

**South African Reserve Bank
Working Paper Series
WP/16/02**

**Estimating South Africa's output gap and
potential growth rate**

Johannes Fedderke and Daniel Mengisteab

Authorised for distribution by Chris Loewald

March 2016



South African Reserve Bank

South African Reserve Bank Working Papers are written by staff members of the South African Reserve Bank and on occasion by consultants under the auspices of the Bank. The papers deal with topical issues and describe preliminary research findings, and develop new analytical or empirical approaches in their analyses. They are solely intended to elicit comments and stimulate debate.

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the South African Reserve Bank or South African Reserve Bank policy. While every precaution is taken to ensure the accuracy of information, the South African Reserve Bank shall not be liable to any person for inaccurate information, omissions or opinions contained herein.

South African Reserve Bank Working Papers are externally refereed.

Information on South African Reserve Bank Working Papers can be found at
<http://www.resbank.co.za/Research/ResearchPapers/WorkingPapers/Pages/WorkingPapers-Home.aspx>

Enquiries

Head: Research Department
South African Reserve Bank
P O Box 427
Pretoria 0001

Tel. no.: +27 12 313-3911
0861 12 SARB (0861 12 7272)

© South African Reserve Bank

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means without fully acknowledging the author(s) and this Working Paper as the source.

Estimating South Africa's output gap and potential growth rate

Johannes W. Fedderke,^{*} Daniel K. Mengisteab[†]

Abstract

This paper estimates the potential output of the South African economy using several univariate filters as well as taking a production function approach. Our aim is to compare the sensitivity of the results to the different methodologies and different measures of output. We find that the potential output is sensitive to the different methodologies and different measurements of output. A Cobb-Douglas specification of the production function is employed, dividing the economy into eight sectors. We find that the production function produced results similar to the band-pass filters but with gaps of lower amplitudes. We then use the Hodrick-Prescott, Christiano-Fitzgerald, and a Kalman filter to observe the natural growth rate of the South African economy from 1960 to 2015. We find estimates of the natural growth rate in the 1.9% - 2.3% range. However, there is also evidence to suggest that the rate is under considerable downward pressure in the post-2010 period. The strongest decline is in the real sectors of the economy (Manufacturing, Mining), the greatest resilience in the service sectors (financial in particular).

^{*}Pennsylvania State University, jwf15@psu.edu

[†]Pennsylvania State University

List of Tables

1	Comparison of Methodologies adopted by International Institutions - note, unless otherwise indicated reference is to corresponding Central Bank	5
2	Cobb-Douglas Production Function Results	21
3	Growth Rates Implied under Alternative Aggregate Economic Activity Measures for Hodrick Prescott and Christiano Fitzgerald Filters	23
4	Growth Rates Implied under Alternative Aggregate Economic Activity Measures - Direct Growth Rates Approach 5 year averages	27
5	Growth Rates Implied for Economic Sectors	28

List of Figures

1	High-Pass Filters - GDP at Market Prices: Hodrick-Prescott (HP) and Butterworth (BW) Filters.	13
2	GDP at Market Prices: Christiano-Fitzgerald (CF) and Baxter-King (BK) Filters.	14
3	Gross Value Added at Basic Prices: Hodrick-Prescott (HP) and Butterworth (BW) Filters.	14
4	Band-Pass Filters - Gross Value Added at Basic Prices: Christiano-Fitzgerald (CF) and Baxter-King (BK) Filters.	15
5	Gross Domestic Expenditure: Hodrick-Prescott (HP) and Butterworth (BW) Filters.	15
6	Band-Pass Filters - Gross Domestic Expenditure: Christiano-Fitzgerald (CF) and Baxter-King (BK) Filters.	16
7	High-Pass Filters - Gross National Income: Hodrick-Prescott (HP) and Butterworth (BW) Filters.	16

8	Gross National Income: Christiano-Fitzgerald (CF) and Baxter-King (BK) Filters.	17
9	Gross Domestic Product: Maximum, Average, Minimum Output Gap Measure across the four time series filters.	18
10	Gross Value Added: Maximum, Average, Minimum Output Gap Measure across the four time series filters.	19
11	Gross Domestic Expenditure: Maximum, Average, Minimum Output Gap Measure across the four time series filters.	19
12	Gross National Income: Maximum, Average, Minimum Output Gap Measure across the four time series filters.	20
13	Production Function Based Output Gap Measures. Production Function refers to the aggregate output gap measure on the weighted average of sectoral production functions. HP_Composite uses the HP filter for sectors in which no plausible production function emerged. CF_Composite uses the CF filter for sectors in which no plausible production function emerged.	22
14	Gross Domestic Product: Natural Rate of Growth.	25
15	Gross Value Added: Natural Rate of Growth.	26
16	Gross Domestic Expenditure: Natural Rate of Growth.	26
17	Gross National Income: Natural Rate of Growth.	27
18	Mining: Potential Rate of Growth	29
19	Manufacturing: Potential Rate of Growth	30
20	Electricity, gas and water: Potential Rate of Growth	30
21	Construction: Potential Rate of Growth	31
22	Wholesale and retail trade, and accommodation: Potential Rate of Growth .	31
23	Transport, storage and communication: Potential Rate of Growth	32
24	Finance, insurance, real estate and business services: Potential Rate of Growth	32
25	Community services: Potential Rate of Growth	33

I. Introduction

The potential output of an economy represents the level of output that an economy can sustain without causing any changes to inflation. The output gap is simply the percentage difference between actual output and potential output. When actual output exceeds potential output there is an indication of excess demand which could lead to inflation. Analysis of the output gaps can provide useful insights for policy makers when assessing the macroeconomic performance of a country and its inflation targeting. Closely associated with potential output is the potential growth rate of the economy - a measure of the long-run growth capacity .

The potential output of an economy is an unobservable variable that may be estimated using a variety of methods, each with their advantages and disadvantages. Broadly speaking, there are two approaches to estimating potential output predominate in policy contexts: statistical filtering and approaches based on the estimation of production functions. In addition, there exist a range of additional multivariate approaches to output gap derivation.

The statistical filtering approach typically involves univariate statistical filters (eg. Hodrick-Prescott, (HP) Christiano-Fitzgerald (CF)) to separate the cyclical and trend components in macroeconomic time series data. High pass filters, such as the HP, filter all stochastic cycles below a specified frequency, thus allowing only cyclical structure which occur more frequently than a pre-specified benchmark. Low pass filters, most frequently used in engineering contexts, offer the opposite structure, in that they filter out all stochastic cycles above a specified frequency, thereby allowing only cyclical structure that occurs less frequently than a specified benchmark. Band pass filters, such as the CF filter, offer the intermediate case, in which stochastic frequencies outside a specified range are excluded, thus allowing cyclical structure below a maximum, and above a minimum frequency. Many central banks also employ bivariate filters, multivariate Kalman filters, or modifications of the univariate filters such as the HP filter.

The benefit of the statistical filtering approach is its simplicity. The disadvantages are that the potential outputs generated are sensitive to the parameterization of filters, the lack

of structural economic theoretical foundations to the filter, and hence the failure to indicate underlying structural, and hence causal, drivers of potential output.

The production function approach involves the estimation of a production function to determine the potential output of an economy. The advantage of using a production function rather than statistical filters, derives from its greater foundation in economic theory, by linking potential output of an economy to factor inputs. Any changes in the factor inputs of the economy would be evident in the potential output generated by the production function. However, production function approaches have their own disadvantages arising from data limitations, particularly as regards the measurement of capital, questions as to the appropriate functional form to be employed in the production function, and econometric concerns surrounding identification.

Other approaches used to estimate potential output include dynamic stochastic general equilibrium (DSGE) models, structural vector autoregressive (SVAR) models, amongst others. The appeal of such models is the greater explicitness with which economic theory is incorporated. On the other hand, the relative complexity of specifications that result, makes inference increasingly sensitive to the underlying parameterization of the model, which is often incompletely verified through econometric testing. Data limitations can further constrain successful implementation.

The purpose of this policy note is to consider the sensitivity of estimated potential output and the implied output gap to alternative methodologies. In doing so, we move beyond previous contributions to the South African debate, by considering a wider range of statistical instruments in deriving potential output and associated gap measures. This allows for a comparison of the sensitivity of inference to the methodology adopted.

Four prior contributions have considered questions closely related to the present policy note. Ehlers et al (2013) and Anvari et al (2014) both considered the potential output and growth rate of the South African economy, while Smit and Burrows (2002) focussed on potential output and the output gap. Kemp (2015) considers the sensitivity of measures

of the output gap to including financial cycle proxies into the Hodrick-Prescott filter. Smit and Burrows (2002) employ a production function (Cobb-Douglas and Constant elasticity substitution), HP filter and adapted multivariate filter loading on employment, the marginal product of labor, and the labor-output elasticity. Sample period is 1970Q1 through 1998Q4. While the study does not undertake a more formal comparison of the results obtained from the alternative approaches, implied potential output and output gap measures do not appear to vary substantively across the alternative methodologies. Ehlers et al (2013) employ production functions (Cobb-Douglas), a multivariate version of the HP filter loading on inflation (Phillips curve), unemployment (Okun's law) and capacity utilization,* and a general equilibrium model to derive potential output and its associated growth rate (from the potential output measure), for the 1998-2011 period. The study reports stronger differences between the alternative methodologies than the earlier Smit and Burrows (2002) study, with the highest potential output emanating from the production function approach, while the HP filter, multivariate HP filter and general equilibrium approaches show only marginal differences. A significant limitation of the study is its choice of sample period. The 1998-2008 period corresponds to a period of rapid growth, that at least potentially is due to rapid growth in the world economy generating a commodity based boom. This is reflected in very strong estimated values for potential growth for South Africa in the study, of at least 3.5%. In evaluating our own results, we will return to a consideration of the significance of the 1995-2008 period, and its role in interpreting the potential growth of the South African economy.

Anvari et al (2014), considering the period 1971Q1 to 2013Q4, adopts the Borio et al (2013, 2014) approach of incorporating financial cycle characteristics into the estimation of potential output in a Kalman filter. Their findings continue to report relatively robust potential growth for South Africa, with the 2013 estimate being 2.5%. The interpretation is that the current low growth performance of the economy is an aberration due to the

*This approach has affinities to that proposed by Beneš et al (2010).

aftermath of the financial crisis, and that the relatively robust growth performance of the early 2000s is a better benchmark of economic performance for South Africa. Kemp(2015) similarly represents an application of the Borio et al (2013, 2014) methodology, loading on credit extension, interest rate and house price information to incorporate financial cycle data into the HP filter. Kemp reports finding evidence of a larger output gap leading up to the 2008 crisis than under the standard HP filter as a result - a somewhat surprising result given the standard interpretation of the 2008 crisis as an exogenous shock to South Africa emanating from the US and European banking systems.

This paper considers a range of alternative methodologies for determining potential output, and the associated potential rate of growth of the South African economy.

Results prove sensitive to methodology. The choice of time series filter carries significant implications both for the volatility and amplitude of the output gap measure. The core distinction is between high-pass and band-pass filters, with the latter showing lower amplitude in the implied business cycle for South Africa. We also consider an approach that uses all of the time series filters, in order to generate an estimate of an upper and a lower bound for the output gap. While there are periods in which the agreement between the filters is strong, there are also time periods in which there is strong divergence between the filters, in the sense that the filters straddle positive and negative values: some filters indicating that the output gap is positive, some that it is negative. This is particularly the case in periods where policy makers need clear information regarding the direction of the economy - such as the post 2010 period of relatively weak economic growth.

In terms of the potential rate of growth of the economy, across the range of methodologies we employ we find the potential rate of growth to lie in the range of 1.9 - 2.3% per annum over the 2010-15 period. However, on at least some filter estimates, the growth deceleration post 2010 now points to a structural growth rate that lies closer to the 1% level, than the 2% level. This makes it critical to establish whether the relatively strong growth spurt over 1995-2007 was a temporary windfall or a reflection of sustainable international demand

	HP	CF	BK	BW	Kalman	Prod. Funct	DSGE	SVAR
Brazil						X		
Canada - Budget Office								X
Bank of Canada	X							X
Croatia						X		
IMF (various)	X	X	X		X	X		
Malaysia						X		
New Zealand						X		
OECD	X					X		
SARB	X				X	X		
US - CBO						X		
US - Fed							X	

Table 1: Comparison of Methodologies adopted by International Institutions - note, unless otherwise indicated reference is to corresponding Central Bank

conditions.

The paper is structured as follows. Section II. reviews the approaches adopted by a range of central banks and international agencies for comparative purposes. Section III. reviews the methodologies employed in the present analysis. Section IV. presents the data employed. Sections V. and VI. present results. Section VII. evaluates and concludes.

II. Review

A review of the approaches employed by a range of central banks and multilateral agencies, indicates that the methodologies outlined in the Introduction are the most frequently used in the estimation of output gaps. Table 1 summarizes the use of the alternative methodologies across a sample of institutions.

Barbosa-Filho, et al (2004) used a disaggregated Cobb-Douglas production function to estimate potential output, and inferred the aggregate growth rate of Brazil from the sectoral growth rates. Bank Negara Malaysia (2012) and the Croatian National Bank (Vrbanc, 2006) similarly used a standard Cobb-Douglas specification as a means to estimate potential output. The Reserve Bank of New Zealand (Lienert and Gillmore, 2015) used a variant of the production function involving capital utilization and different assumptions, but continue

to impose Cobb-Douglas technology structure.

The Bank of Canada (Pichette et al, 2015) compared two approaches. The first employed a multivariate filter derived from the HP filter to filter the cyclical components from a Cobb-Douglas production function. The second modeled output as a function of trend labor input and trend labor productivity. The Canadian Budgeting Office (Barnett and Matier, 2010) also chose to model output as a function of trend labor input and trend labor productivity to estimate potential output.

The US Congressional Budget Office used a highly disaggregated production function and took the sums weighted as a proportion of GDP to estimate potential output. Chermukhin (2013) of the US Federal Reserve in Dallas employed a DSGE model to formulate an estimate of the US's potential output.

Working papers from the IMF (Jain-Chandra and Zhang, 2014) created numerous estimates of potential output using three univariate filters; HP, CF, and Baxter-King (BK), as well as a Kalman filter, a multivariate filter and a Cobb-Douglas production function.

What emerges is thus a strong reliance on production function approaches, though in some cases inputs are measured not as factors of production, but as capacity utilization rates of factors.

III. Methodology

A. *Time Series Filters*

All four filters discussed below are used as a means to separate time series data into its trend and cyclical components:

$$(1) \quad y_t = \tau_t + c_t$$

where y_t denotes the time series of interest, c_t the stationary cyclical component driven by stochastic cycles, τ_t the trend component. These filters are often used to identify and

remove trends and seasonal components from time series data as well as to estimate the business-cycle component of data.

A generic linear filter of an infinitely long time series y_t can be written as:

$$(2) \quad \begin{aligned} y_t^* &= \sum_{j=-\infty}^{\infty} \alpha_j y_{t-j} = \alpha(L) y_t \\ f_{y^*}(\omega) &= |\alpha(e^{i\omega})|^2 f_y(\omega) \end{aligned}$$

where α_j are the filter weights and y_t^* is the smoothed series without any unwanted stochastic frequencies. Under filtering, the smoothed series is defined by the spectral density $f_{y^*}(\omega) = 0$, in which ω denotes the frequency of the independent stochastic cycles that contribute to the variance and autocovariance of y_t . The gain of the filter, $|\alpha(e^{i\omega})|$, determines what is filtered out of the series.

The literature has identified a range of time series filters. We consider four. The Hodrick-Prescott (HP), Butterworth (BW), Christiano-Fitzgerald (CF), and Baxter-King (BK) filters are special cases of the general filters outlined above. High-pass filters, such as HP and BW, only allow for stochastic cycles that meet a minimum level frequency and block the lower frequency stochastic cycles. For example, if ω_θ is the minimum specified frequency, then only $\omega \geq \omega_\theta$ would survive filtering. Band-pass filters, such as CF and BK, by contrast allow only stochastic cycles within a specified range of frequencies, such that $\omega_L \leq \omega_\theta \leq \omega_H$, with any frequency outside the upper, ω_H , and lower, ω_L , bound being filtered out.

Hodrick-Prescott Filter.—The Hodrick-Prescott filter (Hodrick and Prescott, 1997) is defined by:

$$(3) \quad y_{t,HP}^* = \min_{\tau_t} \left[\sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \{(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\}^2 \right]$$

which minimizes the sum of squared deviations of the series, y_t , from the trend, τ_t , subject to the smoothing parameter λ , typically 1600 for quarterly data - though note the alternatives

considered by Pollock (2000). As a two-sided symmetric filter it is subject to end-point bias (Baxter and King, 1995), such that it converges to the observed values of the underlying series at the beginning and end of the series.

Butterworth.—The Butterworth filter (Pollock, 2000) is given by:

$$(4) \quad \hat{h}_{BFT}(B) = \frac{\lambda [1 - B]^d [1 - B^{-1}]^d}{[1 + B]^d [1 + B^{-1}]^d + \lambda [1 - B]^d [1 - B^{-1}]^d}$$

where B is the lag operator, λ the smoothing parameter, and d is the order of the filter. The BW filter is a two parameter estimator where, like the CF and BK filters, the frequency settings of the filter must be predetermined. The order of the filter is the second parameter and is used to estimate the slope of the gain function at the cutoff frequency. Following Burns and Mitchell (1946) the default settings are 1.5 to 8 years (6 and 32 quarters), and the default order is 2.

Christiano-Fitzgerald.—The finite Christiano-Fitzgerald filter (Christiano and Fitzgerald, 2003) given by:

$$(5) \quad c_{t,CF}^* = b_0 y_t + \sum_{j=1}^{T-t-1} b_j y_{t+j} + \tilde{b}_{T-t} y_T + \sum_{j=1}^{t-2} b_j y_{t-j} + \tilde{b}_{t-t} y_1$$

minimizes the mean squared error between the filtered series and the series filtered by the ideal band-pass filter.[†] The cyclical component is given by $c_{t,CF}^*$, the b_0, b_1, \dots, b_j , the weights from the ideal band-pass filter, with:

$$(6) \quad \tilde{b}_{T-t} = -\frac{1}{2}b_0 - \sum_{j=1}^{T-t-1} b_j \quad \tilde{b}_{t-t} = -\frac{1}{2}b_0 - \sum_{j=1}^{t-2} b_j$$

The CF is not symmetric, and assumes the raw data to be a random walk process. The CF filter typically produces better estimates of the cyclical component than the BK filter if the

[†]Under the CF methodology, there is an ideal band pass filter which is of infinite length. Because the observed series is finite the 'ideal' filter cannot be computed precisely and thus the finite CF filter minimizes the mean squared error between the cyclical component derived from the ideal band pass filter and the cyclical component derived from the finite filter.

data resembles a random walk or a random walk with drift. For the CF filter the default business cycle frequency settings are 1.5 to 8 years as per Burns and Mitchell (1946), which is equivalent to 6 and 32 quarters. A disadvantage of the CF filter is the rigidity of the frequency setting: that business cycles are not always pre-determined to be within the 6 to 32 quarter frequency.

Baxter-King.—The finite Baxter-King filter (Baxter and King, 1999) is given by:

$$(7) \quad c_{t,BK}^* = \sum_{j=-q}^{+q} \hat{b}_j y_{t-j}$$

with $c_{t,BK}^*$ denoting the cyclical component, q defines the order of the symmetric moving average (SMA). The ideal coefficients, \hat{b}_j , are defined by:

$$(8) \quad \sum_{j=-q}^{+q} \hat{b}_j = 0 \quad \hat{b}_j = b_j - \bar{b}_q \quad \bar{b}_q = (2q+1)^{-1} \sum_{j=-q}^{+q} b_j$$

Unlike the CF filter, the BK filter is a SMA where the weights on the leads and lags are equal. Hence the filter drops observations both at the beginning and end of the series. The resultant trade off for the BK filter is that choosing a larger q brings the estimation closer to the ideal filter, but increases the number of missing observations. This truncation is the major disadvantage of the BK filter from a policy perspective. Like the CF filter the default business cycle frequency settings are 1.5 to 8 years (6 and 32 quarters) and the default order of the SMA is 12.

B. The Kalman Filter

The Kalman Filter (KF) uses an autoregressive linear state space model (Hamilton, 1994b) given by:

$$(9) \quad \begin{aligned} y_t &= Z_t \alpha_t + d_t + \varepsilon_t \\ \alpha_t &= T_t \alpha_{t-1} + c_t + R_t \eta_t \end{aligned}$$

where t denotes time, y_t is an M dimensional time series, α_t is a vector of unobserved state variables, d_t and c_t are vectors of exogenous variables, ε_t , η_t are error terms, and Z_t , T_t , R_t , are time-invariant matrix coefficients. The state space model estimates the parameters via a maximum likelihood estimator for which the Kalman filter recursively estimates the least squares forecasts of y_t . The Kalman filter which consists of two sets of recursive equations (10, 11, and 12, 13 respectively), determines the optimal parameter estimates, predicting equations and updating equations:

$$(10) \quad a_{t|t-1} = T_t a_{t-1} + c_t$$

$$(11) \quad P_{t|t-1} = T_t P_{t-1} T_{t-1}' + R_t Q_t R_t'$$

$$(12) \quad a_{t|t-1} = P_{t|t-1} Z_t' F_t^{-1} v_t$$

$$(13) \quad P_t = P_{t|t-1} - P_{t|t-1} Z_t F_t^{-1} Z_t' P_{t|t-1}$$

where a_t represents the optimal estimator of α_t and P_t is its conditional covariance matrix. a_{t-1} and P_{t-1} are the optimal predictors of α_t and P_t . For each time t , the Kalman filter produces the conditional expected state vector $a_{t|t-1}$ and $P_{t|t-1}$. Q_t , T_t , R_t are all matrix parameters, and F_t^{-1} is the forecast error. The optimal predictor of the time series y_t is then:

$$(14) \quad y_{t|t-1} = Z_t a_{t|t-1} + d_t$$

where Z_t is the coefficient matrix and d_t is a vector of exogenous variables.

C. Production Functions

For the estimation of the output gap, we considered two alternative production functions.

The first is the standard Cobb-Douglas:

$$(15) \quad Y_t = AK_t^\alpha L_t^\beta e^{\varepsilon_t}$$

which, provides the empirical specification:

$$(16) \quad \ln Y_t = \ln A + \alpha \ln \left(\frac{K_t}{L_t} \right) + (\beta - \alpha) \ln L_t + \varepsilon_t$$

The alternative considered is the translog production function (Berndt and Christensen, 1973):

$$(17) \quad \ln Y_t = \ln A + \alpha_1 \ln K_t + \alpha_2 \ln L_t + \beta_1 (\ln K_t)^2 + \beta_2 (\ln L_t)^2 + \gamma_1 \ln K_t \ln L_t + \varepsilon_t$$

In order to avoid aggregation bias, we chose to estimate the outputs and derive the output gaps sector by sector, deriving the aggregate output gap as the sum of sectoral output gaps weighted by the sectoral share in GDP.

For a number of sectors the production function was not well defined. This held true for Mining, Finance, insurance real estate and business services, Community and social services, and Electricity, gas and water. For these sectors we employed both the most plausible production function, and as an alternative either the HP or the CF filters to estimate the output gap, with aggregation consistently by means of the weighted sum of sectoral output gaps.

IV. Data

All of the data used for this paper was obtained from the South African Reserve Bank. Four different measurements of output were used with the four filters mentioned above for 16 variations of the South African output gap. For the production function the economy was disaggregated into 8 two digit sectors with the associated labor and capital inputs. All

of these variables are given in the appendix with their available years listed as well as their associated Dickey-Fuller test statistics.

Unfortunately the agriculture, forestry and fishing sector had to be omitted due to the lack of data on labor. Unlike the potential output gaps estimated by the filters which are calculated for the years 1960:Q1 to 2015:Q2, the estimated output gaps estimated via the production function were only able to be calculated from 1976:Q3 to 2015:Q1 due to limited labor data in the Electricity, Trade, Transportation, and Financial sectors.

V. Output Gap Results

A. *Filters: Output Gap*

The output gaps on South African aggregate output under the application of the time series filters are reported in Figures 1 through 8.

A number of features are apparent from the results:

- There is negligible difference between the two high-pass filters (HP vs. BW).
- There is negligible difference between the two band-width filters (CF vs BK). Given the truncation of observations at the sample end-points under the BK filter, the CF is therefore preferable in policy application.
- There are strong differences between the high-pass filters and the band-width filters.

Specifically:

- The high-pass filters are noisier, and with greater cyclical variation over the course of the cycle.
- By contrast the two band-width filters generate greater smoothing over the cycle.
- The high-pass filters are currently projecting a negative output gap.
- By contrast the band-width filters are currently projecting a positive output gap.

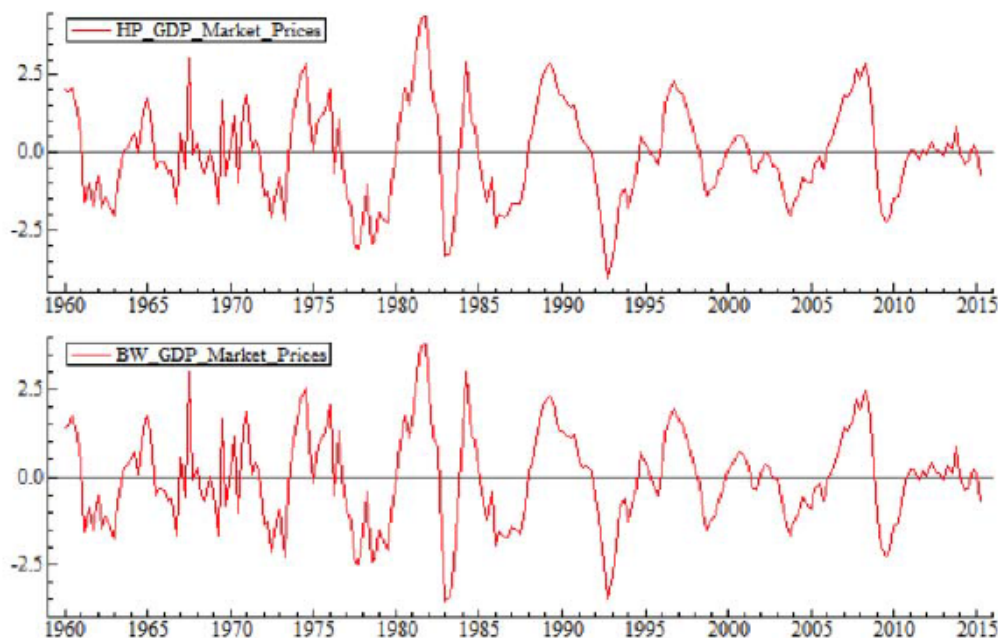


Figure 1: High-Pass Filters - GDP at Market Prices: Hodrick-Prescott (HP) and Butterworth (BW) Filters.

- Volatility of the output gap varies widely across the output measurements. From most to least volatile the output measurements are: Gross Domestic Expenditure; Gross National Income; Gross Value Added; Gross Domestic Product.
- It follows that inferences regarding the output gap are very sensitive to the choice of time series filter. This is true in terms of the amplitude of the implied output gap. It is true with respect to the volatility of the output gap through the cycle. It is currently even true in terms of whether a positive or negative output gap is implied for the South African economy.
- If one filter is to be chosen, in terms of its performance the Christiano-Fitzgerald filter appears to give the most reliable results. It is less volatile than the high-pass filters, and does not suffer from end-point truncation.

We also considered the implications of being agnostic about which filter to employ. Instead of generating a single output gap measurement for a given filter, we considered

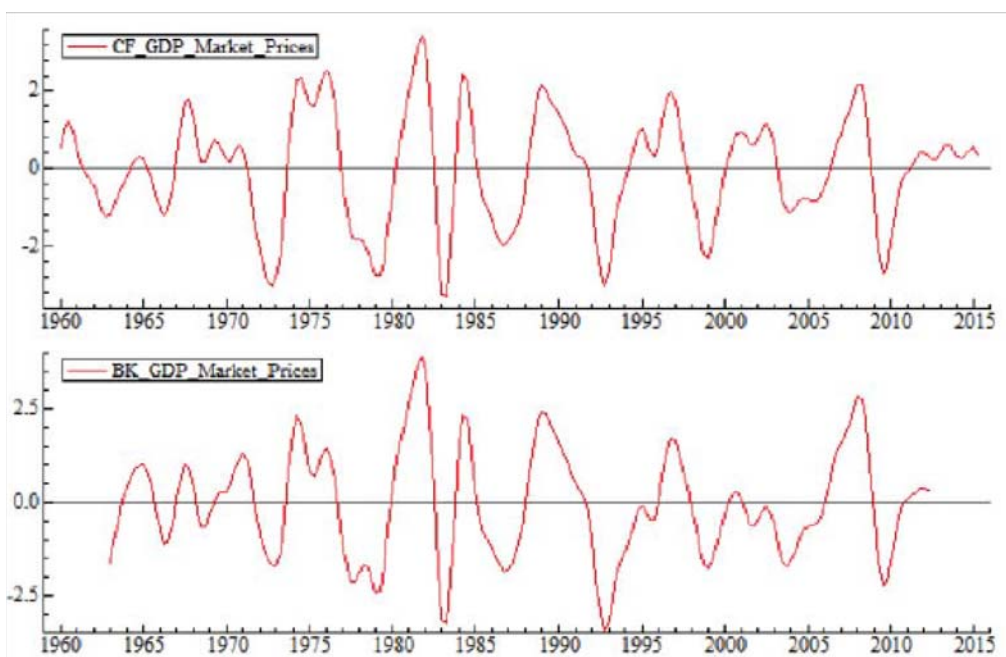


Figure 2: GDP at Market Prices: Christiano-Fitzgerald (CF) and Baxter-King (BK) Filters.

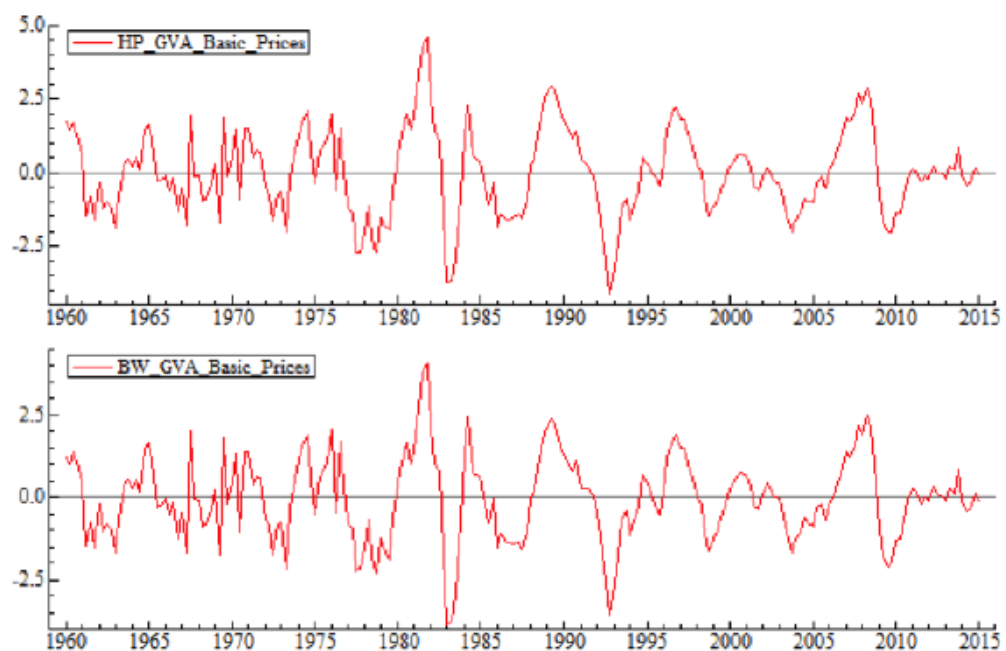


Figure 3: Gross Value Added at Basic Prices: Hodrick-Prescott (HP) and Butterworth (BW) Filters.

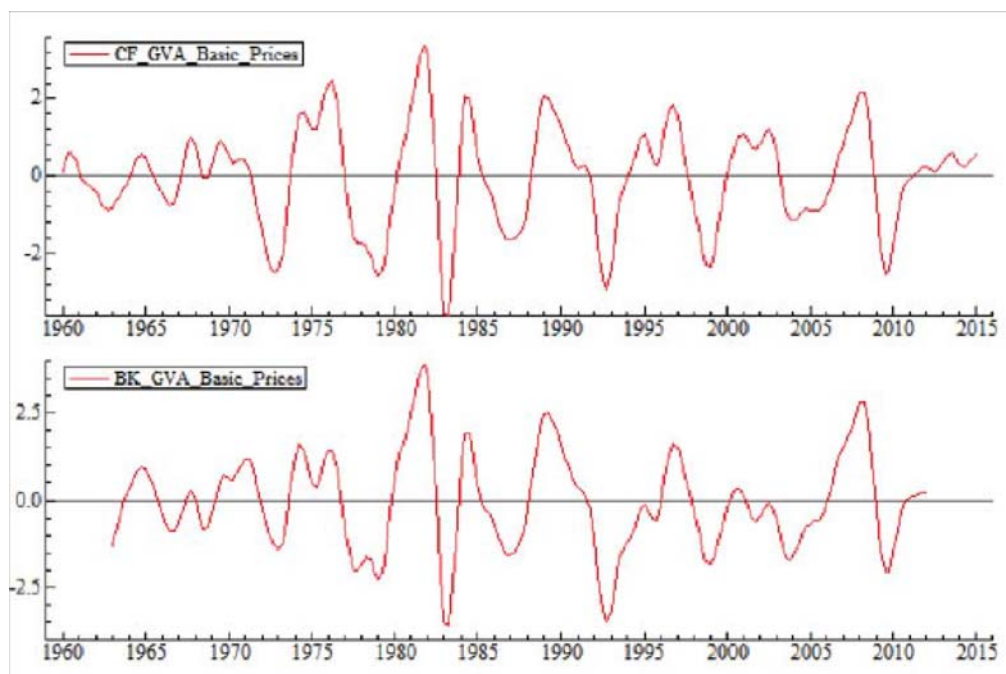


Figure 4: Band-Pass Filters - Gross Value Added at Basic Prices: Christiano-Fitzgerald (CF) and Baxter-King (BK) Filters.

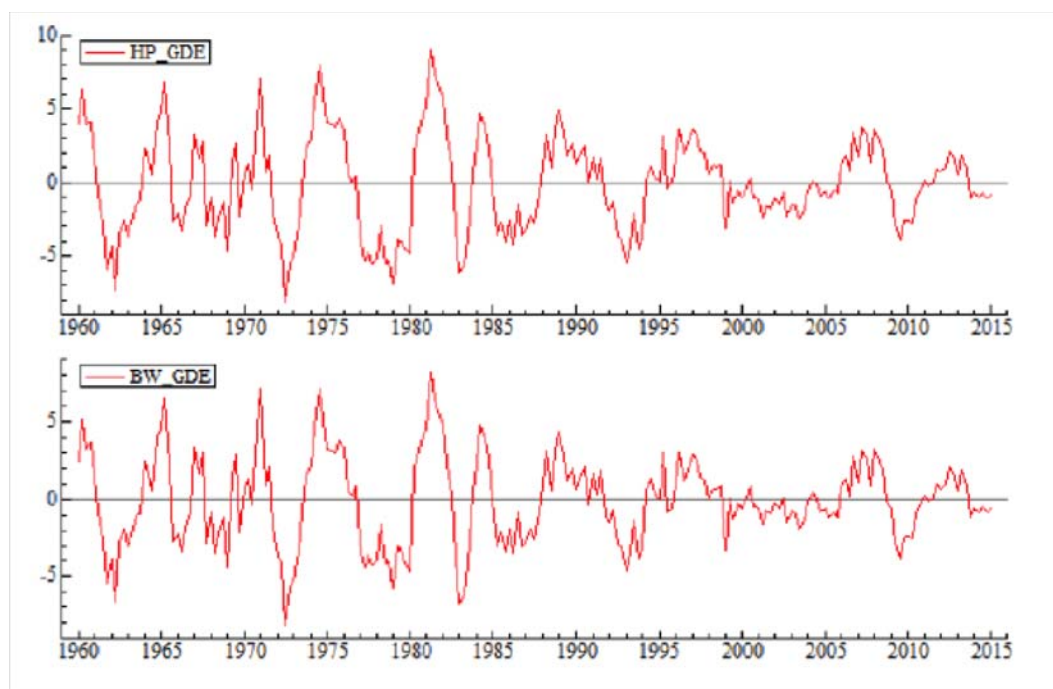


Figure 5: Gross Domestic Expenditure: Hodrick-Prescott (HP) and Butterworth (BW) Filters.

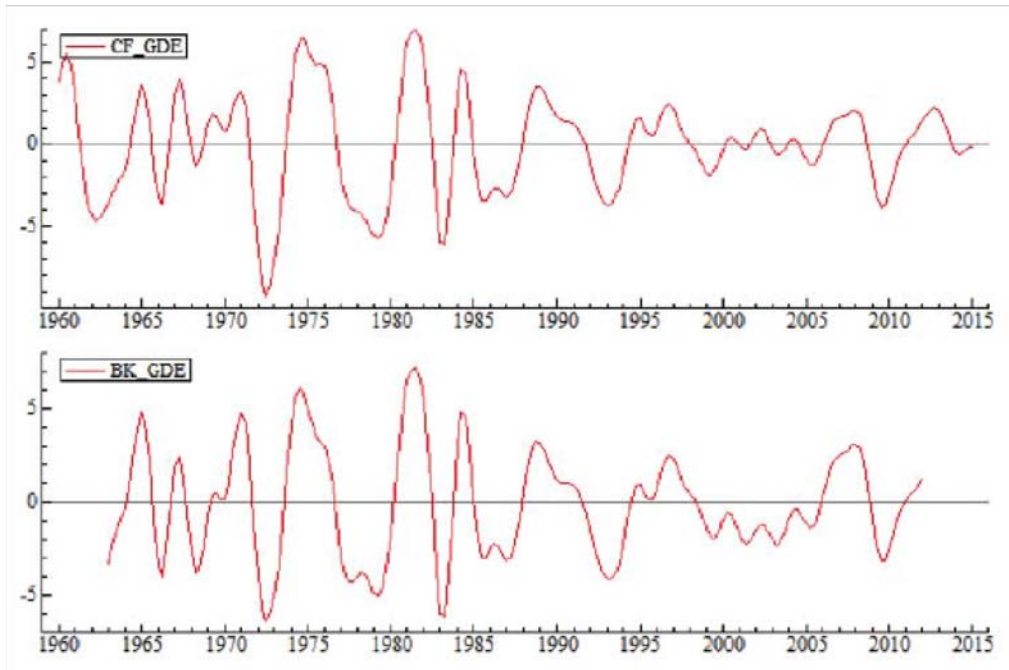


Figure 6: Band-Pass Filters - Gross Domestic Expenditure: Christiano-Fitzgerald (CF) and Baxter-King (BK) Filters.

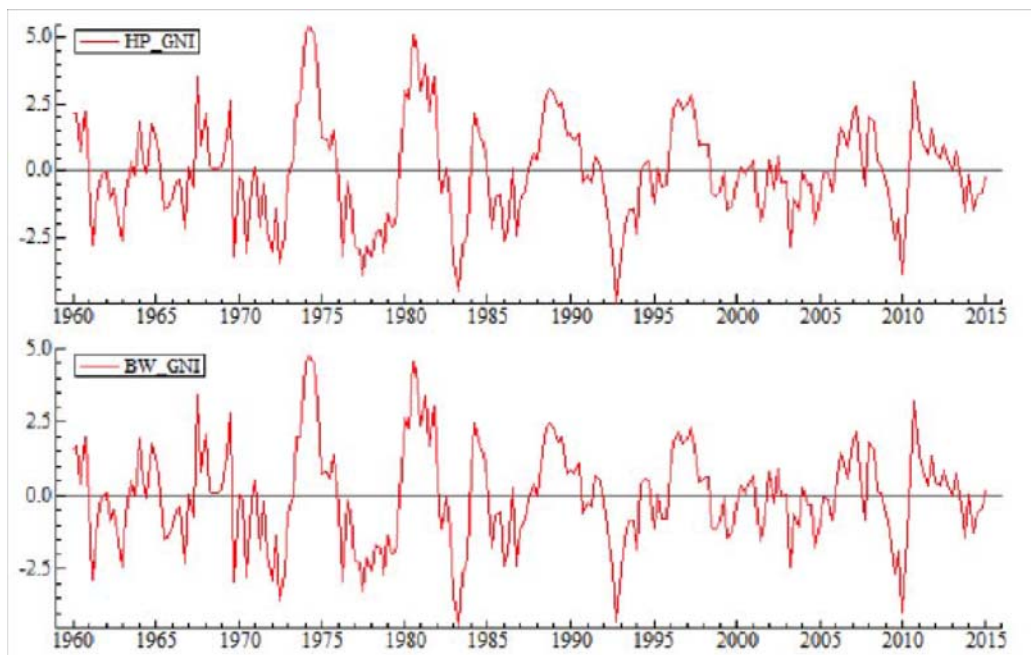


Figure 7: High-Pass Filters - Gross National Income: Hodrick-Prescott (HP) and Butterworth (BW) Filters.

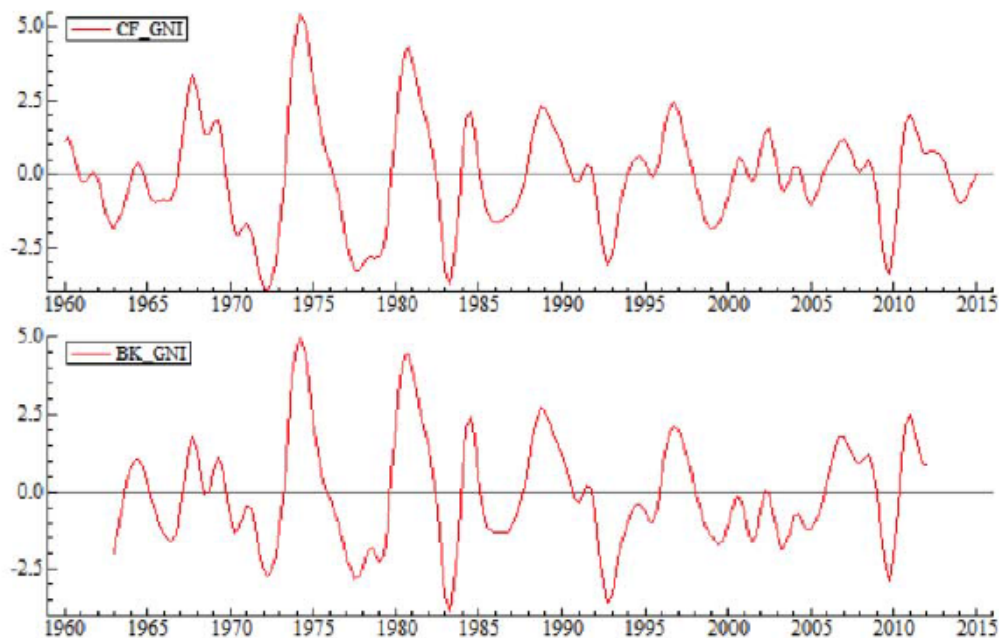


Figure 8: Gross National Income: Christiano-Fitzgerald (CF) and Baxter-King (BK) Filters.

a range of uncertainty generated by the maximum and minimum output gap estimate to emerge across the four time series filters we employ. We also reported the average output gap measurement across the four filters. Note that the two limit values would in general be supplied each of the four filters across the full sample period, depending on which was generating the maximum or minimum estimate in any one time period. The results for the four measurements of aggregate economic activity are reported in Figures 9 through 12.

Two features are striking about these results. There are certainly periods in which the agreement between the filters is strong. Thus the relatively strong period of growth in South Africa in the 2005-7 period is uniformly reported as a period with a positive output gap across all filters. The period following the global financial crisis of 2007/8, is also uniformly reported as a period with a strong negative output gap. But, by contrast, there are also periods in which there is strong divergence between the filters. Both in the early 2000s, surrounding the timing of the emerging markets financial instability (Russia), and the current period of relatively weak economic growth show the filters straddling positive and negative values:

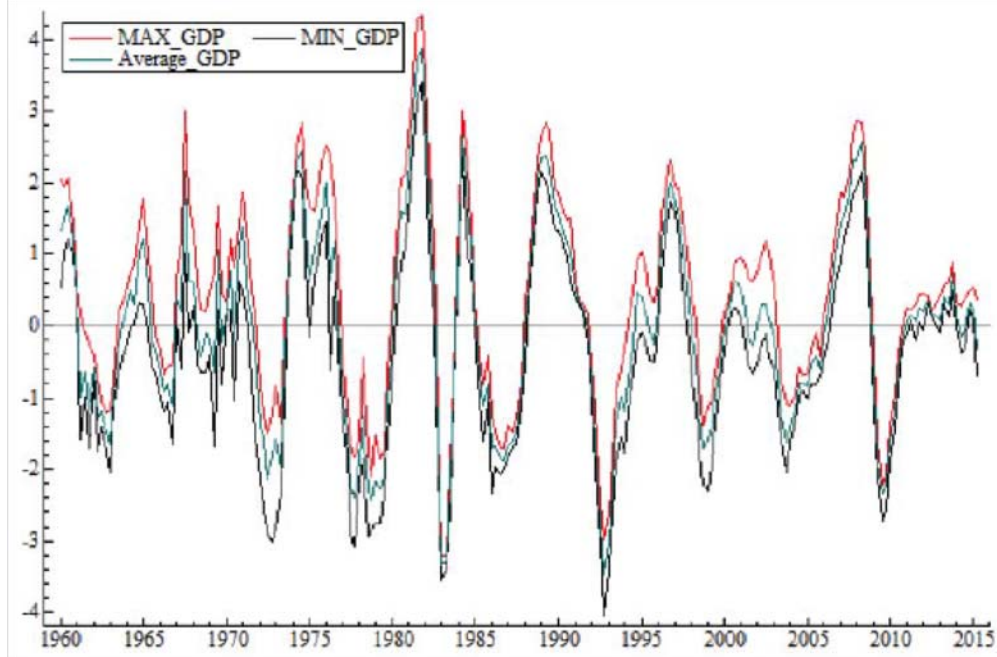


Figure 9: Gross Domestic Product: Maximum, Average, Minimum Output Gap Measure across the four time series filters.

some filters indicating that the output gap is positive, some that it is negative.

A consequence for policy making is that output gap measurements based on time series filters need to be approached with caution. While the methodology appears to give relative unambiguous signals at times, unfortunately the indication is also that particularly at times where close guidance to policy making would be useful, such as the early 2000s and the period after 2011 in South Africa, the filters can give conflicting signals even on the direction (positive, negative) of the output gap.

Finally, note that the greatest volatility in the implied output gap continues to attach to the measurements of Gross Domestic Expenditure and Gross National Income measures.

B. Production Function Approach: Output Gap

Our output, capital stock and employment series are uniformly $\sim I(1)$. The relevant Augmented Dickey-Fuller test statistics are reported in the Appendix.

In Table 2 we report the estimated sectoral production functions. We note that de-

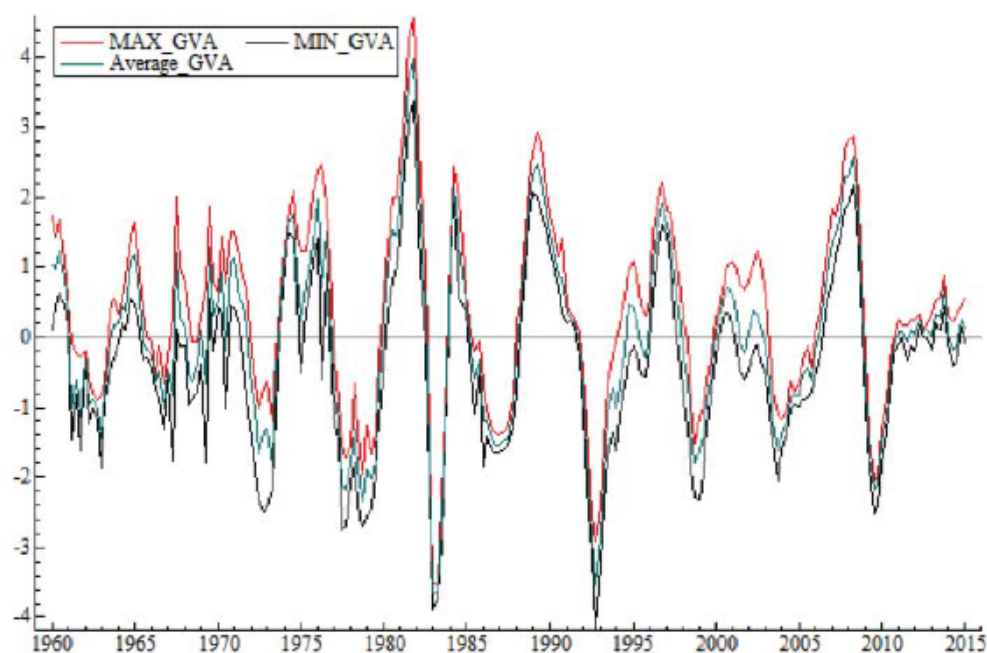


Figure 10: Gross Value Added: Maximum, Average, Minimum Output Gap Measure across the four time series filters.

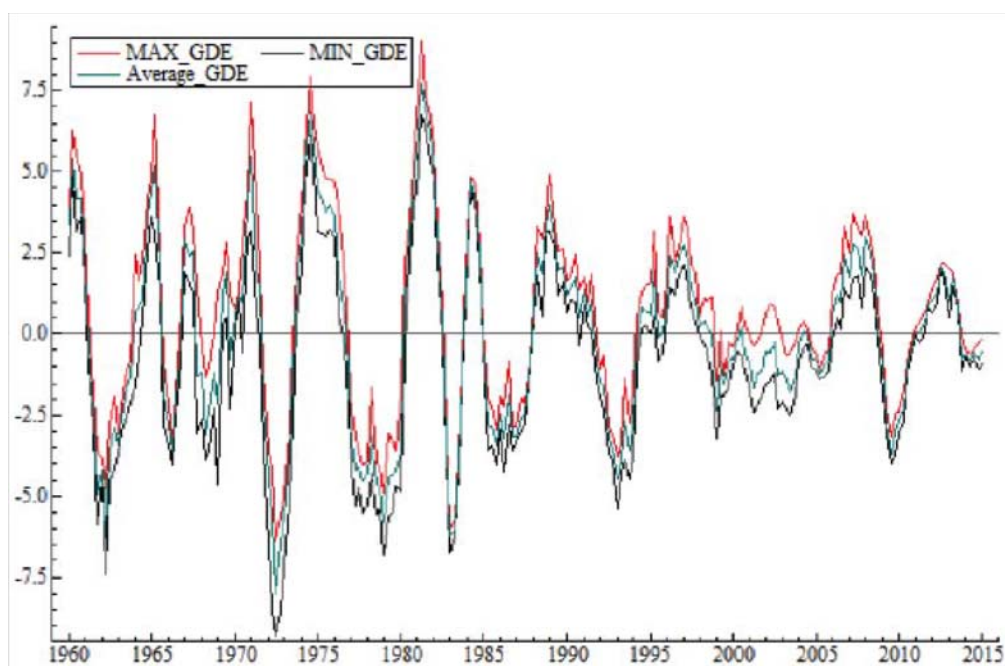


Figure 11: Gross Domestic Expenditure: Maximum, Average, Minimum Output Gap Measure across the four time series filters.

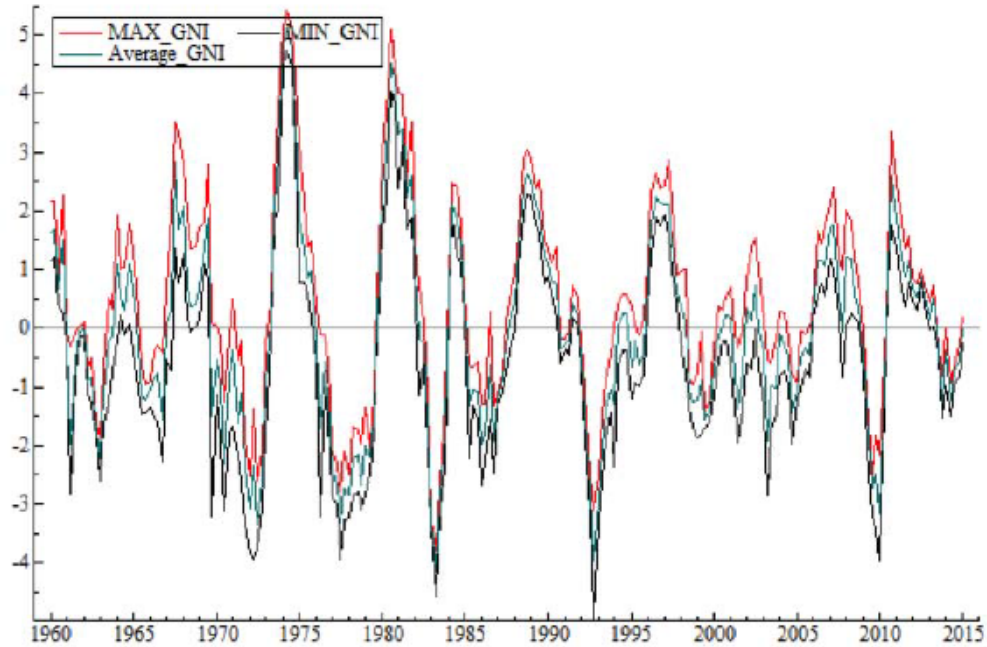


Figure 12: Gross National Income: Maximum, Average, Minimum Output Gap Measure across the four time series filters.

spite extensive specification searches, in general theoretically consistent coefficients were not readily found for a number of the sectors. In addition, translog specifications were subject to multiple cointegrating vectors. Without clear priors by means of which to restrict the coefficient space, this restricts the empirical usefulness of the translog specifications for our purposes. For this reason, we proceed with the Cobb-Douglas technology assumption, as reported in Table 2.

Despite the concerns with the coherence of the results, we nevertheless proceed. Three estimates of the output gap were derived. In the first, we employed the estimated sectoral production functions, irrespective of the theoretical coherence of the estimated coefficients, in order to derive sectoral output gaps. These were then aggregated into an aggregate output gap, employing the weighted sum of sectoral output gaps, with weights determined by the sector's contribution to GDP. As an alternative, we employed time series filters for sectors in which production function coefficients were questionable, and then proceeded with weighted aggregation to an aggregate output gap. We employed both the HP and CF filters.

	Mining	Manuf.	EGW	Constr.	TRA	TSC	FIREBS	CSS
Lags:	4	6	5	5	4	6	5	5
Trace:	40.84*** [0.001]	32.22** [0.03]	39.75*** [0.002]	30.40** [0.04]	38.41*** [0.003]	30.94*** [0.04]	42.26*** [0.001]	23.95 [0.21]
Rank:	1	1	1	1	1	1	1	1
α	-0.02 (0.02)	0.42 (0.25)	0.05 (0.39)	0.47*** (0.07)	0.72*** (0.10)	0.49 (0.38)	1.28*** (0.09)	0.88* (0.47)
$\beta - \alpha$	0.09 (0.06)	-0.88 (0.47)	0.13 (0.28)	1.27*** (0.10)	1.11*** (0.04)	1.81*** (0.16)	1.46 (0.07)	1.84 (0.47)
ecm:	-0.14** (0.03)	-0.01** (0.003)	-0.01** (0.003)	-0.001 (0.01)	-0.06** (0.02)	0.02 (0.01)	-0.01 (0.02)	-0.01 (0.01)
<p>Figures in round parentheses are standard errors. Figures in square parentheses are probability values.</p> <p>*, **, *** denotes significance at the 10%, 5%, and 1% levels respectively.</p>								

Table 2: Cobb-Douglas Production Function Results

The result is reported in Figure 13.

A number of features emerge from these results:

- The amplitude of the output gap measurements under the production function approach is lower than the time series filter approach (the range is approximately $-1\% - +1\%$) - even where the HP and CF filters are used in conjunction with the production function estimates.
- The production function approach gives an output gap structure that is closer to the band-pass filters, than to the high-pass filters. This holds specifically with respect to the current positive output gap, and the lower magnitude of output gaps generated.

VI. Implications for the Potential Rate of Growth of South Africa

Given the estimates generated for potential output under the alternative methodologies, what implications follow for the potential growth rate of South Africa? Given the current anaemic growth performance of the economy, this question is of particular salience.

We considered this question in three different ways.

First, we compute the implied growth rate of the economy from the measurements of potential output generated under Section V. We do so both for the four measurements of aggregate economic activity, and the potential output estimated through the production

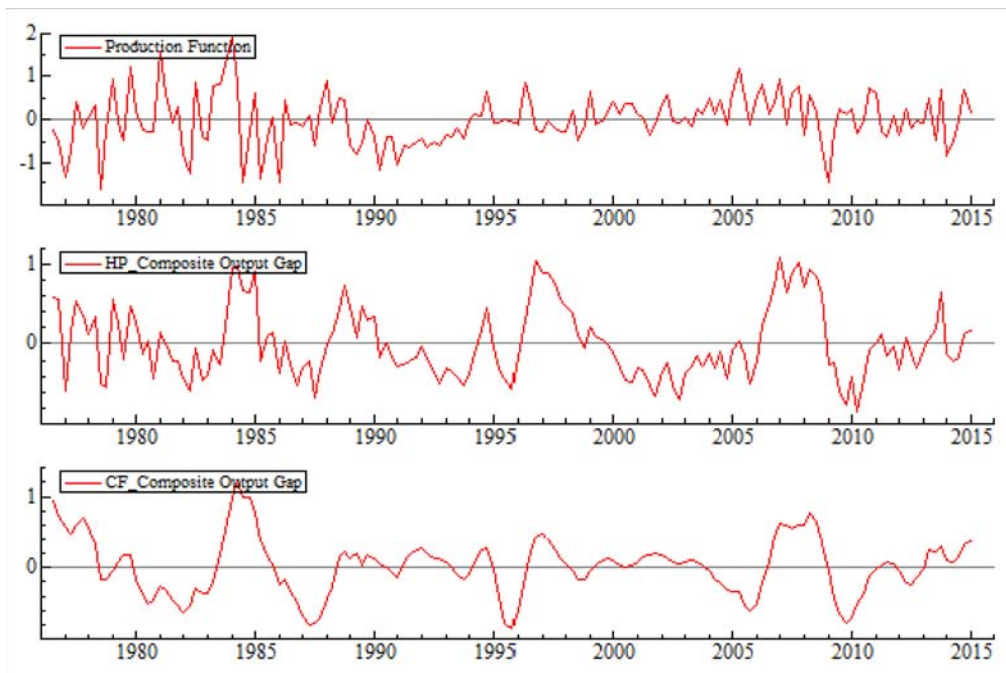


Figure 13: Production Function Based Output Gap Measures. Production Function refers to the aggregate output gap measure on the weighted average of sectoral production functions. HP_Composite uses the HP filter for sectors in which no plausible production function emerged. CF_Composite uses the CF filter for sectors in which no plausible production function emerged.

	1960-65	1965-70	1970-75	1975-80	1980-85	1985-90	1990-95	1995-2000	2000-05	2005-10	2010-15
Gross Domestic Product											
HP	6.4	5.4	3.8	2.9	2.1	1.2	0.7	2.6	3.7	3.6	2.1
CF	5.8	5.1	3.9	3.0	1.9	1.4	0.3	3.0	3.5	3.7	1.9
Gross Value Added											
HP	6.0	5.2	3.8	2.9	2.1	1.2	0.7	2.6	3.7	3.6	2.1
CF	5.8	5.1	3.9	3.0	1.9	1.4	0.3	3.0	3.5	3.7	1.9
Gross Domestic Expenditure											
HP	6.9	6.4	4.9	2.4	2.1	0.9	1.1	2.3	4.3	4.8	2.5
CF	7.4	5.6	5.4	2.4	1.9	1.1	0.9	2.6	4.2	5.0	2.3
Gross National Income											
HP	6.1	5.5	4.8	3.0	2.0	1.4	0.7	2.3	3.9	4.5	2.2
CF	6.1	5.1	4.9	3.1	1.9	1.5	0.4	2.6	3.6	4.8	2.0
Production Function											
	-	-	-	1.7	3.1	1.5	0.4	2.3	3.2	3.7	2.2

Table 3: Growth Rates Implied under Alternative Aggregate Economic Activity Measures for Hodrick Prescott and Christiano Fitzgerald Filters

function approach. In doing so we consider the potential output from one high-pass (HP) and one band-pass (CF) filter. Results are reported in Table 3 in the form of five year averages. The results confirm the steady slow-down of the growth rate through the 1990s that has often been remarked upon for South Africa, the subsequent recovery over 1995-2010, and a slow down post 2010. The HP filter based potential output measurement consistently generates the higher growth estimates than does the CF filter. The implied growth estimated from the Gross Domestic Expenditure and Gross national Income measurements generally exceed those that are obtained from Gross Domestic Product and Gross Value Added measurements. On the GDP measurement, the growth rates implied by the two filters narrowly straddle a 2% per annum growth rate on average for the 2010-15 period.

The second was to employ filters directly on the actual growth rate of the economy given by four-quarter logged differences of the four measurements of aggregate economic activity we employ for this study. The filters we employ are the Kalman filter using only the measurement of aggregate economic activity as a state variable - denoted Kalman_Uni. Second, the Kalman filter using four state variables, output, capital, labor (employment) and

human capital[‡] - denoted Kalman_Multi. Finally, we also employed the Hodrick-Prescott and Christiano-Fitzgerald time series filters. Results are reported in Figures 14 through 17. Given the virtually identical results between the univariate and multivariate Kalman filters, we report only the univariate Kalman in the Figures - the implied long run growth values for both Kalman Filters appear in Table 4 which reports five year averages..

Noteworthy are the following features of the results:

- We again observe the long term decline in the growth rate from 1960-95, the reversal over 1995-2008, and a subsequent sharp decline after 2010. This is true for all measurements of economic activity. It is also true across all three reported filters, with the Kalman filter showing the highest degree of volatility.
- By the close of the sample period, implied potential growth rates had declined to very low levels - in some instances below 1%, though the 1-2% range is the more prevalent finding. See the graphical evidence.
- The differences between the univariate and multivariate state-space Kalman filters are negligible - see Table 4.
- A crucial question is whether the growth recovery of the 1995-2008 period was merely a temporary reflection of unusually buoyant international demand conditions, or whether it reflects greater fundamental strengths in the economy. The concern is that if the former interpretation is correct (the 1995-2008 period as non-repeatable aberration), the implied long-run growth performance of the economy is implied to be below 2% - barely sufficient to make headway against population growth in terms of improvements in per capita GDP.
- The growth potential of South Africa would then have approximately be a third of that observed at the start of the sample period.

[‡]The use of employment truncated the start of the sample period to 1967Q1 due to data availability. The human capital input was proxied by the average years of schooling in the population, obtained from Barro and Lee (2010).

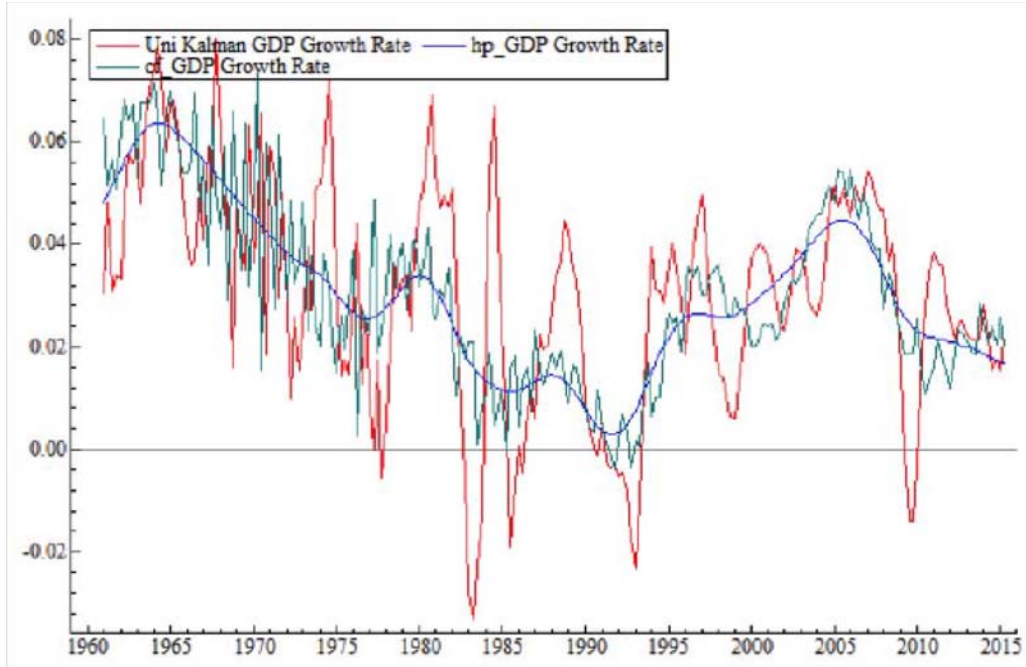


Figure 14: Gross Domestic Product: Natural Rate of Growth.

- The implied growth in Gross Domestic Expenditure and Gross National Income again exceeds that of Gross Domestic Product and Gross Value Added, in general.
- Contrasting the results derived from the measurement of potential output (Table 3) with those derived directly from observed growth (Table 4), the potential rates derived from directly from actual observed growth deliver higher estimates. The potential output measurements imply growth rates at or below 2%. Those from observed growth in the range of 2-3% (average: 2.3%).
- Note however that where we do not employ period averages, the implied decrease in the potential growth rate is stronger than reported in Table 4.

Finally, we considered the implied growth rates across the two digit sectors employed in constructing the output gap measurements under the production function approach. From the sectoral growth rates, we then inferred a growth rate for the economy based on the sectoral weight in aggregate GDP. The sectoral growth rates are illustrated in Figures 18

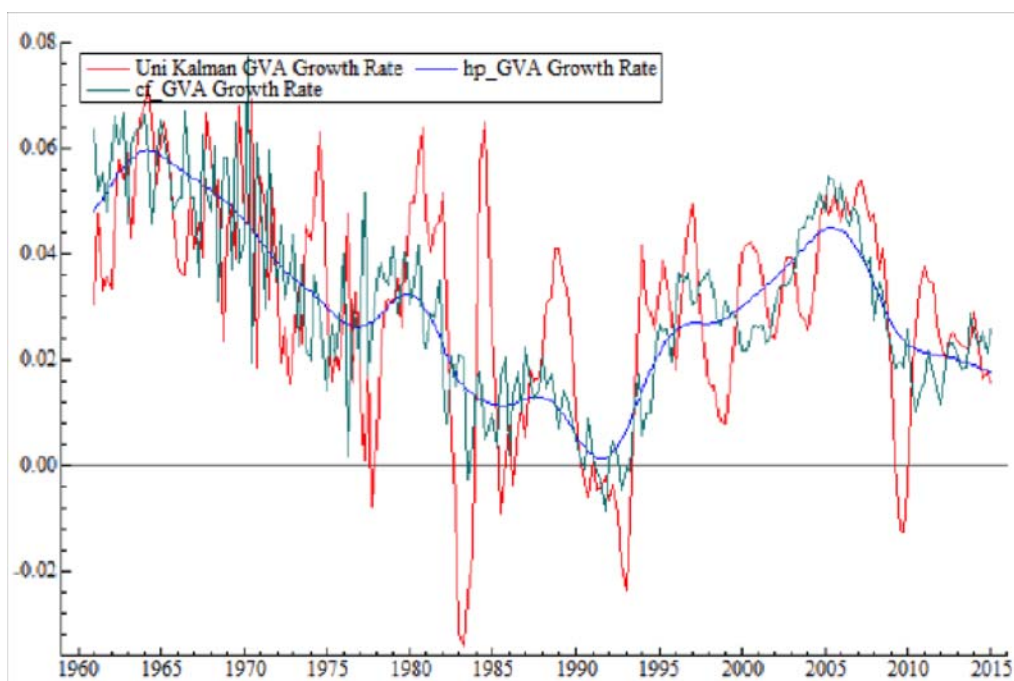


Figure 15: Gross Value Added: Natural Rate of Growth.

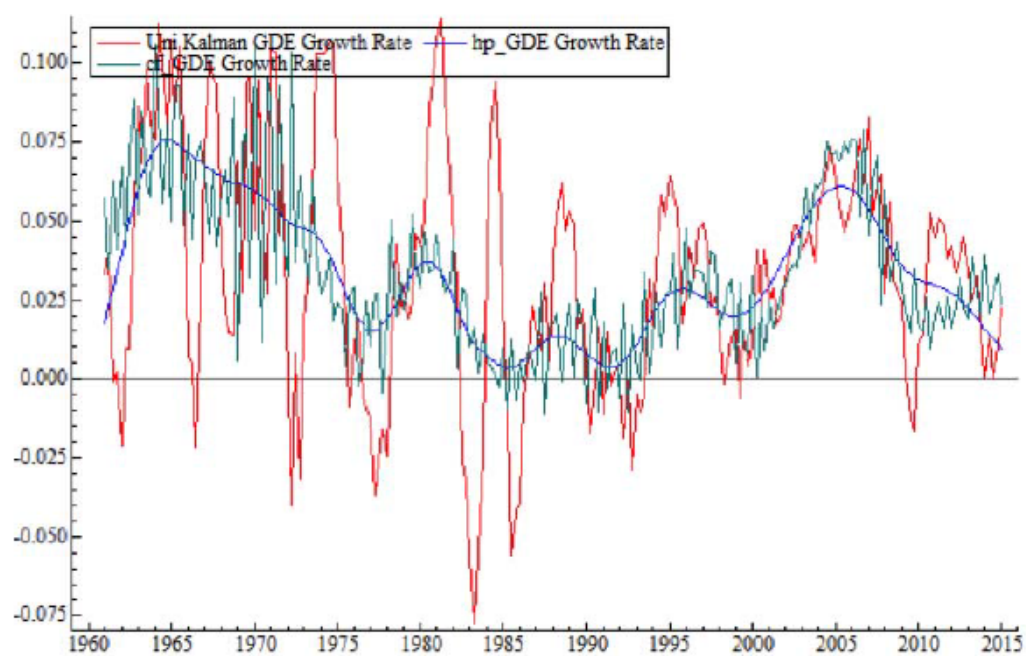


Figure 16: Gross Domestic Expenditure: Natural Rate of Growth.

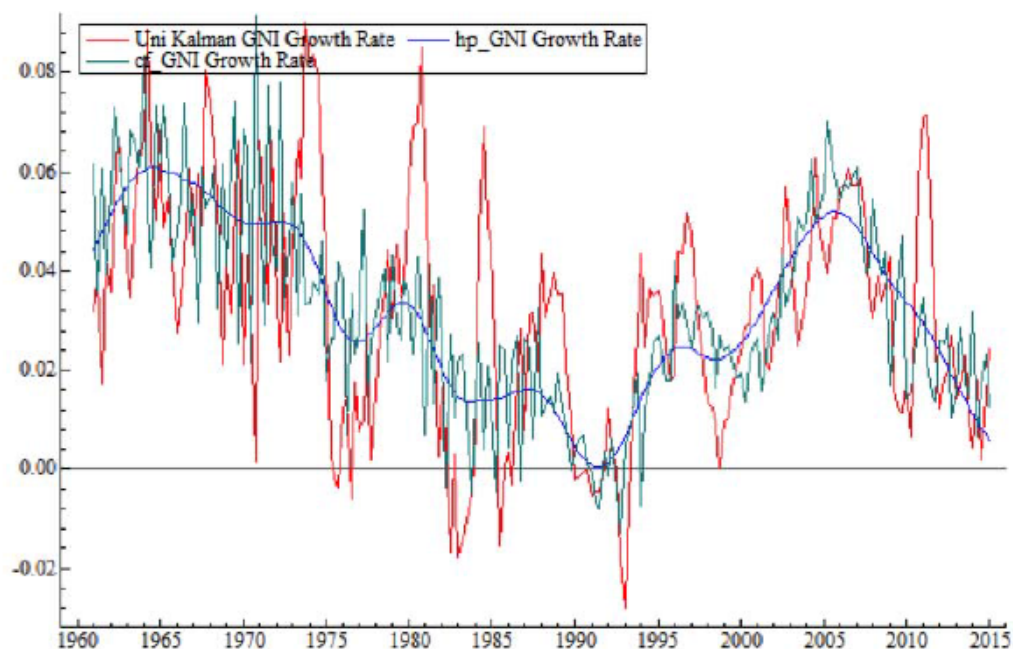


Figure 17: Gross National Income: Natural Rate of Growth.

	1960-65	1965-70	1970-75	1975-80	1980-85	1985-90	1990-95	1995-2000	2000-05	2005-10	2010-15
Gross Domestic Product											
Kalman_Uni	5.4	5.0	4.1	2.2	3.0	1.8	0.5	2.6	3.4	3.7	2.4
Kalman_Multi		4.1	4.1	2.5	3.2	1.9	0.7	2.6	3.0	3.5	2.4
HP	5.8	5.5	3.8	2.9	2.2	1.3	0.8	2.6	3.6	3.7	2.0
CF	6.2	5.3	3.9	2.9	2.1	1.4	0.6	2.9	3.3	3.8	2.0
Gross Value Added											
Kalman_Uni	5.1	4.9	3.9	2.3	2.8	1.8	0.4	2.6	3.5	3.7	2.3
Kalman_Multi		4.32	4.0	2.6	3.0	1.9	0.7	2.6	3.1	3.6	2.4
HP	5.6	5.3	3.8	2.8	2.1	1.2	0.7	2.6	3.7	3.7	2.0
CF	5.8	5.2	3.8	2.9	2.0	1.3	0.4	3.0	3.4	3.8	2.0
Gross Domestic Expenditure											
Kalman_Uni	5.2	5.6	5.5	1.1	3.7	1.5	0.7	2.7	4.0	4.4	2.9
Kalman_Multi		4.2	5.3	1.9	3.7	1.8	0.8	2.8	3.2	4.2	2.9
HP	5.4	6.7	4.9	2.3	2.1	0.9	1.1	2.4	4.2	4.9	2.3
CF	6.6	6.1	5.3	2.3	2.2	0.9	1.0	2.7	3.9	5.2	2.4
Gross National Income											
Kalman_Uni	5.0	5.1	5.0	2.0	2.8	2.2	0.6	2.5	3.6	4.2	2.5
Kalman_Multi		4.6	4.9	2.5	3.0	2.3	0.8	2.5	3.2	3.9	2.7
HP	5.5	5.6	4.8	3.0	2.1	1.4	0.7	2.3	3.8	4.5	2.0
CF	5.9	5.3	4.9	3.0	2.0	1.4	0.4	2.7	3.5	4.9	2.0

Table 4: Growth Rates Implied under Alternative Aggregate Economic Activity Measures - Direct Growth Rates Approach 5 year averages

	1960-65	1965-75	1975-85	1985-95	1995-2005	2005-15
	Mining					
Kalman_Uni	4.0	0.5	0.1	-0.2	0.1	-0.5
HP	5.5	0.2	0.2	-0.5	0.1	-0.6
CF	4.8	0.2	0.2	-0.4	0.2	-0.8
	Manufacturing					
Kalman_Uni	9.6	6.5	3.3	0.5	2.7	2.4
HP	10.8	6.9	3.4	0.3	2.8	1.8
CF	11.0	6.9	3.3	0.2	2.9	1.7
	Electricity, gas and water					
Kalman_Uni		6.9	5.5	3.6	2.2	1.7
HP		7.5	6.1	3.4	2.0	1.0
CF		8.0	6.1	3.4	1.9	1.1
	Construction					
Kalman_Uni	10.3	8.7	-0.3	-1.7	3.2	6.0
HP	11.1	9.4	-0.8	-1.8	3.2	6.0
CF	10.1	9.7	-1.1	-2.1	2.9	6.2
	Wholesale, retail trade and accommodation					
Kalman_Uni		5.7	3.2	0.3	3.5	3.4
HP		6.1	2.6	0.7	3.8	3.1
CF		6.8	2.5	0.5	3.8	3.1
	Transport, storage and communication					
Kalman_Uni			3.5	1.9	6.1	3.3
HP			3.3	1.8	6.5	3.2
CF			3.1	1.8	6.6	3.2
	Finance, insurance, real estate and business services					
Kalman_Uni		5.2	3.3	2.0	4.9	4.2
HP		4.9	3.4	1.9	5.0	4.1
CF		5.2	3.4	1.9	5.0	4.1
	Community Services					
Kalman_Uni	3.2	3.5	3.4	2.5	2.0	3.2
HP	3.6	4.0	3.5	2.3	1.3	3.3
CF	3.7	4.0	3.5	2.3	1.2	3.3

Table 5: Growth Rates Implied for Economic Sectors

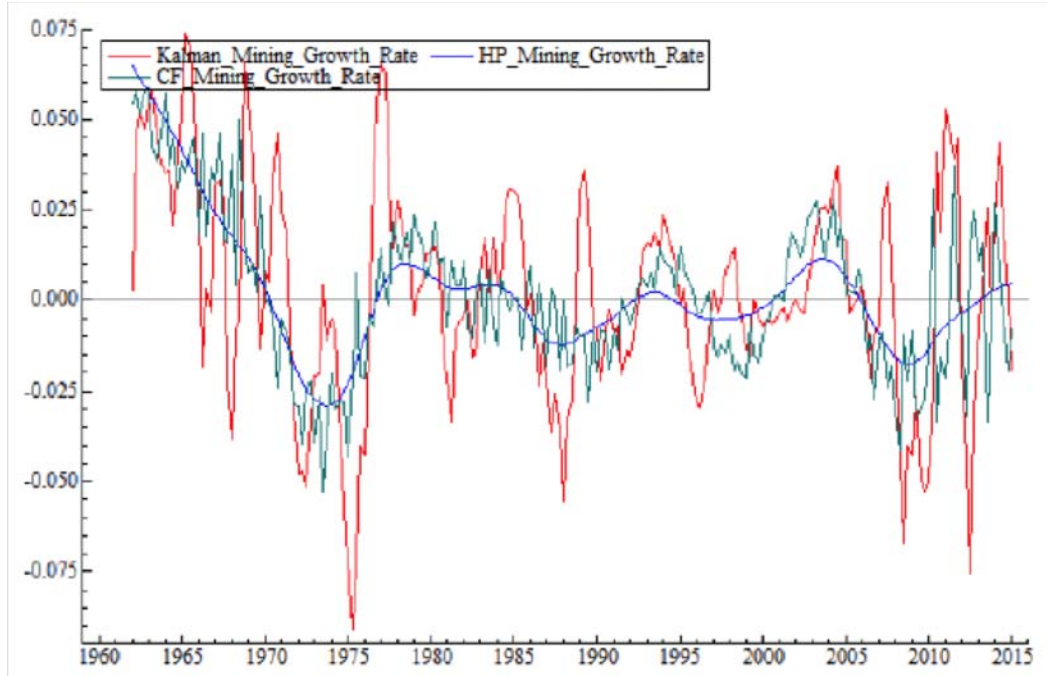


Figure 18: Mining: Potential Rate of Growth

through 25, and in Table 5 for ten year averages. The core implication to emerge is that the slow-down in the economy is concentrated in the real sectors, Mining and Manufacturing in particular, but also the public utilities sector. By contrast, growth in the service sectors has been relatively buoyant - especially in the financial services sectors. Even the sectors that are growing relatively strongly, have shown a dramatic decrease in growth performance post 2008. Construction is the one real sector that has continued to report relatively strong growth over the 2005-15 period, though here too there has been a dramatic downturn post 2010.

VII. Evaluation and Conclusion

This paper considered a range of alternative methodologies for determining potential output, and the associated potential rate of growth of the South African economy.

Results are sensitive to methodology. Specifically, the choice of time series filter carries significant implications both for the volatility and amplitude of the output gap measure-

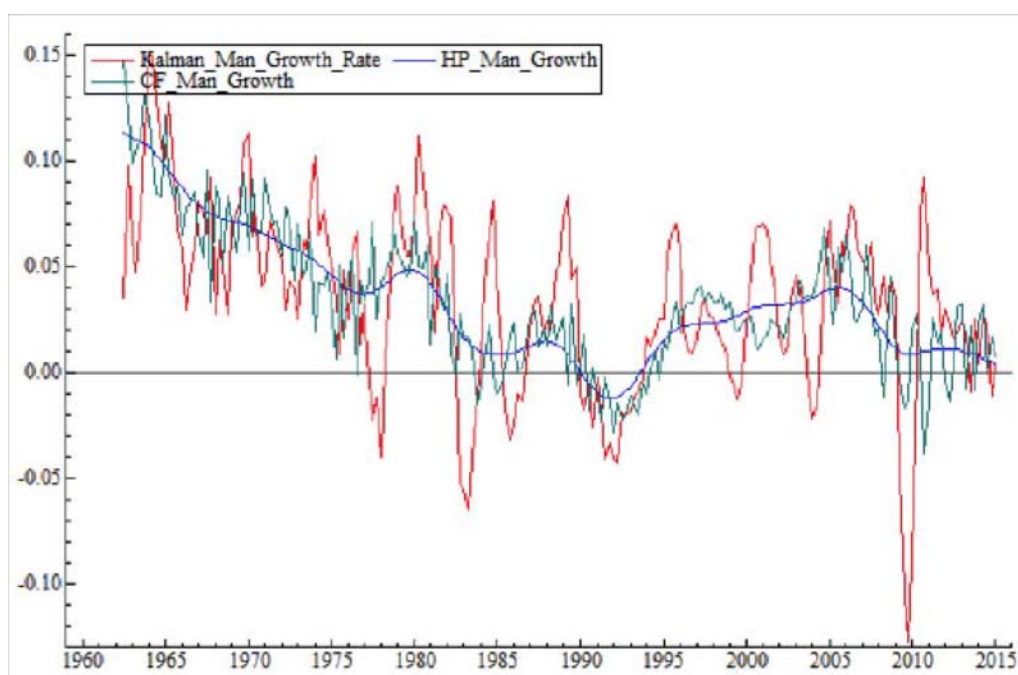


Figure 19: Manufacturing: Potential Rate of Growth

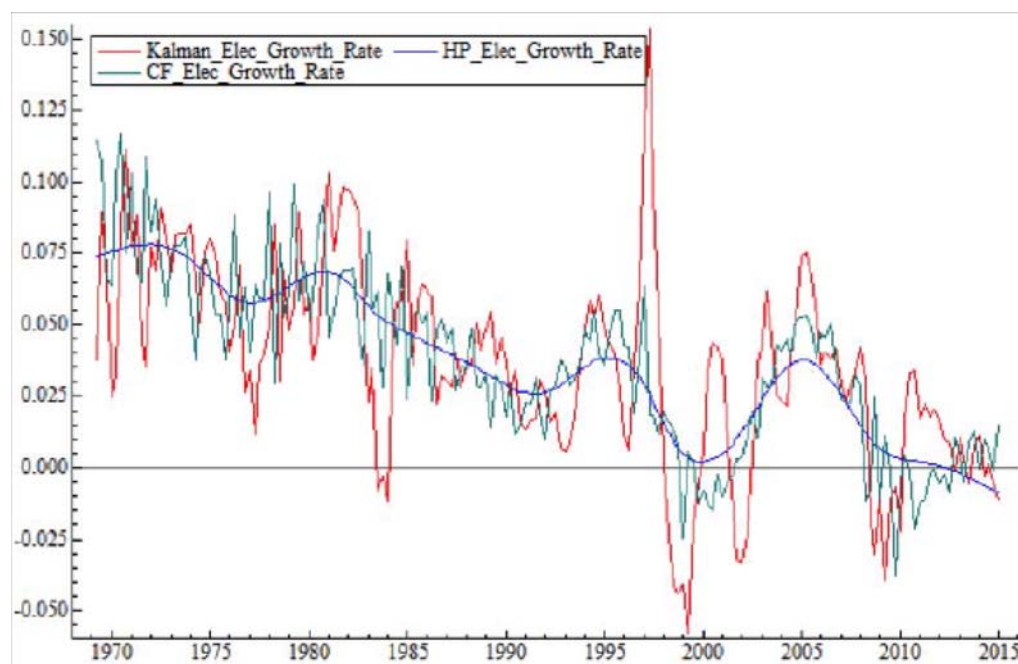


Figure 20: Electricity, gas and water: Potential Rate of Growth

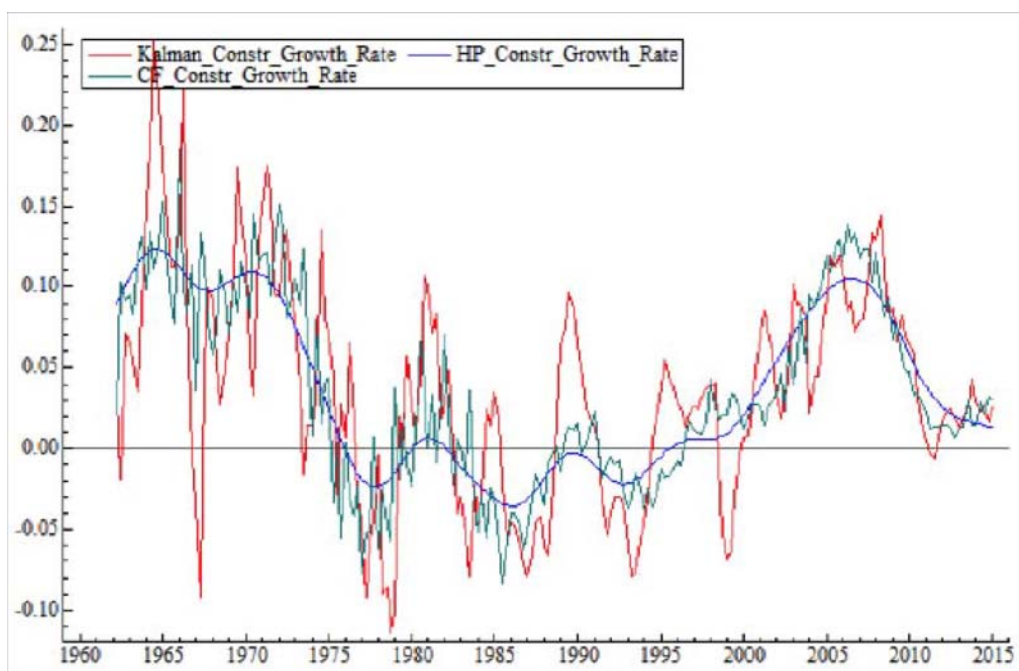


Figure 21: Construction: Potential Rate of Growth

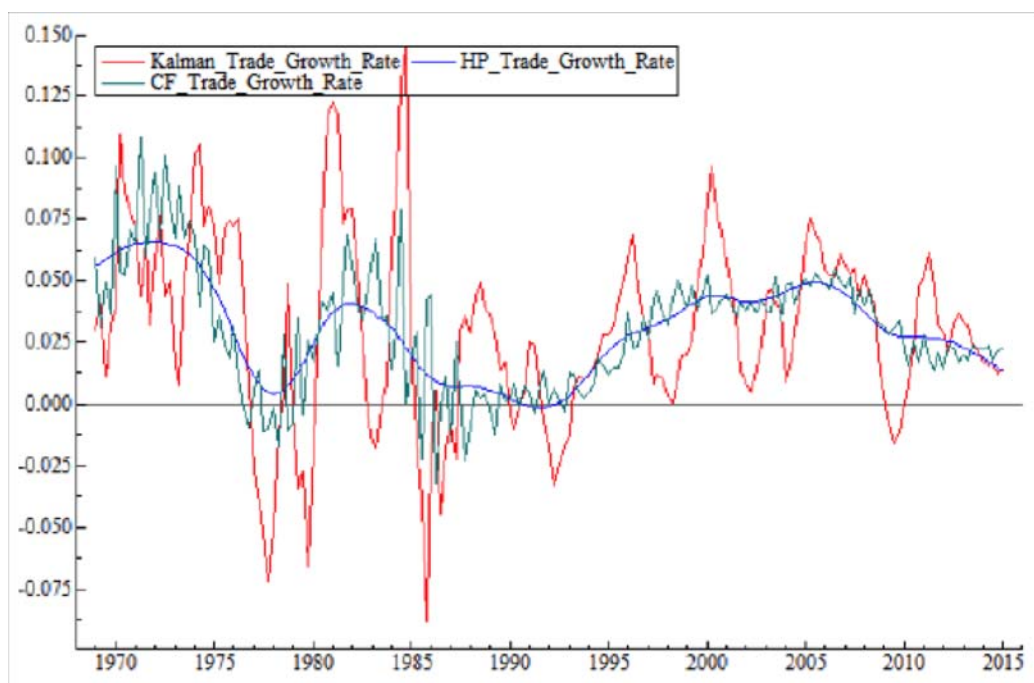


Figure 22: Wholesale and retail trade, and accommodation: Potential Rate of Growth

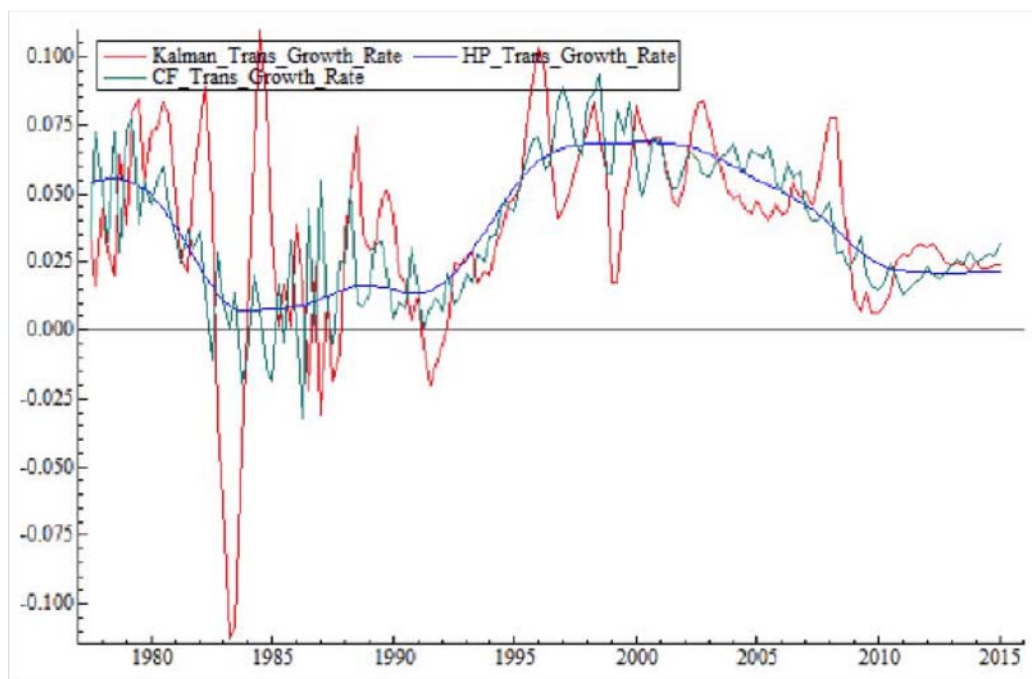


Figure 23: Transport, storage and communication: Potential Rate of Growth

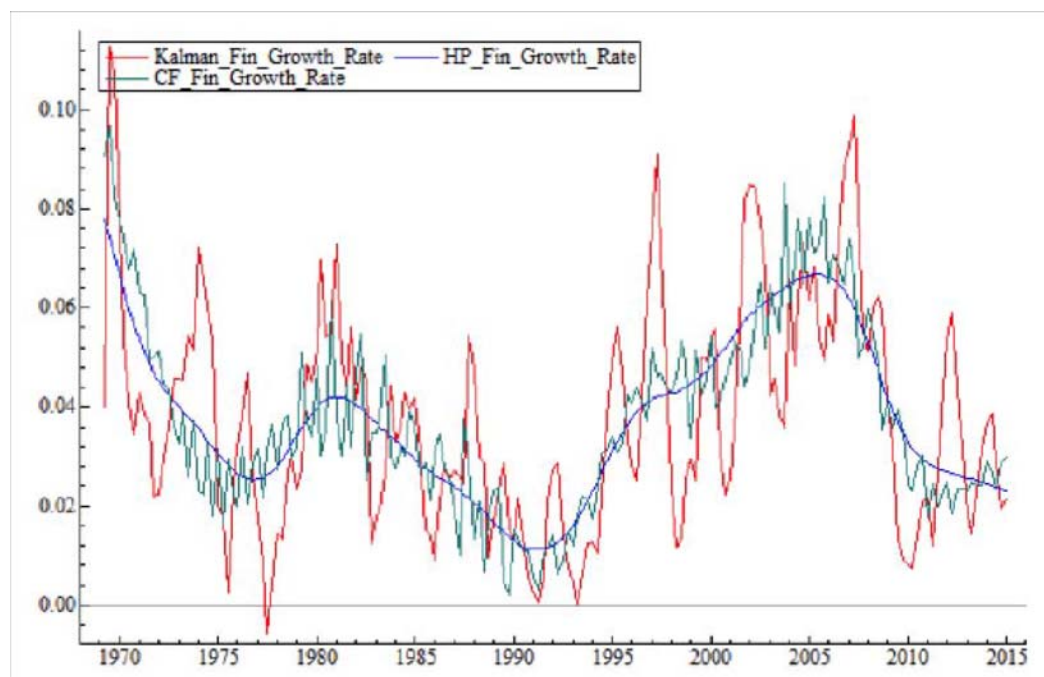


Figure 24: Finance, insurance, real estate and business services: Potential Rate of Growth

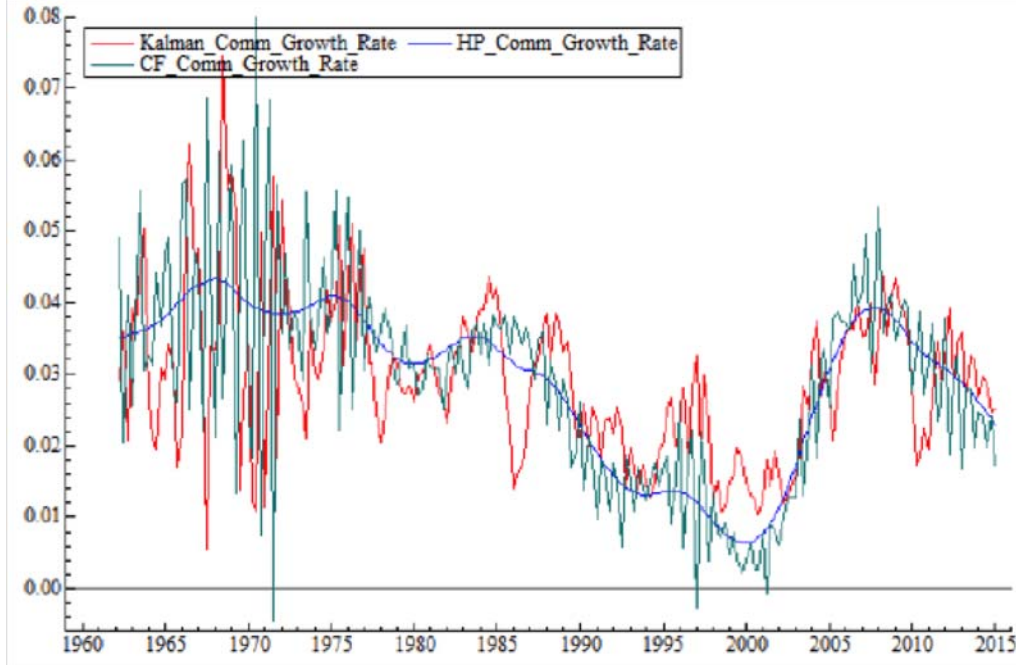


Figure 25: Community services: Potential Rate of Growth

ment. A core distinction is between high-pass and band-pass filters, with the latter showing lower amplitude in the implied business cycle for South Africa. Specifically the Christiano-Fitzgerald filter provides results that do not generate extreme amplitudes in the output gap. Since it also does not suffer from the end-point truncation of the Baxter-King band-pass filter, we have a moderate preference for the Christiano-Fitzgerald filter.

We also considered an approach that uses all of the time series filters, in order to generate an estimate of the upper and the lower bound for the output gap. These are supplied by the maximum and the minimum values generated across the four time series filters for any time period. While there are periods in which the agreement between the filters is strong, there are also time periods in which there is strong divergence between the filters, in the sense that the filters straddle positive and negative values: some filters indicating that the output gap is positive, some that it is negative. This is particularly the case in periods where policy makers need clear information regarding the direction of the economy - such as the post 2010 period of relatively weak economic growth.

In terms of the potential rate of growth of the economy, we employed three alternative approaches in estimation. In the first, we inferred the structural growth rate from estimates of potential output as estimated by time series filters. In the second, we apply filters directly to actual growth rates in output. Third, we employ filters in determining sectorally specific output growth rates. The inference under the derivations from the potential output measurements, is that the potential rate of growth lies in the range of 1.9 - 2.1% per annum over the 2010-15 period. Estimating the potential rate of growth directly from actual growth of output, returns an average estimate of 2.3% per annum for 2010-15. In terms of sectoral growth rates, the strongest deceleration in growth has been in the real sectors of the economy (Mining, Manufacturing particularly), and the most resilient growth has been returned by the service sectors (Particularly financial sectors). Finally, on at least some filter estimates, the growth deceleration of the economy as a whole, post-2010, now points to a structural growth rate that lies closer to the 1% level, than the 2% level. Regardless of which estimate is correct (i.e. even if the more optimistic 2.3% is correct), nonetheless the implication is of considerable welfare constraints on development prospects in South Africa, whose population continues to grow at 1.33% per year. Even under the best long run growth performance of the economy (which has not been realized since the early 1970s), the implication is that per capita welfare will continue to grow only very moderately. This is all the more concerning under a scenario in which the relatively strong growth spurt over 1995-2007 proves to be have been a temporary windfall. Under these circumstances the likely potential rate of growth of the economy will lie closer to the population growth rate, offering virtually no prospect of welfare improvement over time.

Which of the two interpretations is correct will emerge with additional evidence. But there are two reasons that suggest that the less optimistic scenario may be more probable. First, even the estimate of 2% potential growth in the economy is a product of averaging over the past 5-10 years. Point estimates for the last time points, particularly from the time series filters, are considerably more pessimistic (including predictions of below 1%). More

fundamentally, an exploration of the structural change in the South African economy and the associated unbalanced growth path (Fedderke, 2014), confirms a redistribution of labor from high to low productivity growth sectors, due to a price elasticity of demand below unity. Employment growth in South Africa is thus concentrated in low efficiency sectors. The implication is that the single most underutilized resource in the South African economy, labor, is unlikely to experience rapid growth in employment rates, barring fundamental structural reform of the labor market. In effect, an economy which allows 26-40% of the labor factor of production to remain unemployed, is not likely to realize sustained rapid growth in output.

REFERENCES

- Anvari, V., Ehlers, N., and Steinbach, R., 2014, A semi-structural approach to estimate South Africa's potential output, South African Reserve Bank Working Paper Series WP/14/08.
- Anderson, Brian D.O., and John B. Moore., 2012, *Optimal filtering*. Courier Corporation.
- Arora, Vivek B., and Ashok Bhundia., 2003, "Potential output and total factor productivity growth in post-apartheid South Africa." No. 3-178. International Monetary Fund.
- Bank Negara Malaysia., 2012, "Potential Output and the Output Gap in Malaysia." *Quarterly Economic Bulletin*, no. 2nd Quarter.
- Barnett, Russell and Matier, Chris., 2010, Office of the Parliamentary Budget Officer, *Estimating Potential GDP and the Government's Structural Budget Balance*.
- Barro, Robert and Jong-Wha Lee, 2010, "A New Data Set of Educational Attainment in the World, 1950-2010." *Journal of Development Economics*, 104, 184-198.
- Baxter, M., and R. G. King., 1995, Measuring business cycles approximate band-pass filters for economic time series., NBER Working Paper No. 5022, National Bureau of Economic Research.

- Baxter, Marianne, and Robert G. King., 1999, "Measuring business cycles: approximate band-pass filters for economic time series." *Review of Economics and Statistics*, 81(4), 575-593.
- Beneš, J., K. Clinton, R. Garcia-Saltos, M. P. Johnson, D. Laxton, P. B. Manchev, and T. Matheson, 2010, *Estimating Potential Output with a Multivariate Filter*, *International Monetary Fund*.
- Berndt, Ernst R., and Laurits R. Christensen., 1973, "The translog function and the substitution of equipment, structures, and labor in US manufacturing 1929-68." *Journal of Econometrics*, 1(1), 81-113.
- Borio, C., P. Disyatat, and M. Juselius, 2013, Rethinking potential output: Embedding information about the financial cycle, *BIS Working Paper*.
- Borio, C., P. Disyatat, and M. Juselius, 2014, A parsimonious approach to incorporating economic information in measures of potential output, *BIS Working Paper*.
- Burns, Arthur F., and Wesley C. Mitchell., 1946, *Measuring business cycles*. NBER Books.
- Cheremukhin, Anton A., 2013, "Estimating the output gap in real time." Federal Reserve Bank of Dallas Staff Papers, no 22, December.
- Christiano, Lawrence J., and Terry J. Fitzgerald., 2003, "The band pass filter." *International Economic Review*, 44(2), 435-465.
- De Jong, P., 1988, "The likelihood for a state space model." *Biometrika*, 75, 165–169.
- Ehlers, N., Mboji, L., and Smal, D., 2013, The pace of potential growth in the South African economy, South African Reserve Bank Working Paper WP/13/01.
- Fedderke, J.W., 2014, Exploring Unbalanced Growth in South Africa: Understanding the Sectoral Structure of the South African Economy, South African Reserve Bank Working Paper WP/14/07.

- Giorno, Claude, Pete Richardson, Deborah Roseveare, and Paul Van den Noord., 1995, "Estimating Potential Output, Output Gaps and Structural Budget Balances", OECD Economics Department Working Papers, No. 152, OECD Publishing. <http://dx.doi.org/10.1787/533876774515>
- Gómez, Víctor., 1999, "Three equivalent methods for filtering finite nonstationary time series." *Journal of Business & Economic Statistics*, 17(1), 109-116.
- Gomez, Victor., 2001, "The use of Butterworth filters for trend and cycle estimation in economic time series." *Journal of Business & Economic Statistics*, 19(3), 365-373.
- Hamilton, James Douglas., 1994a, *Time Series Analysis*. Princeton: Princeton University Press.
- Hamilton, James D., 1994b, "State-space models.", *Handbook of Econometrics* 4, 3039-3080.
- Hodrick, Robert J., and Edward C. Prescott., 1997, "Postwar US business cycles: an empirical investigation." *Journal of Money, Credit, and Banking* , 29(1), 1-16.
- Jain-Chandra, Sonali, and Longmei Zhang., 2014, "How can Korea Boost Potential Output to Ensure Continued Income Convergence?", No. 14/54. International Monetary Fund, 2014.
- Kemp, J.H., 2015, Measuring Potential Output for the South African Economy: Embedding Information About the Financial Cycle, *South African Journal of Economics*, 83(4), 549-68.
- King, Robert G., and Sergio T. Rebelo., 1993, "Low frequency filtering and real business cycles." *Journal of Economic Dynamics and Control*, 17(1), 207-231.
- Lienert, Ashley, and David Gillmore., 2015, The Reserve Bank's method of estimating "potential output". No. AN2015/01. Reserve Bank of New Zealand.
- Nelson Filho, H. Barbosa., 2004, "Estimating potential output: a survey of the alternative methods and their application to Brazil." Economic Commission For Latin America and the Caribbean, No. 0503003. EconWPA, 2005.

Otsu, Takeshi., 2007, "Time-Invariant Linear Filters and Real GDP: A Case of Japan." Mimeo: Seijo University, Faculty of Economics.

Pichette, Lise, Pierre St-Amant, Ben Tomlin, and Karine Anoma., 2015, Measuring Potential Output at the Bank of Canada: The Extended Multivariate Filter and the Integrated Framework. No. 15-1. (2015).

Pollock, D. S. G., 2000, "Trend estimation and de-trending via rational square-wave filters." *Journal of Econometrics*, 99(2), 317-334.

Scheibe, Jorg., 2003, "The Chinese Output Gap During the Reform Period 1978-2002." Mimeo: No. 179. University of Oxford, Department of Economics.

Smit, B.W., and Burrows, L R., 2002, Estimating Potential Output and Output Gaps for the South African Economy, Stellenbosch Economic Working Papers, 5/2002.

United States Congressional Budget Office, and Robert Arnold., 2001, CBO's Method for Estimating Potential Output: An Update. Congressional Budget Office.

Vrbanc, Igeta., 2006, "Estimate of potential gross domestic product using the production function method." In the *Dubrovnik Economic Conference*, organized by Croatian National Bank on June. 2006.

Zivot, Eric., 2006, "State-space Models and the Kalman Filter." Mimeo: University of Washington.

VIII. Appendix

Output	Years Available	SARB Code
Gross value added at basic prices of all industries	1960;Q1 - 2015;Q2	KBP6645D
Gross domestic product at market prices	1960;Q1 - 2015;Q1	KBP6006D
Gross domestic expenditure	1960;Q1 - 2015;Q1	KBP6012D
Gross national income	1960;Q1 - 2015;Q1	KBP6016D

Data Sources: Output Series

Sector	Yearly/Quarterly	Years Available	SARB Code
Gross value added at basic prices of manufacturing	Quarterly	1960;Q1 - 2015;Q2	KBP6634D
Gross value added at basic prices of mining and quarrying	Quarterly	1960;Q1 - 2015;Q2	KBP6632D
Gross value added at basic prices of electricity, gas and water	Quarterly	1960;Q1 - 2015;Q2	KBP6635D
Gross value added at basic prices of construction (contractors)	Quarterly	1960;Q1 - 2015;Q2	KBP6636D
Gross value added at basic prices of wholesale and retail trade, catering and accommodation	Quarterly	1960;Q1 - 2015;Q2	KBP6638D
Gross value added at basic prices of transport, storage and communication	Quarterly	1960;Q1 - 2015;Q2	KBP6639D
Gross value added at basic prices of finance, insurance, real estate and business services	Quarterly	1960;Q1 - 2015;Q2	KBP6640D
Gross value added at basic prices of community, social and and personal services	Quarterly	1960;Q1 - 2015;Q2	KBP6642D
Employment in the Private Sector: Manufacturing	Quarterly	1960;Q1 - 2015;Q1	KBP7004L
Employment in the Private Sector: Mining	Quarterly	1960;Q1 - 2015;Q1	KBP7003L
Employment in the Private Sector: Electricity	Quarterly	1967;Q1 - 2015;Q1	*
Employment in the Private Sector: Construction	Quarterly	1960;Q1 - 2015;Q1	KBP7005L
Employment in the Private Sector: Trade	Quarterly	1967;Q1 - 2015;Q1	KBP7006L
Employment in the Private Sector: Transport	Quarterly	1975;Q1 - 2015;Q1	*
Employment in the Private Sector: Finance	Quarterly	1967;Q1 - 2015;Q1	KBP7007L
Employment in the Public Sector: Community	Quarterly	1960;Q1 - 2015;Q1	KBP7002L
Fixed Capital Stock: Manufacturing	Yearly	1946 - 2014	KBP6142Y
Fixed capital stock : Mining & quarrying	Yearly	1946 - 2014	KBP6141Y
Fixed capital stock : Electricity, gas & water	Yearly	1946 - 2014	KBP6143Y
Fixed capital stock : Construction (contractors)	Yearly	1946 - 2014	KBP6144Y
Fixed capital stock : Wholesale & retail trade, catering and accommodation	Yearly	1946 - 2014	KBP6145Y
Fixed capital stock : Transport, storage & communication	Yearly	1946 - 2014	KBP6146Y
Fixed capital stock : Financial intermediation, insurance, real estate and business services	Yearly	1946 - 2014	KBP6147Y
Fixed capital stock : Community, social & personal services	Yearly	1946 - 2014	KBP6148Y

Data Sources; Production Functions

Sector	Y/K/L	Levels	1st Difference
Manufacturing	Output	-4.114** (0.0156)	-7.472** (0.0185)
	Capital	-3.738** (0.0000)	-2.885* (0.0155)
	Labor	-4.244** (0.0008)	-6.650** (0.0004)
Mining	Output	-2.909* (0.0004)	-7.824** (0.0256)
	Capital	-0.4378 (0.0000)	-2.919* (0.0237)
	Labor	-0.5769 (0.0078)	-5.092** (0.0007)
Electricity	Output	-3.901** (0.0008)	-7.761** (0.0000)
	Capital	-0.6054 (0.0189)	-0.605 (0.0068)
	Labor	-2.632 (0.0054)	-6.426** (0.0005)
Construction	Output	-2.152 (0.0019)	-7.924** (0.0013)
	Capital	-2.854 (0.0000)	-3.070* (0.0023)
	Labor	-3.140* (0.0094)	-7.790** (0.0170)
Trade	Output	-2.286 (0.0006)	-7.190** (0.0001)
	Capital	-2.757 (0.0000)	-2.022 (0.0060)
	Labor	0.2645 (0.5111)	-5.282** (0.4877)
Transport	Output	-1.465 (0.0301)	-7.501** (0.0016)
	Capital	-1.824 (0.0000)	-1.781 (0.0387)
	Labor	-1.661 (0.2700)	-9.514** (0.2582)
Finance	Output	-1.073 (0.0128)	-5.113** (0.0012)
	Capital	-2.041 (0.0000)	-3.815** (0.0476)
	Labor	-0.2591 (0.6031)	-3.925** (0.0311)
Community	Output	-2.264 (0.0045)	-15.10** (0.0074)
	Capital	-2.329 (0.0000)	-1.384 (0.0028)
	Labor	-2.376 (0.0067)	-5.530** (0.0031)

Augmented Dickey-Fuller Test Statistics