The modelling of inflation expectations

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1. Introduction

As described in some detail in an article in the *Quarterly Bulletin* for March 1994[1], the price-formation process in South Africa is particularly dependent on changes in labour costs, which in turn are largely driven by inflation expectations, i.e. the rate of inflation that people expect to prevail in the future. Unfortunately, there is no unanimity about the uniqueness of inflation expectations, i.e. on how such expectations are formed and how they should be measured. In econometric analyses a subjective assumption must therefore be made about the formation of expectations which can then be used as an auxiliary hypothesis in the estimation of behavioural equations.

Econometric analyses would, of course, be improved considerably if inflation expectations were directly observable and could be measured. In some countries surveys of price expectations are conducted on a regular basis and these can be used directly in structural relationships as a proxy for the true underlying expectations. Although some opinion surveys on inflation expectations are made in South Africa, no consistent time series on price expectations are available for use in the estimation of equations.

The aim of this paper is, therefore, to investigate some formulations of inflation expectations and to test their validity based on South African data. These hypotheses on the formation of inflation expectations are first described in Section 2 of this document and then tested empirically in Section 3. The forecast results as well as a test of the accuracy of the different inflation expectations hypotheses are tabulated in Section 4. Some concluding remarks are finally made in Section 5.

2. Models of inflationary expectations

The inflation hypotheses investigated in this study, are;

- expectations based on recent actual inflation rates;
- expectations based on distributed lags of actual inflation rates;
- adaptive expectations;
- extrapolative expectations;
- rational expectations;
- expectations based on time series analysis.

[1] Pretorius, C.J. and Smal, M.M.: "A macro-economic examination of the price-formation process in the South African economy", *Quarterly Bulletin* of the South African Reserve Bank, March 1994.

Actual inflation rate of previous period

One of the most elementary methods of describing inflation expectations is to assume that the rate of increase in the general price level in period "t" is expected to be equal to the actual inflation rate of the previous period, "t-1". This method can be represented by the following equation:

$\dot{p}_t^e = \dot{p}_{t-1}$, where

$\dot{p}_t^e$ = inflation expectations in period "t", and

$\dot{p}_{t-1}$ = the inflation rate in the immediately preceding period.

One of the disadvantages of this approach is that it puts too much emphasis on the inflation rate of a single period. For example, the negative effects of non-recurrent events (such as the introduction of the value-added tax system) on the inflation rate in a specific period are assumed to "pass through" fully into the expected inflation rate for the immediately succeeding period, even though the events concerned are unlikely to be repeated in the succeeding period.

Distributed lags of actual rates of inflation

In view of the disadvantages of basing inflation expectations on the result of a single period, a different approach has been adopted in which the expected rate of inflation is assumed to be a weighted average of past inflation rates. It is usually assumed that the weights decline over time, implying that inflation expectations adapt systematically over time. This method of calculating inflation expectations can be represented in the following manner:

$$\dot{p}_t^e = \sum_{i=1}^{n} w_i \dot{p}_{t-i}$$

The sum of the coefficients (w_i) is often assumed to be equal to one. This means that continued inflation at a uniform rate will eventually be fully anticipated.

Adaptive expectations

One of the most popular and widely used hypotheses concerning the formation of inflation expectations is the adaptive expectations formula used by Cagan[2] in his studies of hyperinflation. This approach entails expressing inflation expectations as a geometrically declining weighted average of past inflation. The

[2] Cagan, P and Lipsey, R.: *The Financial Effects of Inflation*, Cambridge, Massachusetts, National Bureau of Economic Research, 1978.

Koyck transformation technique can then be applied to the equation to convert it to the following equation:

$$\dot{p}_t^e = a_0 + a_1\dot{p}_{t-1}^e + a_2\dot{p}_t$$

If it is assumed that $a_1 + a_2 = 1$ and $a_0 = 0$, the adaptive expectations can also be indicated by:

$$\dot{p}_t^e - \dot{p}_{t-1}^e = a_2 (\dot{p}_t - \dot{p}_{t-1}^e) , 0<a_2<1$$

This equation implies that expectations are revised upwards or downwards according to the most recent error in the prediction of the inflation rate. However, in the case of a rising trend in inflation, this formulation of inflationary expectations should lead to a persistent underestimation of the actual inflation rate. Conversely, when inflation is falling, expected inflation should always exceed actual inflation.

Extrapolative expectations

The extrapolative expectations as described by Hirsch and Lovell[3] can be written as:

$$\dot{p}_t^e = a_0 + a_1(\dot{p}_{t-1}^e - \dot{p}_{t-2}^e) + a_2\dot{p}_{t-1}$$

Inflation expectations are described in this equation as a function of the previous actual inflation rate as well as the difference between the inflation rate of the previous two periods. One characteristic of the extrapolative hypothesis is the regressivity of expectations. If $a_1>0$, the past trend is extrapolated and expected to continue. On the other hand, if $a_1<0$, the past trend is expected to be reversed.

Rational expectations

The rational-expectations model of inflation suggests that the expected inflation rate in any single period is based on all information available to decision-makers at that moment. This information includes the past performance of the economy and expectations of future policies that are likely to be followed by the authorities. Changes in selected key variables, such as the money supply, the exchange rate of the currency and the government's budget deficit, can be regarded as indicators of economic policies which may influence inflation expectations.

Based on the assumption that changes in the money supply are an indicator of monetary policy and of concurrent or future inflation, economic agents would set their rational expectations of inflation in period "t" according to the rate of expected monetary expansion for period "t". This leads to the following expression for price expectations:

[3] Hirsch, A.A. and Lovell, M.C.: *Sales Anticipations and Inventory Behaviour*, New York, John Wiley and Sons.

$$\dot{p}_t^e = \dot{m}_t^e$$

The variable $\dot{m}_t^e$ represents the expected rate of monetary expansion. The expected rate of monetary expansion is determined by the perception of economic agents of what factors the monetary authorities respond to when they determine $\dot{m}_t$, and the likely results of the authorities' actions. The central bank may, for instance, have a rule or target in setting the rate of monetary expansion. Economic agents will study the behaviour of monetary authorities and use this information to predict the systematic components of the central bank's policies. The actual rate of monetary expansion that is realised will depend on the average rate of growth in money supply and a random factor that represents random fluctuations in the rate of monetary growth, which are not controllable by the central bank. This can be illustrated by the following equation:

$$\dot{m}_t = \dot{m} + e_t$$

where $\dot{m}$ represents the average rate of monetary expansion and e_t represents the random component in the rate of growth in the money supply.

The idea that economic agents can identify the systematic behaviour of the central bank and predict the results of such behaviour, can be extended to their prediction of any other systematic policy and the results of any such policy. Price expectations can also be influenced by perceived changes in fiscal policy and observed and expected changes in the exchange rate.

Time-series models

The distinguishing feature of a time-series model, as opposed to an econometric model, is that normally no attempt is made to relate a specific time series to other variables; the movements of the time series are explained solely in terms of own past values. A stochastic process can be used to describe the movements of a time series fluctuating around its mean as:

$$y_t - \mu = \phi (y_{t-1} - \mu) + \varepsilon_t$$

where y_t represents the observation of the time series in period "t", μ the mean of the series and ε_t is a random disturbance term.

A model that contains lagged values of both the observed variable and the disturbance term is known as an autoregressive-moving-average (ARMA) process. If $|\phi|<1$, the observations fluctuate around the mean of the series and the process is stationary. Although very few time series are stationary, the notion of stationarity is fundamental to time-series analysis, and stationary ARMA processes form the basis on which a more general series can be built. The majority of time series do not fluctuate around a constant level, but show some kind of systematic upward or downward movement instead. In order to make a time series stationary, transformations such as logarithms and differences of the observations are calculated.

If a classical linear regression model (with time as the independent variable) is fitted to the observations of a time series, constant parameters will be estimated and a global trend line can be determined. This is assumed to hold good at all points in time. The ARMA process, in contrast, is a forecasting procedure which allows the parameters to adapt to changes in the data. More weight is placed on the most recent observations, basing forecasts on the estimate of the recent trend rather than the trend over the entire estimation period.

Box and Jenkins made use of the proposition that the optimal forecast for a time series in general depends on the particular form of the stochastic process which characterises the series that is being forecast. If the series is described by an ARIMA (an autoregressive-integrated-moving-average) process, the current period's forecast is the result of a weighted average of all previous observations.

3. Statistical estimation of inflation-expectations equations

The empirical results of the estimated equations are described in this section. All the econometric calculations were carried out with quarterly, seasonally adjusted data. T-values of the estimated coefficients as well as the following summary statistics are tabulated:

$\bar{R}^2$ = Adjusted coefficient of determination;

D-W = Durbin-Watson d-statistic;

RHO = Autocorrelation coefficient.

The period of estimation is stated below the summary statistics for each equation.

The percentage change over four quarters in the derived deflator for private consumption expenditure is used as the indicator of inflation and of expected inflation.

Actual inflation rate of previous period

If inflation expectations can only be explained by the previous period's actual inflation rate, the function can be presented in the following manner:

$$\dot{p}^e = 1{,}0\cdot\dot{p}_{-1}$$

This kind of expectations formulation is obviously not suitable in an environment where actual inflation is highly volatile. In South Africa, with its relatively stable inflation rate in the 1980s, this rate could have been used by many decision-makers as an acceptable technique for forecasting inflation in that period.

Distributed lags of actual rates of inflation

An Almon lag structure was used to describe the pattern of the actual rates of inflation in the period specified. It was assumed that the weights of the lagged variable followed a pattern dictated by a second-degree polynomial with an end restriction, i.e. the weights reach zero after a number of time periods.

The function was estimated as follows:

$$\dot{p}^e = \underset{(2{,}06)}{0{,}54} + \underset{(40{,}57)}{0{,}48}\cdot\dot{p}_{-1} + \underset{(40{,}57)}{0{,}32}\cdot\dot{p}_{-2} + \underset{(40{,}57)}{0{,}16}\cdot\dot{p}_{-3}$$

$\bar{R}^2$ = 0,93

D-W = 1,30

Estimation period: 1962.I to 1993.IV

Adaptive expectations

It is not possible to test the adaptive-expectations approach as described in Section 2 empirically, because of the lack of observations on inflation expectations. The adaptive expectations cannot be equated to the observed inflation rate, because the observed inflation rate will then appear on both sides of the equation to be estimated.

Under the assumption that expected inflation is proportionally equal to actual inflation, the equation for adaptive expectations, namely:

$$\dot{p}_t^e = a_0 + a_1\dot{p}^e_{t-1} + a_2\dot{p}_t$$

can be reduced to be a function of only the previous period's inflation rate. The function was calculated as follows:

$$\dot{p}^e = \underset{(1{,}88)}{0{,}45} + \underset{(44{,}83)}{0{,}96}\cdot\dot{p}_{-1}$$

$\bar{R}^2$ = 0,94

D-W = 2,22

Estimation period: 1961.II to 1993.IV

The hypothesis of extrapolative expectations

According to the extrapolative hypothesis, the expected inflation rate in a period is determined by the actual inflation rate in the previous period, adjusted for the actual change in the inflation rate. The function was estimated as follows:

$$\dot{p}^e = \underset{(1,98)}{0,47} - \underset{(1,99)}{0,18} \cdot (\dot{p}_{-1} - \dot{p}_{-2}) + \underset{(45,15)}{0,96} \cdot \dot{p}_{-1}$$

$\bar{R}^2$ = 0,94
D-W = 1,99

Estimation period: 1961.III to 1993.IV

$$\dot{p}^e = \underset{(3,04)}{0,04} \cdot \dot{M}y_{-3} + \underset{(2,15)}{0,03} \cdot \dot{p}f_{-2} + \underset{(37,23)}{0,91} \cdot \dot{p}_{-1}$$

$\bar{R}^2$ = 0,94
D-W = 2,26

Estimation period: 1962.III to 1993.IV

Rational expectations

On the assumption that changes in the money supply are representative of monetary policy, that changes in the prices of imported goods are indicative of exchange rate changes, and that government's budget deficit is representative of fiscal policy, economic agents are assumed to base their rational expectations of inflation in period "t" on :

- the growth rate over four quarters in the M3 money supply per unit of production ($\dot{M}y$); [4]
- the percentage change over four quarters in the price of imported goods ($\dot{p}f$);
- the inflation rate in the preceding period ($\dot{p}$); and
- a four-quarter moving average of the government's budget deficit relative to nominal gross domestic product (gd).

$$\dot{p}^e = \underset{(2,34)}{0,03} \cdot \dot{M}y_{-4} + \underset{(2,05)}{0,03} \cdot \dot{p}f_{-2} + \underset{(1,70)}{0,08} \cdot gd_{-3} + \underset{(30,18)}{0,89} \cdot \dot{p}_{-1}$$

$\bar{R}^2$ = 0,94
D-W = 2,27

Estimation period: 1962.III to 1993.II

As a result of the substantial increase in the government's budget deficit in the past few years, forecasts with the above equation resulted in a considerable overestimation of inflation expectations. This can be ascribed to the fact that the equation was estimated over an extended period in which the budget deficit was relatively small, resulting in a relatively large coefficient for the budget deficit. When the above equation was re-estimated over a shorter and more recent period, the coefficients changed considerably, but they were no longer statistically significant.

A different rational expectations model was therefore specified that excluded the government's budget deficit and which improved the forecast ability considerably. The appropriate equation was estimated as:

ARIMA-models

An inflation-expectations ARIMA-model was estimated, using the Box-Jenkins methodology in which expectations were assumed to be a weighted function of their own past values.

The inflation expectations process was identified as an ARIMA(0,1,1) model, which implies:

- that no autoregressive structure was applicable;
- a differencing factor of one; and
- a moving average factor ε, with a four-period lag.

The calculated coefficients of the ARIMA-model can be summarised as:

$$\dot{p}^e - \dot{p}^e_{-1} = \underset{(3,12)}{0,11} + (\varepsilon - \underset{(8,91)}{0,6314}_{\varepsilon-4})$$

$\bar{R}^2$ = 0,95

Estimation period: 1961.II to 1993.II

4. Summary of results

Summary of regression results

In Table 1 the regression results of the various expectations hypotheses are compared. From this table it is obvious that the adjusted coefficients of determination ($\bar{R}^2$) of the different hypotheses are all of the same high order of magnitude. The $\bar{R}^2$ would thus not be a sufficient criterium to differentiate between the various hypotheses.

Table 1
Summary of regression results

Expectations hypothesis	$\bar{R}^2$	D-W
Distributed lags of actual inflation rates ...	0,93	1,30
Adaptice expectations..........................	0,94	2,22
Extrapolative expectations	0,94	1,99
Rational expectations...........................	0,94	2,27
Rational expectations (without budget deficit)..	0,94	2,26
ARIMA-model	0,95	–

4 Experiments were made with different definitions of the money supply in the specification of the rational expectations equation. In the end the M3 money-supply definition clearly proved to be the superior variable to use in these calculations.

Table 2
Forecast based on past inflation

	Actual value	Prediction	Percentage error
1992 q1	16,95	16,61	-2,0
1992 q2	16,37	16,61	1,5
1992 q3	15,66	16,61	6,1
1992 q4	15,71	16,61	5,7
1993 q1	12,71	16,61	30,7
1993 q2	11,14	16,61	49,1
1993 q3	10,48	16,61	58,5
1993 q4	8,21	16,61	102,3
Mean absolute percentage error			32,0

Table 4
Forecast based on adaptive-expectations model

	Actual value	Prediction	Percentage error
1992 q1	16,95	16,39	-3,3
1992 q2	16,37	16,18	-1,2
1992 q3	15,66	15,98	2,0
1992 q4	15,71	15,79	0,5
1993 q1	12,71	15,60	22,7
1993 q2	11,14	15,43	38,5
1993 q3	10,48	15,26	45,6
1993 q4	8,21	15,09	83,8
Mean absolute percentage error			24,7

Summary of forecast results

It was consequently decided to compare forecasts of the various hypotheses with the estimated equations over a sample period for which actual data exist. Dynamic simulations were executed over the period from the first quarter of 1992 to the fourth quarter of 1993.

The hypothesis according to which inflation predictions are based on actual inflation rates in the past, was able to forecast inflation for 1992 fairly accurately because the actual inflation rate throughout 1992 remained more or less at the same level as the actual inflation rate in the fourth quarter of 1991. When inflation started to subside in 1993, the percentage error increased noticeably. This proves that the time horizon of the forecast, especially in the case of a dynamic forecast, plays a prominent role in the accuracy of the forecast. If the forecast is performed for only one period ahead, the actual inflation rate of the previous period instead of the previous forecast value can be used, which enables the expectations to adapt more easily to actual changes in inflation.

Table 3
Forecast based on distributed-lag model

	Actual value	Prediction	Percentage error
1992 q1	16,95	15,90	-6,2
1992 q2	16,37	16,01	-2,2
1992 q3	15,66	15,97	2,0
1992 q4	15,71	15,87	1,0
1993 q1	12,71	15,83	24,5
1993 q2	11,14	15,77	41,6
1993 q3	10,48	15,72	50,0
1993 q4	8,21	15,66	90,7
Mean absolute percentage error			27,3

The distributed-lag hypothesis was also unable to predict the turning-point in the inflation rate during the 1990s (see Table 3). This shortcoming in the forecasting ability of this hypothesis can be attributed to the fact that the equation is based on the weighted average of fairly stable past inflation rates.

The simplified adaptive-expectations hypothesis was able to forecast the inflation rate fairly accurately in 1992. However, this method was also not able to predict the deceleration in the inflation rate during 1993. The accuracy of the forecast under the adaptive-expectations hypothesis was, however, slightly better than that obtained by means of the two methods already described above.

As indicated in Table 5, the extrapolative hypothesis performed marginally weaker than the adaptive-expectations and the distributed-lag hypotheses. It also predicts the inflation rate based on previous values of inflation only and was therefore unable to forecast the turning-point in the inflation rate during 1993.

Table 5
Forecast based on extrapolative-expectations model

	Actual value	Prediction	Percentage error
1992 q1	16,95	16,32	-3,7
1992 q2	16,37	16,25	-0,7
1992 q3	15,66	16,15	3,1
1992 q4	15,71	16,05	2,2
1993 q1	12,71	15,96	25,6
1993 q2	11,14	15,86	42,4
1993 q3	10,48	15,78	50,6
1993 q4	8,21	15,69	91,1
Mean absolute percentage error			27,4

Table 6
Forecast based on rational-expectations model, incorporating fiscal policy

	Actual value	Prediction	Percentage error
1992 q1	16,95	16,1	-5,0
1992 q2	16,37	15,6	-4,7
1992 q3	15,66	15,1	-3,6
1992 q4	15,71	14,7	-6,4
1993 q1	12,71	14,3	12,5
1993 q2	11,14	13,8	23,9
1993 q3	10,48	13,5	28,8
1993 q4	8,21	13,3	62,0
Mean absolute percentage error			18,4

In Tables 6 and 7 forecasts are furnished of the inflation rate based on two versions of the rational-expectations model. Both versions of the rational expectations model clearly outperformed the other hypotheses which are based only on historical inflation rates, in terms of the forecast accuracy of inflation-expectations. In Table 6 the rational-expectations model used includes a variable representing fiscal policy as indicated by the government deficit before borrowing. The results still show a considerable overestimation of the inflation rate for 1993, which clearly accentuates the fact that the actual lower inflation rate was brought about despite a large increase in the government deficit before borrowing and debt repayment. A re-estimation of this equation over a shorter sample period (including the most recent observations) reduced the forecast error because the coefficient of the budget deficit was markedly smaller than the one calculated over a longer time period.

Table 7
Forecast based on rational-expectations model, disregarding fiscal policy

	Actual value	Prediction	Percentage error
1992 q1	16,95	16,0	-5,6
1992 q2	16,37	15,3	-6,5
1992 q3	15,66	14,7	-6,1
1992 q4	15,71	14,1	-10,2
1993 q1	12,71	13,4	5,4
1993 q2	11,14	12,7	14,0
1993 q3	10,48	12,2	16,4
1993 q4	8,21	11,5	40,1
Mean absolute percentage error			13,0

The best forecasting results were, however, obtained with the rational-expectations model in which the effects of fiscal policy on inflation expectations were disregarded (see Table 7). The fact that other policy variables were included in this model in addition to the historical inflation rates, enabled it to predict the decline in the actual inflation rate during 1993. This improved forecasting accuracy seems to confirm that fiscal policy was not fully taken into consideration by economic agents when they formed expectations of future changes in the aggregate price level. However, with a consistently growing public sector deficit, the effect may be different.

The ARIMA-model specified inflation expectations as a moving average of its own past values (see Table 8). As was the case with the other hypotheses which also explain inflation expectations based on inflation rates recorded in the past, it clearly overestimated inflation in 1993 and was unable to predict the decline in the inflation rate.

From these results it can be seen that most of the hypotheses failed to predict the deceleration in the inflation rate during 1993. This can, among other things, be attributed to the fact that the forecasts were carried out in a dynamic way. The lagged value of the forecasts, and not the actual data of the previous period, were used when performing such dynamic simulations. This emphasises to some extent the disadvantage of explaining a variable only by its values in previous periods, because no correction is made once the particular variable is overestimated or underestimated. From the results obtained, it was also apparent that the rational-expectations model (excluding the government's budget deficit) clearly outperformed the other hypotheses regarding the forecast accuracy of inflation expectations.

Table 8
Forecast based on ARIMA-model

	Actual value	Prediction	Percentage error
1992 q1	16,95	16,5	-2,7
1992 q2	16,37	16,8	2,6
1992 q3	15,66	16,7	6,6
1992 q4	15,71	16,6	5,7
1993 q1	12,71	16,8	32,2
1993 q2	11,14	16,9	51,7
1993 q3	10,48	17,0	62,2
1993 q4	8,21	17,1	108,3
Mean absolute percentage error			34,0

5. Conclusion

The inflation expectations of agents depend on inflation rates recorded in the past and the behaviour of policy-makers. The results reported in this paper indicate fairly conclusively that expectations formed in a rational way by duly recognising the intentions of policy-makers are likely to be closer to the mark than those based on historical inflation rate patterns only. The first-mentioned method is therefore applied in the Reserve Bank's econometric model to take inflation expectations into consideration.

When visible success is demonstrated in reducing actual inflation, the adjustment of expectations in response to policy changes is increased. This reconfirms the importance of applying monetary policy measures in a consistent manner in order to achieve greater price stability. If policies are conducted in a time-consistent manner, they will be perceived as credible and thus succeed in influencing inflation expectations in the desired direction. Conversely, policies which are pursued in an inconsistent way are unlikely to influence expectations downwards because they may be regarded with suspicion, and may often elicit private-sector responses contrary to those intended by public-sector decision-makers.

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