t captures the impact of inflation expectations on inflation whereas the slope α_t captures the degree of the response of inflation to excess demand factors and the extent of the short run tradeoff faced by policy makers. The more anchored inflation expectations are to the central bank target level π_t^*, the smaller ρ_t and the smaller the effect of inflation expectations on inflation. As shown in Figure 6 our smoothed estimates suggest that the persistence parameter ρ_t is time-varying and has increased from 1994 to 2001, remained constant from 2001 to 2008, and eventually declined after around 2008. This indicates that inflation expectations in South Africa have been relatively more anchored and stable since 2008.[15] Note however that, the SARB's implicit target (π_t^*) estimates are consistent with financial market analysts inflation expectations. Thus, this result would suggest that financial analysts inflation expectations have been relatively more anchored than those of the price setters (unions and businesses) as found in Kabundi, Schaling, and Some (2015). Moreover, the financial crisis in 2008 followed by the recession in 2009 in South Africa open the question of whether this behaviour of inflation expectations is the result of "good policy or good luck".

This can be formalized as follows. Lagging equation (1) by one period and substituting in (2) we get

$$\pi_t^e = \frac{\rho_t}{1-\rho_t L}\left(\varepsilon_{t-1} - \alpha_{t-1}\left(u_{t-1} - u_{t-1}^n\right)\right) + \frac{1-\rho_t}{1-\rho_t L}\pi_t^* \qquad (12)$$

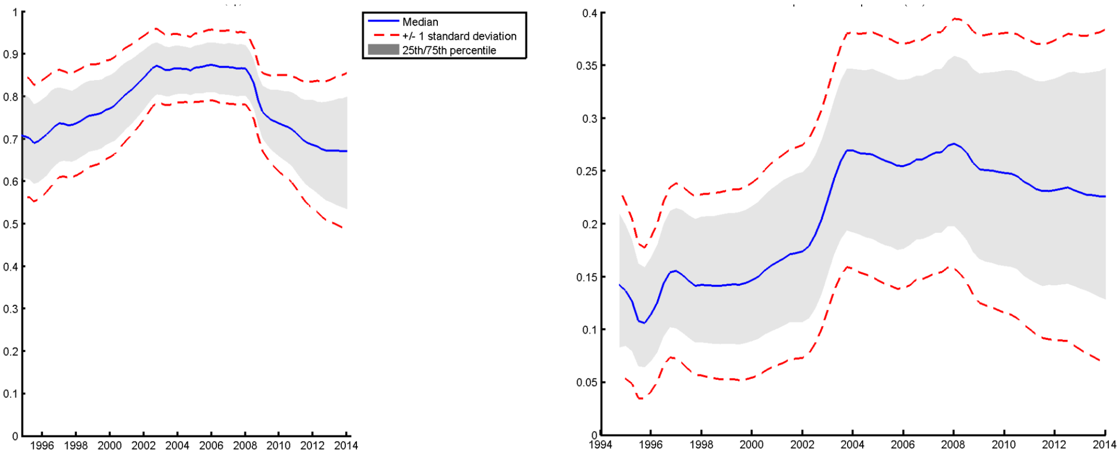
From this moving average presentation of inflation expectations it can be seen that these are a weighted average of, on the one hand, the central bank's inflation target, and on the other hand past cyclical unemployment and past supply shocks. The hypothesis of good policy (better anchoring of expectations) is supported by the data. We have seen a decline in ρ_t after 2008. However, independently of this development Figure 7 shows that there has also been a decrease in the conditional volatility of the inflation residual term around 2008 associated with a global decline in energy and food prices. This means that the variance of has fallen.[16] Thus the drop in inflation expectations was not only driven by good policy but also by good luck.

[15] Matheson and Stavrev (2013) find that during the 1970s inflation expectations became more backward looking and volatile. As the Federal Reserve began moving towards inflation targeting in the early 1980s, long-run inflation expectations began drifting downward and eventually settled at a lower level (around 2%) around 2000.

[16] Formally it can be shown that $Var\left(\varepsilon_t\right) = h_t^2 + \left(\left(\rho_{t-1}\right)^2 + t\sigma_{\varepsilon\rho}^2\right)\sigma_{\varepsilon^*}^2$

Figure 6: Inflation Persistence and Slope Parameters Estimates

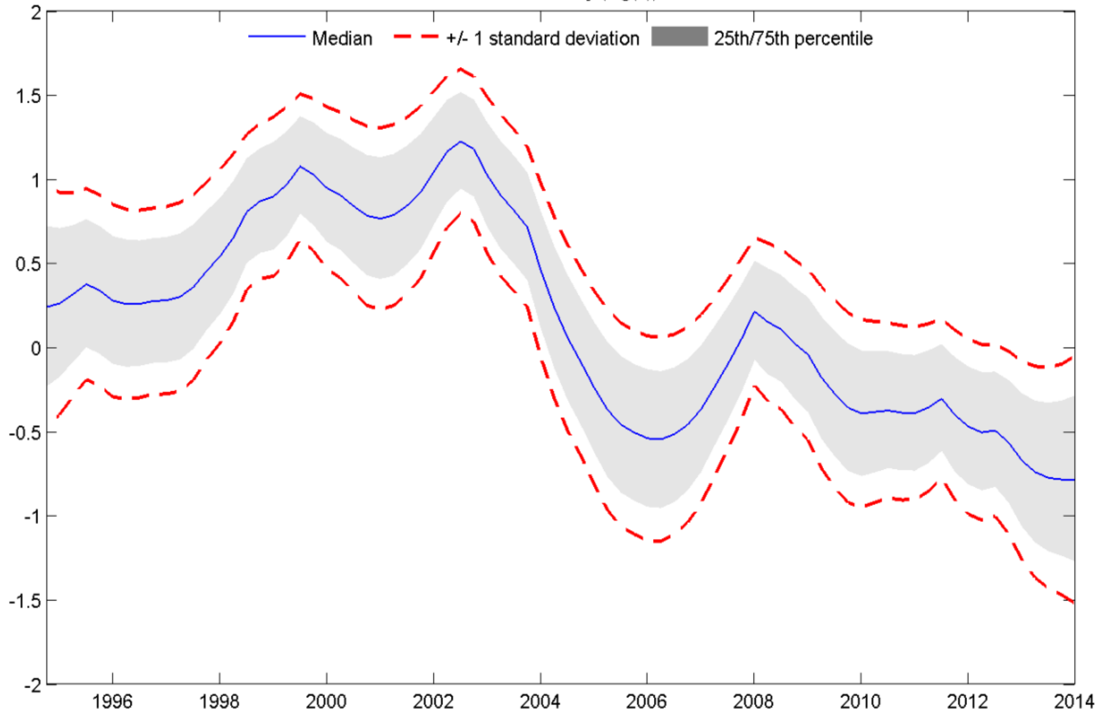
Inflation Persistence (ρ_t) Slope of the Phillips Curve (α_t)



Although at lower levels, the slope parameter α_t has increased from 1994 to the mid 2000s and since then exhibits a slight downward trend. The decline in the slope parameter means that inflation has become less responsive to demand side factors. In the context of our model this suggests that the importance of unemployment as a driver of the inflation process has decreased. The implications for disinflation policy is that the sacrifice ratio, which captures the increase in unemployment above the natural rate due to each percentage point decline in inflation, is becoming higher. This suggests that the SARB should focus on anchoring inflation expectations to further reduce the tradeoff that still exists. Note that this pattern of the slope parameter is in line with the findings in most advanced economies whereby the slope of the Phillips curve has significantly flattened particularly after the great recession.[17]

[17] These results are in line with Blanchard et al. (2015) who also find that the effect of the unemployment gap on inflation has steadily decreased over time. However, their results indicate that all the decrease took place *before* the crisis.

Figure 7: Inflation Residual Conditional Volatility $(\log(h_t))$



5 Conclusion

In this paper we have estimated a time-varying Phillips curve for South Africa with quarterly data from 1994 to 2014. The inflation persistence, the slope of the Phillips curve, the natural rate of unemployment and the central bank's inflation target band were modelled as time-varying parameters and variable, respectively. This framework has enabled us to answer several questions central to the conduct of monetary policy. Our results indicate that the slope of the Phillips curve has flattened since the mid 2000s - particularly after the Great Recession - which is in line with the findings in most advanced countries. We find that our estimated inflation target band from 3.25 to 6.41% is slightly higher than the official SARB target bounds of 3-6% band. This suggests that monetary policy has been relatively loose and that there has been little attempt to hit the lower bound of the official band. Related to the band is the central bank's perceived inflation target. The estimated target is relatively constant with a mean of 5.12%. As it lies closer to the upper bound than the lower bound of the official target band this may be indicative of some accommodation of cost-push shocks.

References

[1] Anvari, V., Ehlers., N. and Steinbach, R. (2014), "A semi-structural approach to estimate South Africa's potential output", SARB Working Papaer 14/08.

[2] Ball, L. and Mazumder, S. (2011), "Inflation Dynamics and the Great Recession", *Brookings Papers on Economic Activity*, 42, pp. 337-405, Spring.

[3] Ball, L. (2015), "Discussion of Blanchard, Cerrutti and Summers", ECB Forum on Central Banking, 21-23 May, Sintra Portugal.

[4] Blanchard, O., Cerutti, E. and Summers, L. (2015), "Inflation and Activity - Two Explorations and their Monetary Policy Implications", Paper Presented at the ECB Forum on Central Banking, 21-23 May, Sintra Portugal.

[5] Bullard, J. and Schaling, E. (2001), "New Economy: New Policy Rules?", *Quarterly Review Federal Reserve Bank of St. Louis*, Volume 83, No. 5, September/October, pp. 57-66.

[6] Chan, J., Koop, G., and Potter, S. (2015), "A Bounded Model of Time Variation in Trend Inflation, NAIRU and the Phillips Curve," forhtcoming in *Journal of Applied Econometrics.*

[7] Gali, J., Smets, F. and Wouters, R. (2012), "Unemployment in an Estimated New Keynesian Model", NBER Macroeconomics Annual 2011, pp. 329-360.

[8] Gali, J. (2015a), "Hysteresis and the European Unemployment Problem Revisited", Paper Presented at the ECB Forum on Central Banking, 21-23 May, Sintra Portugal.

[9] Gali (2015b), *Monetary Policy, Inflation and the Business Cycle. An Introduction to the New Keynesian Framework*, Second Edition, Princeton University Press, forthcoming.

[10] Gordon, R. J. (1998), "Foundations of the Goldilocks Economy: Supply Shocks and the Time-Varying NAIRU", *Brookings Papers on Economic Activity, 2, 297-333*

[11] Gordon, R. J. (2011), "The History of the Phillips Curve: Consensus and Bifurcation", *Economica*, 78, pp. 10-50.

[12] Kabundi, A. and Schaling, E. (2013), "Inflation and Inflation Expectations in South Africa: An Attempt at Explanation," *South African Journal of Economic*, 81(3): 346-355.

[13] Kabundi, A., Schaling, E., and Some, M. (2015), "Monetary Policy and Heterogeneous Inflation Expectations in South Africa," *Economic Modelling*, 45: 109-117

[14] Kydland, F.E. and Prescott, E.C. (1977), "Rules rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy,* 85(3): 473-491.

[15] Matheson, T. and Stavrev (2013), "The Great Recession and the Inflation Puzzle", *Economics Letters*, 120, pp. 468-472.

[16] Mishkin, F., (2007), "Does Inflation targeting Make a Difference? ", in Frederik Mishkin and Klaus Schmidt- Hebbel, eds., Monetary Policy Under Inflation Targeting, pp. 291-372

[17] Phillips, A.W. (1958), "The Relation Between Unemployment and the Rate of Money Wage Rates in the United Kingdom, 1861-1957, *Economica*, 25, pp. 283-299.

[18] Samuelson, P.A. and Solow, R.M. (1960), "Analytical Aspects of Anti-Inflation Policy, *American Economic Review*, Papers and Proceedings, 50, pp. 177-194.

[19] Sargent, T. (1982), "Rational expectations and the reconstruction of macroeconomics," *Quarterly Review*, Federal Reserve Bank of Minneapolis, issue Sum.

[20] Seedwell, H., Albert, T.M, Fulbert, T., (2015), "Term of Trade Shocks and Inflation Targeting in Emerging Market Economies", forthcoming *South African Journal of Economics.*

[21] Stella, A. and Stock, J., (2013), "A state-dependent model for inflation forecasting," Board of Governors of the Federal Reserve System: Washington, DC.

[22] Stock, J.and Watson, M. (2007), "Why has U.S. inflation become harder to forecast," *Journal of Money, Credit and Banking*, 39: 3–33.

[23] Svenson, L.E.O, (2010), Inflation Rate Targeting, NBR Working Paper 16654

[24] Svensson, L.E.O., (1999), Inflation targeting: Some extensions, Scandinavian Journal of Economics 101(3), 337–361.

[25] Viegi, N. (2015), Labour Market and Monetary Policy in South Africa. SARB Working Paper 15/01

Figure A.1: Atheoretical and Phillips Curve NAIRU

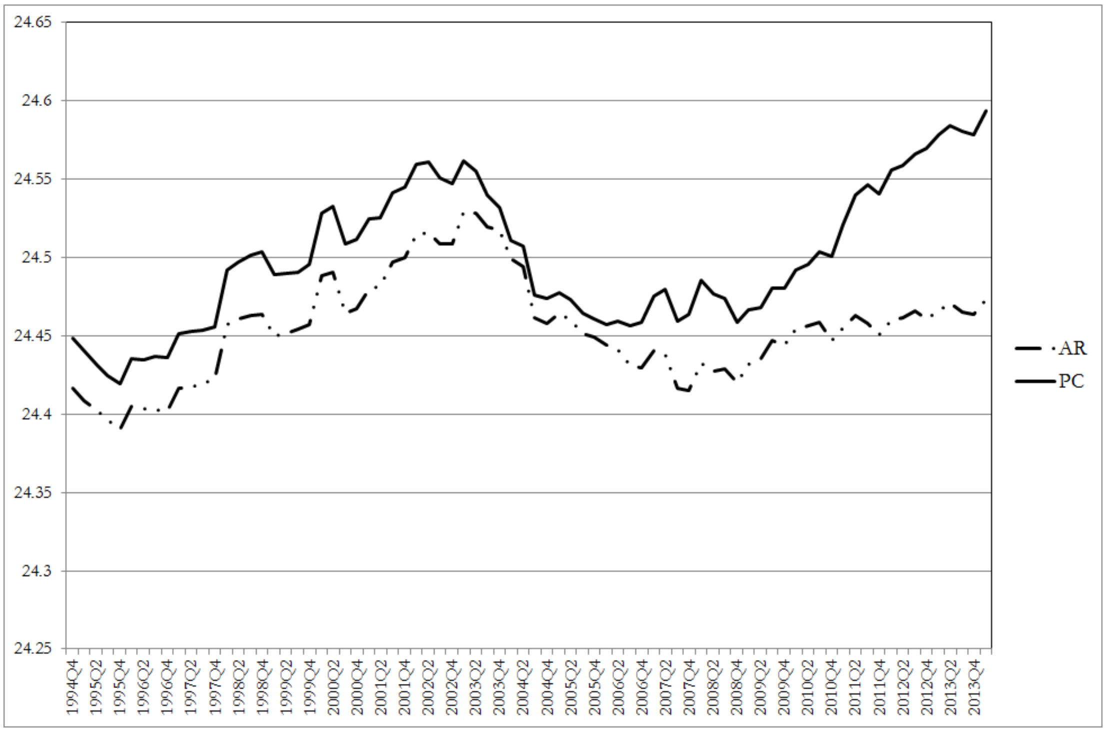


Table A.1: Posterior distribution statistics

Parameter	Mean	Std	Min	Max
φ_1	0.86	0.1	0.35	1.28
φ_2	0.01	0.1	-0.41	0.54
σ_u^2	1.11	0.15	0.64	2.21
σ_h^2	0.10	0.02	0.042	0.28
$\sigma_{u^n}^2$	0.01	0.002	0.004	0.03
$\sigma_{\pi^*}^2$	0.02	0.005	0.008	0.07
σ_ρ^2	0.002	0.001	0.001	0.007
σ_α^2	0.002	0.001	0.001	0.006
a_{u^n}	17.50	1.45	15	20
a_{π^*}	3.25	0.14	2.98	3.50
b_{π^*}	6.41	0.40	5.50	7.03
b_{u^n}	27.40	1.42	25	30