Gross fixed investment in the macro-econometric model of the Reserve Bank

by C.J. Pretorius

Introduction

A sustained increase in productivity or an expansion of production capacity represent two of the most important factors needed to achieve long-term economic growth. The expansion of a country's production capacity requires additional investment or capital formation. International studies have indicated that a high ratio of investment relative to gross domestic product is one of the most important preconditions for achieving sustained high economic growth.

South Africa's poor investment performance during the 1980s and first part of the 1990s can be depicted as one of the main reasons for the restricted expansion in the country's growth potential. Fixed investment relative to gross domestic product declined from 27,8 per cent in 1981 to 15,5 per cent in 1993, and has since increased to 17,0 per cent in 1996.

The forced correction of external imbalances in South Africa during the 1980s, as in many other developing countries, was accompanied by large cuts in investment expenditure and a decline in domestic savings. The lower levels of investment resulted in lower economic growth rates and declining job opportunities. Global integration and greater access to international markets as sources of capital, coupled with lower public-sector deficits since 1994 which made more domestic savings available to finance private-sector investment, resulted in an upward trend in private investment during the last few years and subsequent higher economic growth.

Although investment currently accounts for less than 20 per cent of overall gross domestic product, this spending category is very important in the determination of economic growth as fluctuations in the level of investment expenditure over the course of the business cycle tend to be much larger than fluctuations in any of the other macroeconomic expenditure aggregates.

Investment expenditure is normally divided into four categories, namely infrastructural investment in the public sector, residential construction, business fixed investment and the net change in business inventories. The reason for this distinction is that each of these categories is determined by substantially different factors and they respond differently to fluctuations in economic activity. This article focuses mainly on fixed investment in the business sector.

The purpose of the article is to describe the fixed investment equation of the private business sector as contained in the Reserve Bank's macroeconometric model. The first section gives an overview of Jorgenson's neoclassical fixed investment theory that underpins the estimated investment equations in the model. The second section presents an overview of medium-term trends in gross fixed investment as recorded in the national accounts of South Africa. Subsequent sections deal with explanatory variables appearing in the behavioural equation as well as the specification and econometric estimation of the investment equation. Some concluding comments are made in the final section.

The neoclassical fixed investment theory

The neoclassical theory of optimal capital accumulation, as formulated by Irving Fisher (Fisher, 1930.) in the 1920s, has been extensively modified by Jorgenson and others. The biggest attraction of Jorgenson's methodology is that it developed a model of investment spending, which incorporates interest rates, output volumes, capital goods prices and existing stock of capital goods into a coherent framework. The central theoretical feature of neoclassical theory is the concept of user cost of capital, which can be described as the price of the services provided by capital.

The Cobb-Douglas production function can be used to illustrate the production process of an individual firm. The process can be simplified by assuming that the firm produces only one product, using two inputs, namely capital and labour. If the volume of production is indicated by Q, labour by L and the capital stock by K, then the production function of the firm in period t is summarised by the following equation:

$$Q_t = F(L_t, K_t)$$

Neoclassical theory assumes that the objective of a firm is to maximise its profits over the economic life of the assets. Profit is defined as gross revenue less the cost of current inputs and taxes. Revenue is defined as the product of the price per unit of output and quantity produced. Labour costs are defined by the wage rate times the quantity of labour and the capital costs defined by the product of the unit cost of capital services and the quantity of capital services. The profit of an organisation can consequently be represented by:

$$R = pQ - wL - qK - T \qquad (1)$$

where R = net profit;

p = price of the product;
Q = quantity produced;
w = wage rate;
L = quantity of labour;
q = cost of capital services;
K = quantity of capital services; and
T = taxes.

The value of the firm in period t = 0 can be illustrated in the following way:

$$V = \int_0^\infty e^{-rt}(pQ - wL - qK - T)\,dt \qquad (2)$$

where r represents a discounting rate independent of t.

The present value of the firm is equal to the discounted value of the expected future net yields. The objective of the firm is to maximise equation 2 subject to the following constraints:

$$Q_t = F(L_t, K_t)$$

$$I_t = \delta K_t + \dot{K}, \text{ where } \dot{K} = \frac{dK}{dt} \qquad (3)$$

I_t represents gross fixed investment and δ the depreciation rate. Equation 3 assumes that depreciation is proportional to the capital stock.

The solution to the firm's optimisation problem and the derivation of a formula for the user cost of capital are summarised in Appendix A. Profit over the life of the asset will be maximised when the marginal product of labour is equal to the ratio of the price of labour services to the price of the product, also referred to as the real wage:

$$\frac{\partial f}{\partial L} = \frac{w}{p}$$

The optimal capital stock is obtained where the value of the marginal product of capital is equal to the ratio of the user cost of capital to the price of the product, also referred to as the real user cost of capital:

$$\frac{\partial F}{\partial K} = \frac{c}{p}$$

where c indicates the user cost of capital and is defined as:

$$c = q(\delta + r \frac{\dot{q}}{q}) \qquad (8)$$

The user cost of capital can also be seen as the implicit price of the costs of using capital in the production process. The firm should act as if it is renting capital to itself and make decisions that reflect the implicit charges of using the capital. In elementary models the implicit costs have three elements:

- the costs associated with the acquisition of the asset;
- the financing costs related to the loan capital; and
- the depreciation of the capital over the period in use, less any capital gains of the firm during that period.

The impact of tax policy on investment

Robert Coen (1968: 200-211) conducted a study to determine the impact of tax concessions in the manufacturing sector. Tax incentives can stimulate capital expenditure in two ways:

- Tax incentives increase the after-tax rate of return on capital by reducing the amount of taxes that has to be paid on income from assets.
- Tax incentives reduce tax liabilities and therefore increase the cash flow and funds available for investment expenditure.

The neoclassical model provides a useful framework to evaluate tax policy and its impact on investment spending. Coen extended the concept of the user cost of capital to accommodate tax liabilities of firms and the effect of tax concessions on investment.

The effect of tax rates on investment behaviour is analysed in Appendix B. It is shown that the firm will continue to expand its capital stock as long as the marginal product of capital exceeds the real user cost of capital.

$$\therefore\ p\frac{\partial Q}{\partial K} > q\ (r + \delta)\ (1 - uB)\ /\ (1 - u) = c \qquad (11)$$

That is $\frac{\partial Q}{\partial K} > [q\ (r + \delta)\ (1 - uB)\ /\ (1 - u)]\ /\ p$

or $\frac{\partial Q}{\partial K} > \frac{c}{p}$

An increase in the rate of taxation, either through a rise in the tax rate or a decrease of incentives, would raise the user cost of capital and discourage fixed investment. Conversely, a decrease in the rate of taxation would lower the user cost of capital and encourage fixed investment.

If u = 0, that is if no direct corporate tax is paid, equation 11 reduces to an equation similar to equation 8, although the capital gain term was not considered in this equation.

Tax credits

This analysis can easily be extended to include tax credits. In order to promote investment, a proportion k of capital outlays can be credited against tax liabilities.

The tax discount is then kq and the unit price of capital goods after provision for tax credits is (1-k)q. The user cost of capital then reduces to:

$$c = q\ (r + \delta)(1 - k - uB) / (1 - u) \tag{12}$$

If the tax credit concession is accompanied by a depreciation allowance on new capital outlays with a factor of say b, the discounted value of the tax savings generated by the depreciation allowance B, is replaced by (1 - b)B. The expression for the user cost of capital then changes to:

$$c = q\ (r + \delta)\ (1 - k - u(1 - b)B) / (1 - u) \tag{13}$$

The influence of tax credits on the user cost of capital can thus be captured by:

- changes in the corporate tax rate (u);
- the impact of depreciation allowances (uB); and
- the granting of tax credits on new investments (k).

For the purpose of this study, it was assumed that no provision is made for tax credits and that b = 0; that is, the definition of equation 12 is used for the concept of the user cost of capital (c).

The calculation of the user cost of capital.

The formula used for the calculation of the user cost of capital is given by the following equation:

$$c = q\ (r + \delta)(1 - uB) / (1 - u) \tag{12}$$

where c = the user cost of capital;
q = the unit price of new capital equipment;
r = an interest rate at which the firm can borrow or lend financial resources;
δ = the depreciation rate;
u = corporate tax rate; and
B = the discounted value of depreciation allowances associated with capital expenditures of R1.

However, it is not always possible to calculate or measure all the elements of the expression for the user cost of capital directly, and the following approximations were therefore used:

q: The unit cost of new capital goods is approximated by the derived deflator for fixed investment.

r: The yield on long-term government bonds traded on the bond exchange is used as a long-term interest rate.

δ: The depreciation rate for a time period t is calculated as:

$$\delta_t = \frac{W_t}{K_{t-1}}$$

where W_t = depreciation allowed for in period t and
K_{t-1} = capital stock at the end of period t-1.

B: $$B = \sum_{i=1}^{\infty} di\ (1 + r)^{-i} \tag{10}$$

where di = depreciation allowances permitted for tax purposes.

If it is assumed that the assets are written off over θ periods, then the equation for the discounted value of depreciation allowances can be simplified, as is shown in Appendix C, to:

$$B = \frac{1}{r\theta}\ (1 - e^{-r\theta})$$

The lifetime of the asset θ is equivalent to the inverse of the depreciation rate $\frac{1}{\delta}$.

From the foregoing it emerges that the concept of user cost of capital is a much wider notion than an interest rate which is normally used in fixed investment equations. The notion of user cost includes variables such as the general price level, interest rates, tax rates and depreciation rates. The influence of these variables on investment expenditure is captured by the user cost of capital variable.

The lag structure of fixed investment

Fixed investment normally takes place over a long period, and therefore net investment in the current period can be viewed as the result of changes in the capital stock and user cost over a number of preceding periods. Real net investment in period t can be presented by a capital stock adjustment model in the following way:

$$I_t = u(S)(K_t^* - K_{t-1}^*) + \delta K_{t-1} \tag{14}$$

where I_t = real gross fixed investment;
δ = depreciation rate;
K = real capital stock;
K* = desired capital stock;
u(S) = the lag structure S, with $Sx_t = x_{t-1}$

The expression δK_{t-1} signifies the depreciation written off in period t. Equation 14 can be simplified to describe how changes in investment spending are likely to respond, with a time delay, to changes in the desired capital stock, past changes in investment expenditure and past changes in the actual capital stock. It has become customary in empirical work to constrain the distribution of the weights of the lagged variables to follow a pattern that can be approximated by a higher order polynomial function.

It normally takes a long time from the moment the investment decision is taken until the actual investment process materialises. Since the actual capital stock does not react immediately to changes in the desired capital, it is possible to restrict the lower order coefficients of the polynomial structure to zero and the polynomial structure can, for example, look like this:

$$\alpha(S) = \alpha 4S^4 + \alpha 5S^5 + \alpha 6S^6$$

Current period changes in the variable involved can therefore be constrained to have no influence on net investment for a prespecified number of periods.

A further problem is that the desired capital stock is not directly measurable and must therefore be approximated through another set of assumptions.

The determination of the desired capital stock

According to equation 11, the desired capital stock is obtained when the marginal production of capital equals the real user cost of capital, that is when:

$$\frac{\partial Q}{\partial K} = \frac{c}{p}$$

In order to derive an expression for the demand for capital, a specific demand function like the Cobb-Douglas production function is assumed:

$$Q = AL^{\alpha}K^{\beta}$$

where A, α and β are constants and α represents the elasticity of output with respect to labour and β represents the elasticity of output with respect to capital. Optimisation decision rules indicate that profit maximisation occurs when:

$$K = \beta\frac{pQ}{c}$$

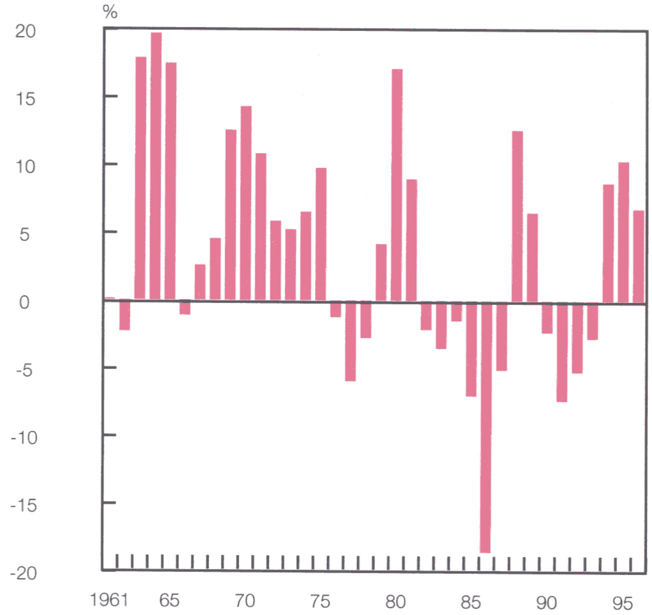
The desired capital stock can thus be approximated by assuming that it is proportional to the ratio of the value of output and the user cost of capital:

The desired capital stock then equals $K^* = \frac{pQ}{c}$, where pQ is the value added in a specific sector and c represents the user cost of capital in that sector.

Medium-term trends in gross fixed investment

The annual average growth rate in total real gross domestic fixed investment has slowed down considerably from the 1960s. It decelerated from an average of nearly 8 per cent in the 1960s to an average of 4,7 per cent in the 1970s and then to an average of less than 1 per cent in the 1980s and early 1990s. The economic upswing that started in May 1993 was characterised by a strong recovery in the growth of real gross fixed investment during 1994 and 1995, but investment spending slowed down somewhat towards the end of 1996. The declining growth trend in gross domestic fixed investment is illustrated in Graph 1.

Graph 1: Growth in real fixed investment, 1961-1996

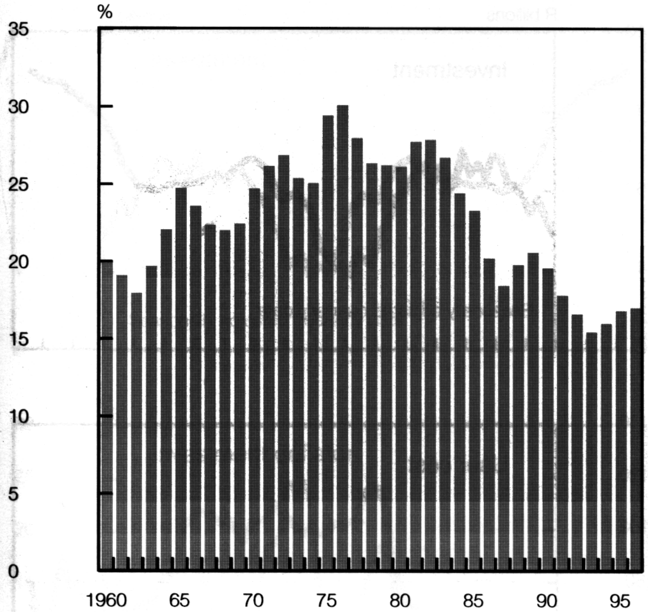


The declining contribution of fixed investment to the gross domestic product is depicted in Graph 2. As a percentage of gross domestic product, gross fixed investment decreased from an average of almost 27 per cent in the 1970s to an average of 17 per cent in the early 1990s. This ratio reached a high point of 30,1 per cent in 1976 before assuming a downward trend and reaching a low point of only 15,5 per cent in 1993. It has since recovered somewhat from this very low level to 17,0 per cent in 1996. This level of fixed investment is nevertheless still well below the more than 25 per cent which is widely regarded as a precondition for a sustained high rate of economic growth.

Table 1 indicates that decreases in real fixed investment by public authorities and public corporations since the 1980s were mainly responsible for the lower growth in total investment. The growth in private sector fixed investment also slowed down, but periods of continued declining investment were not encountered.

The continued contraction in real fixed capital formation by public authorities can be ascribed to significant reductions in real fixed investment by both the general departments and the business enterprises of general government. The decline in the investment

Graph 2: Gross fixed investment as percentage of gross domestic product



of general government in part reflected the increased emphasis by government on current expenditure on social services and the difficulty of government to reduce public consumption.

The contraction in the fixed capital outlays of public corporations was partially a correction of the earlier creation of excess capacity by some of these enterprises. It also formed part of determined cost-cutting plans aimed at improving the cost effectiveness of public corporations.

The apparent reluctance of the private sector to invest in new projects during the 1980s and early 1990s could be ascribed to a number of factors, including:

- a high level of unutilised production capacity in the manufacturing sector;
- the relatively high level of taxation in South Africa;
- rationalisation of the gold-mining industry;
- periods of drought and a high level of farm debt;
- general lack of business confidence as a result of uncertainty about future political developments;
- the civil unrest and violence in many parts of the country; and
- more recently, the high incidence of violent crime.

Since 1993, capital expenditure on a few major projects like Columbus and Alusaf has contributed to the revival of fixed investment. As the recovery in general economic activity in 1993 became more widespread and gathered momentum, a large part of new investment spending was devoted to the replacement of obsolete equipment which had been deferred previously.

Table 2 gives an indication of the contribution of the various subsectors of the economy to total gross domestic fixed investment. The contribution

Table 1. Average annual growth rates in real fixed investment by type of institution

Per cent

	Public authorities	Public corporations	Private sector	Total
1961-1969	7,6	15,0	7,4	7,9
1970-1979	2,8	14,9	3,3	4,7
1980-1989	-2,4	-1,9	3,7	0,7
1990-1996	-6,3	-0,5	3,4	1,2
1960-1996	0,8	7,2	4,4	3,7

Table 2. Investment by type of asset as percentage of total fixed investment

	1960 to 1969	1970 to 1979	1980 to 1989	1990 to 1996
Agriculture, forestry and fishing	8,3	5,9	4,5	3,6
Mining and quarrying	8,7	7,4	12,2	10,5
Manufacturing	19,0	19,9	19,6	24,9
Electricity, gas and water	8,1	9,4	13,3	8,2
Construction	1,1	1,8	1,5	1,3
Wholesale and retail trade, catering and accommodation ...	6,4	6,5	5,7	6,9
Transport, storage and communication	14,1	14,6	10,6	8,4
Finance, insurance, real estate and business services..	17,2	17,6	20,7	23,4
Community, social and personal services	17,0	16,8	12,0	13,0

of the agricultural sector to total gross fixed investment fell from 8,3 per cent in the 1960s to 3,6 per cent in the 1990s. Other sectors in which the relative contributions of fixed investment declined notably were the transport, storage and communication sector and the sector providing community, social and personal services, the latter reflecting the strong declines in fixed investment spending by general government.

Major contributions to fixed investment in the 1990s were made by the manufacturing sector and the sector finance, insurance, real estate and business services. These two sectors were also responsible for the strongest long-term increases in their respective contributions to total fixed investment spending: manufacturing increased its contribution from 19 per cent in the 1960s to 24,9 per cent in the 1990s, while the finance sector increased its contribution from 17,2 per cent in the 1960s to 23,4 per cent in the 1990s.

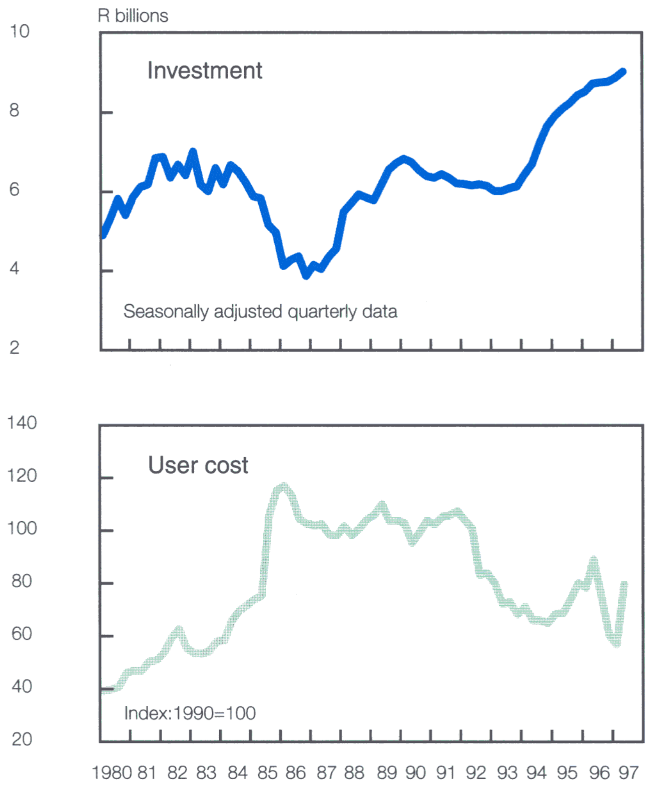
Explanatory variables of the fixed investment function

The theoretical overview in Section 2 provides a number of possible explanatory variables that can be used to explain fixed investment behaviour. The variables used in the calculation of the user cost of capital, along with other explanatory variables that were introduced into the equation explaining changes in fixed investment, are briefly described in this section. Private-sector fixed investment in this section refers to investment in the private non-agricultural sector, excluding private residential construction, fixed capital outlays by public corporations and the gold-mining industry. All the investment aggregates were measured at constant 1990 prices.

User cost of capital

The inverse relationship between real private-sector fixed investment and real user cost of capital is portrayed in Graph 3. The real user cost of capital is defined as the user cost of capital divided by the value added or output price deflator for the private sector. The sharp increase in the real user cost of capital since 1985 coincided with the rapid depreciation of the rand and the subsequent increase in inflation and interest rates, as well as increases in the effective corporate tax rate. The real user cost of capital remained at a relatively high level throughout the 1980s. When the real user cost of capital began to decline from 1992, it was soon followed by a rising level of real gross fixed investment.

Graph 3: Real private-sector fixed investment and real user cost of capital



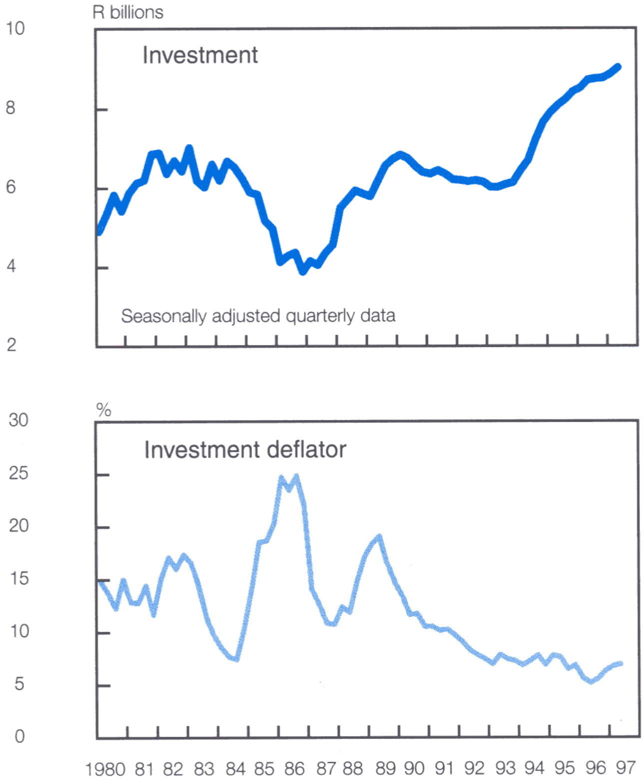
The derived price deflator for fixed investment

The derived price deflator for fixed investment was used to represent the unit price of new capital assets. Graph 4 depicts the generally inverse relationship between real fixed investment and the percentage change over four quarters in the price deflator for fixed investment. The effect of the sharp depreciation of the rand in 1984 and 1985 on the prices of capital equipment in the mid-1980s can clearly be seen from the graph. The annual increase in the average price of capital goods fell below 10 per cent in 1992 and has since then continued to slow down gradually, thereby adding to the recovery in fixed investment activity which began in 1993.

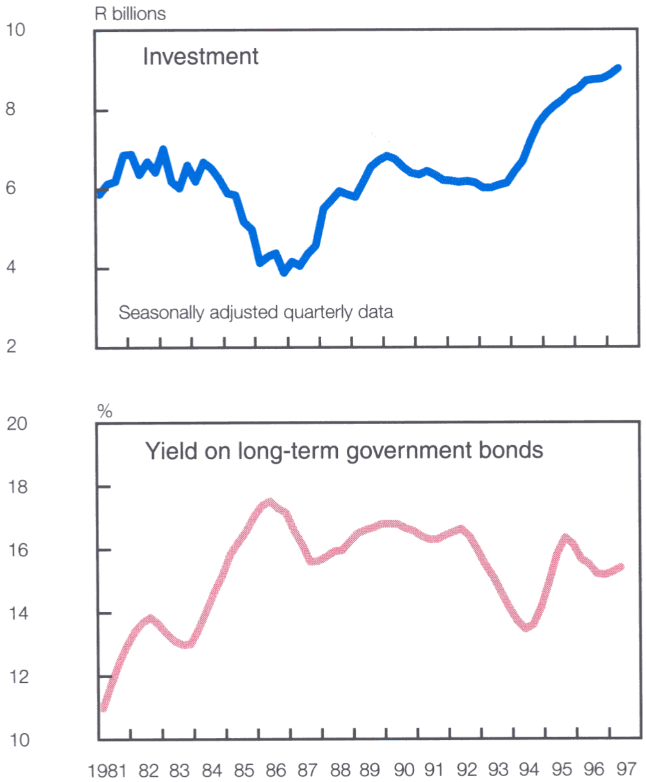
Long-term interest rates

The yield on the long-term government bonds was used as a close approximation of the long-term interest charge at which firms can borrow or lend money. In Graph 5 a five-term moving average of the yield on long-term government bonds is compared with real fixed investment in the private sector. A consistently

Graph 4: Real private-sector fixed investment and the deflator for fixed investment



Graph 5: Real private-sector fixed investment and long-term interest rates



inverse lagged relationship is revealed in the graph. Despite the relative high real interest rates from 1995, real fixed investment was still moving upwards.

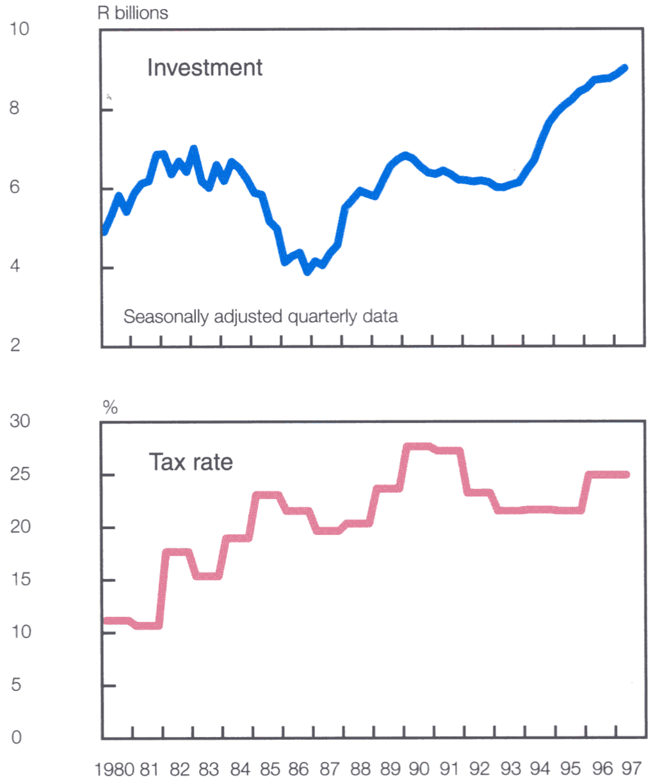
Effective corporate tax rate in the private sector

The negative effect of a high effective corporate tax rate on real fixed investment is illustrated in Graph 6. The lowering of the corporate tax rate from the early 1990s can be linked to the upward trend in fixed investment over the same period. The tax incentives and tax holidays announced in the macroeconomic strategy for growth, employment and redistribution are therefore likely to contribute to a higher level of fixed investment spending.

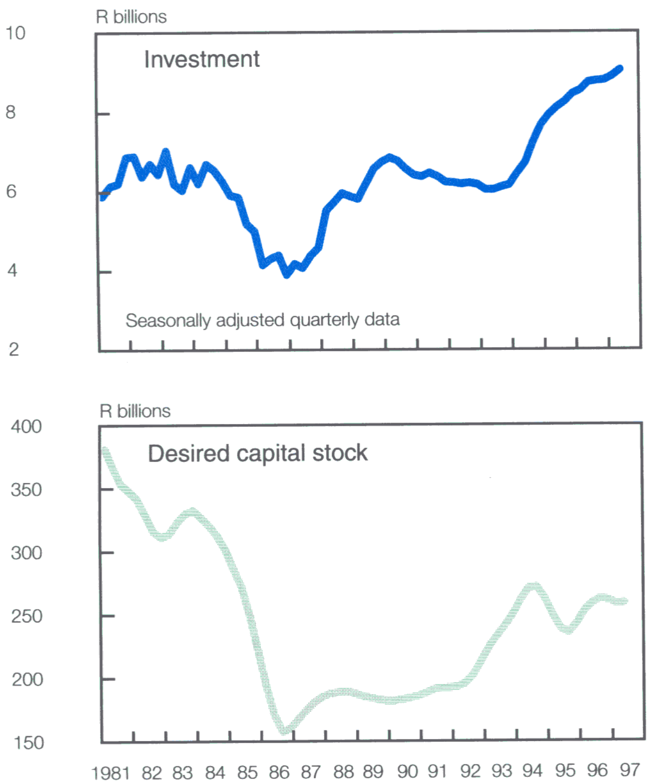
The desired capital stock

As described earlier, the desired capital stock can be approximated by the ratio between the value added in a specific sector and the user cost of capital in that sector. For the purpose of this paper, the sectors involved include all private enterprises in the economy, excluding those in the agricultural and gold-mining sectors and private residential

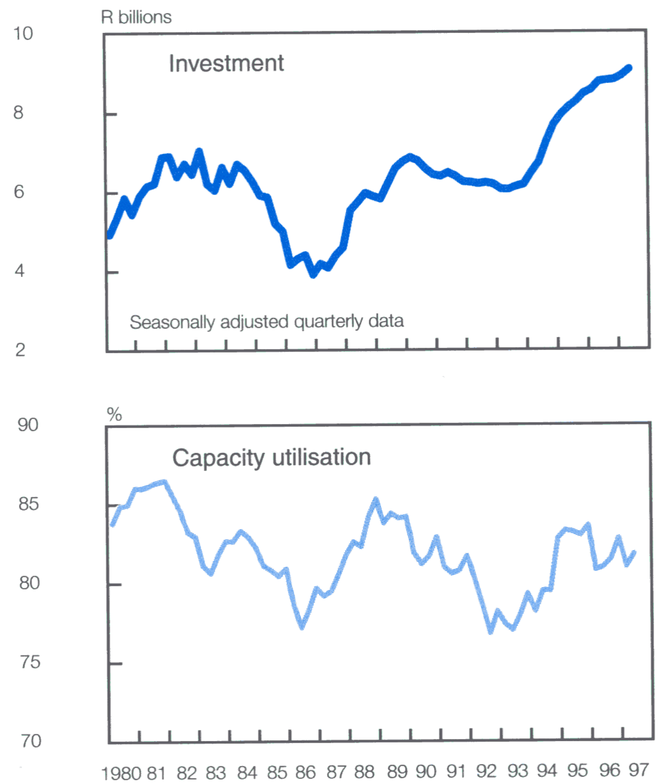
Graph 6: Real private-sector fixed investment and the effective corporate tax rate



Graph 7: Real private-sector fixed investment and the desired capital stock



Graph 8: Real private-sector fixed investment and capacity utilisation



construction. Graph 7 shows a positive relationship between real fixed investment and the assumed desired capital stock. The declining tendency in the real value added by the private sector during the 1980s relative to the user cost of capital, was accompanied by a decline in private-sector fixed investment. These trend movements were broadly reversed from 1993.

The capacity utilisation rate in manufacturing

The positive relationship between the capacity utilisation rate in the manufacturing sector and real fixed investment in the private sector is shown in Graph 8. Low economic growth rates in the late 1980s and early 1990s were accompanied by declining capacity utilisation rates, indicating the existence of some spare capacity in the economy. Since 1993 the economic growth rate has turned positive and has started to accelerate while the capacity utilisation rate increased. Weaker growth in 1996 and 1997 was also reflected in lower utilisation rates. The current capacity utilisation rate of 81 per cent is still well below the 86 per cent level achieved in 1981. The recent rise in private-sector fixed investment from 1996 in the face of a low capacity utilisation rate can partly be ascribed to the replacement of relatively obsolete machinery and equipment.

Statistical estimation of fixed investment in the private sector

The empirical results of the estimated equation for fixed investment in the private sector are described in this section. All the econometric calculations were carried out with quarterly, seasonally adjusted data. T-values of the estimated coefficients as well as the following summary statistics are provided:

$\bar{R}^2$	=	Adjusted coefficient of determination;
D-W	=	Durbin-Watson d-statistic;
RHO	=	Autocorrelation coefficient.

The period of estimation is stated immediately below the summary statistics for each equation.

The equations were specified in logarithmic format in order to allow the parameter estimates to reflect elasticities. All variables used in the estimation of the equations were measured in constant 1990 prices.

The equations were estimated with cointegration techniques comprising the two-step Engle-Granger procedure (Engle and Granger, 1987). The first step involves the estimation of a long-run equation, supported by relevant economic theory. The order of integration of the variables involved was determined. In practice few macroeconomic time series are stationary in level terms, but most are stationary in first or second differences. The Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) test statistics were used to determine stationarity. The test results indicate that all the variables tested were non-stationary. However, the test statistics based on the differences over four quarters of these variables all exceeded the critical values on a 5 per cent significant level and were thus stationary.

The order of integration of the residuals obtained from the long-term equation was determined. The test results indicated stationarity and the residual item could therefore be included in the short-term error correction model.

The second step involved the estimation of a short-term equation or error correction model. The same variables used in the long-term equation were used as explanatory variables in the error correction model (ECM) equation. The ECM equation were estimated in differences over four quarters and the coefficient of determination $\bar{R}^2$ was consequently lower than in the case of the long-term equation.

The following general notation is used in the specification of the equations:

- Δ: the percentage change over four quarters of a variable;
- L: the subscript indicating the long-term equation;
- S: the subscript indicating the short-term equation; and
- ln: the natural logarithm of a variable.

Gross fixed investment in the private sector[1]

The following explanatory variables are included in the equation explaining changes in gross fixed investment in the private sector (GFI):

- the desired capital stock (DCS);
- the capacity utilisation rate in the manufacturing sector (CAP); and
- the capital stock in the private sector, excluding the agricultural and gold-mining sector and private residential buildings, at constant 1990 prices (CS).

The weights of the lagged desired capital stock were determined by running a separate regression of the real fixed investment on the real capital stock and the desired capital stock. The coefficients of the lagged variables were constrained to follow a second degree polynomial with both starting and end points restricted to zero.

The weights for the lagged desired capital stock were determined as:

Period	Weights
0	0,143
1	0,229
2	0,257
3	0,229
4	0,143

Long-term equation:

$$\ln(GFI_L) = BO + B1*\ln(DCS) + B2*\ln(CAP(-1)) + B3*\ln(CAP(-2)) + B4*\ln(CS(-1))$$

COEFFICIENT	ESTIMATE	T-STATISTIC
B0	-23,37	8,69
B1	0,43	7,34
B2	1,56	3,02
B3	1,72	3,57
B4	1,01	11,37

$\bar{R}^2$ = 0,93
D.W = 2.01
RHO = 0,49
Sample period = 81:q1 - 97:q2
Augmented Dickey-Fuller test on residual item = -4,76
[Critical value = -4,53 (1% level)]

Short-term equation:

The dependent variable in the short-term equation is the change over four quarters in gross fixed investment (ΔGFI). All the explanatory variables in the equation resemble the change over four quarters in the variables. The explanatory variables of the long-term equation was also used to explain changes in private-sector fixed investment in the short-term equation:

$$\Delta\ln(GFI_S) = B1*\Delta\ln(DCS) + B2*\Delta\ln(CAP(-2)) + B3*\Delta\ln(CS(-1)) - B4*(RESIDUAL_L(-4))$$

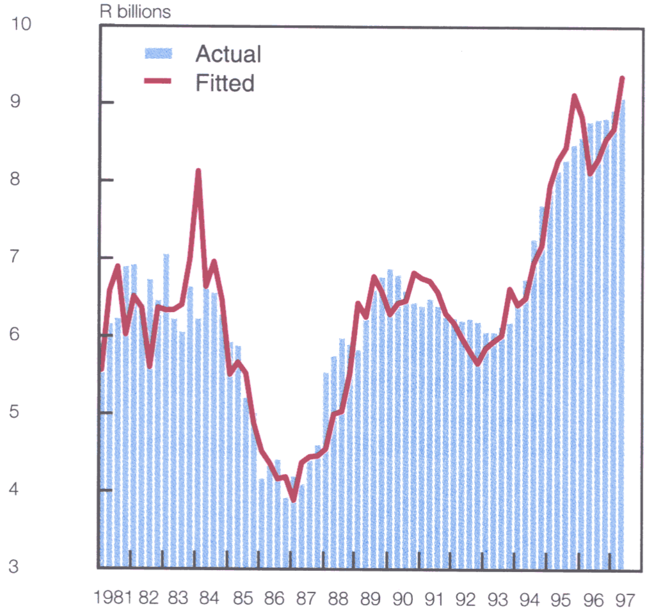
COEFFICIENT	ESTIMATE	T-STATISTIC
B1	0,50	8,54
B2	3,05	10,29
B3	1,02	5,48
B4	-1,07	6,29

$\bar{R}^2$ = 0,78
D.W = 1,25
Sample period = 82:q1 - 97:q2

The actual and fitted values of the equation for private business-sector fixed investment are compared in Graph 9.

1 Excluding the agricultural sector, private residential construction, fixed capital outlays by public corporations and the gold-mining industry, at constant 1990 prices.

Graph 9: Actual and fitted values of fixed investment in the private sector



Concluding remarks

The neoclassical theory of optimal capital accumulation as described by Jorgenson proved to be a helpful model to explain private business-sector fixed investment in South Africa. The user cost of capital is central to this model because it encapsulates the impact changes of interest rates, the prices of capital goods and tax rates in a single variable. Generally speaking, the model provides a reasonably accurate description of private-sector investment behaviour in the South African economy over the period from 1980.

The equation used to describe fixed investment in the private sector in the macro-econometric model of the Reserve Bank is predominantly based on the neoclassical investment theory of Jorgenson. In addition to the desired capital stock and the user cost of capital, the capacity utilisation rate was also utilised to explain changes in fixed investment.

The importance of investment in productive assets was officially recognised when the Ministry of Finance announced the macroeconomic strategy for employment, redistribution and growth. A package of tax incentives was announced to encourage investment, including special allowances for qualifying plant and equipment which will be purchased up to September 1999, as well as a tax holiday available to approved projects on the basis of regional location, job creation and for priority industries. The new tax incentives and the relaxation of exchange control measures will enhance South Africa's attractiveness for local and foreign investors. In so far as these incentives will reduce the user cost of capital, it will contribute to a higher rate of investment spending and the strengthening of the growth capacity of the South African economy.

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Appendix A. The firm's optimisation problem

The maximising problem can be simplified by assuming T= 0, that is taxes are not considered for the time being. The Lagrange equation for maximising Equation 3 can then be described as:

$$L = \int_0^{\infty} e^{-rt}R_t + \lambda_0(t)(Q - F(L,K)) + \lambda_1(t)(\dot{K} - I + \delta K)dt$$

$$\text{Let } f = e^{-rt}R_t + \lambda_0(t)(Q - F(L,K)) + \lambda_1(t)(\dot{K} - I + \delta K)$$

$$f = e^{-rt}(pQ - wL - qK) + \lambda_0(t)(Q - F(L,K)) + \lambda_1(t)(\dot{K} - I + \delta K) \quad (4)$$

The first order conditions for maximising the firm's present value are obtained when the first derivatives of the function f with respect to Q, L, I, K, $\dot{K}$, λ_0, and λ_1 are equated to zero.

$$\frac{\partial f}{\partial Q} = e^{-rt}p + \lambda_0(t) = 0$$

$$\therefore \lambda_0(t) = -e^{-rt}p \quad (5a)$$

$$\frac{\partial f}{\partial L} = -e^{-rt}w - \lambda_0(t)\frac{\partial F}{\partial L} = 0$$

$$\therefore \frac{\partial F}{\partial L} = \frac{-e^{-rt}w}{\lambda_0(t)} \quad (5b)$$

$$\frac{\partial f}{\partial I} = -e^{-rt}q - \lambda_1(t) = 0$$

$$\therefore \lambda_1(t) = -e^{-rt}q \quad (5c)$$

$$\frac{\partial f}{\partial K} = -\lambda_0(t)\frac{\partial F}{\partial K} + \lambda_1(t)\delta = 0$$

According to Allen (1967: 529) it can be assumed that

$$\frac{\partial f}{\partial K} = \frac{d}{dt}\frac{\partial f}{\partial \dot{K}}$$

$$\text{thus } \frac{\partial f}{\partial K} - \frac{d}{dt}\frac{\partial f}{\partial \dot{K}} = -\lambda_0(t)\frac{\partial F}{\partial K} + \delta\lambda_1(t) - \frac{d}{dt}\lambda_1(t) = 0 \quad (5d)$$

$$\frac{\partial f}{\partial \lambda_0} = Q - F(L,K) = 0$$

$$\therefore Q = F(L,K) \quad (5e)$$

$$\frac{\partial f}{\partial \lambda_1} = \dot{K} - I + \delta K = 0$$

$$I = \delta K + \dot{K} \quad (5f)$$

The marginal return on labour can be obtained by substituting equation 5a into equation 5b.

$$\frac{\partial f}{\partial L} = \frac{-e^{-rt}w}{\lambda_0(t)} \quad (5b)$$

$$\text{thus } \frac{\partial f}{\partial L} = \frac{-e^{-rt}w}{-e^{-rt}p}$$

$$\frac{\partial f}{\partial L} = \frac{w}{p} \quad (6)$$

The present value of the firm is therefore maximised when the marginal product of labour is equal to the ratio of the price of labour services to the product price, also referred to as the real wage.

The marginal return on capital can similarly be derived in the following way:

$$\lambda_1(t) = -e^{-rt}q \quad (5c)$$

$$\frac{d}{dt}\lambda_1(t) = e^{-rt}qr - e^{-rt}\frac{dq}{dt} \qquad (7)$$

Substitute equations 5a, 5b, and 7 in equation 5d:

$$-\lambda_0(t)\frac{\partial F}{\partial K} + \delta\lambda_1(t) - \frac{d}{dt}\lambda_1(t) = 0 \qquad (5d)$$

$$-e^{-rt}p\frac{\partial F}{\partial K} - \delta q e^{-rt} - (-e^{-rt}\dot{q} + e^{-rt}qr) = 0$$

$$\therefore\ p\frac{\partial F}{\partial K} = \delta q - \dot{q} + qr$$

$$p\frac{\partial F}{\partial K} = q(\delta + r - \frac{\dot{q}}{q})$$

$$\therefore\ \frac{\partial F}{\partial K} = \frac{q\left(\delta + r - \frac{\dot{q}}{q}\right)}{p}$$

Let $c = q(\delta + r - \frac{\dot{q}}{q})$

then $\frac{\partial F}{\partial K} = \frac{c}{p}$ (8)

where c indicates the user cost of capital.

Appendix B. The effect of tax incentives

The effect of tax incentives can be illustrated in the following way:

Suppose a firm wishes to increase its capital stock by one unit, and the cost of one unit of capital is q and that the depreciation rate of capital is δ per period over the economic life of the asset. The initial outlay of the firm is q and the replacement expenditure in each future period is δq.

The output in each period will be increased by the marginal product of capital, that is $\frac{\partial Q}{\partial K}$. If the additional unit of output can be sold at a price p, then the gross revenue in each period will be increased by $p\frac{\partial Q}{\partial K}$. The net revenue will be increased by $p\frac{\partial Q}{\partial K} - \delta Q - T_i$, where T_i is the increase in direct taxes in period i. If the increase in depreciation charges for tax purposes is indicated by D_i, then the increase in direct taxes is indicated by $T_i = u(p\frac{\partial Q}{\partial K} - D_i)$, where u represents the tax rate on firms. The firm will keep on adding one unit to its existing capital stock as long as the discounted value of the increases in net revenue exceeds the price of a unit of capital. The process can be illustrated in the following way:

$$\sum_{i=1}^{\infty}[p\frac{\partial Q}{\partial K} - \delta q - u(p\frac{\partial Q}{\partial K} - D_i)]\ (1 + r)^{-i}$$

$$= \sum_{i=1}^{\infty}[(1-u)\,p\frac{\partial Q}{\partial K} - \delta q + u D_i]\ (1 + r)^{-i}$$

$$= [(1-u)\,p\frac{\partial Q}{\partial K} - \delta q] \sum_{i=1}^{\infty}(1 + r)^{-i} + u\sum_{i=1}^{\infty} D_i (1 + r)^{-i}$$

but $\sum_{i=1}^{\infty}(1 + r)^{-i} = \frac{1}{r}$

The firm will therefore keep on investing as long as

$$(1-u)\,p\frac{\partial Q}{\partial K}\frac{1}{r} - \delta q\frac{1}{r} + u\sum_{i=1}^{\infty} D_i (1 + r)^{-i} > q \qquad (9)$$

where r is the interest rate at which the firm may borrow or lend.

Let d_i be the tax depreciation permitted on an investment of one unit, i periods after the investment has been made. The discounted value of depreciation charges stemming from an investment of one unit is indicated by $\sum_{i=1}^{\infty} d_i (1 + r)^{-i}$.

The increase in the tax depreciation charges is calculated in the following way:

Periods after investment has taken place	**Tax depreciation charges**
1	qd_1
2	$qd_2 + \delta qd_1$
3	$qd_3 + \delta qd_2 + \delta qd_1$
4	$qd_4 + \delta qd_3 + \delta qd_2 + \delta qd_1$

If the tax depreciation charges are discounted and added, the following result is obtained:

$$\sum_{i=1}^{\infty} D_i (1 + r)^{-i} = \sum_{i=1}^{\infty} qd_i(1 + r)^{-i} + \sum_{i=1}^{\infty}\delta qd_i (1 + r)^{-i}(1 + r)^{-1} +$$

$$\sum_{i=1}^{\infty}\delta qd_i (1 + r)^{-i}(1 + r)^{-2} + \ldots$$

Let $B = \sum_{i=1}^{\infty} d_i(1 + r)^{-i}$, then (10)

$$\sum_{i=1}^{\infty} D_i (1 + r)^{-i} = qB + \delta qB (1 + r)^{-1} + \delta qB (1 + r)^{-2} + \ldots$$

$$= qB + \delta qB \sum_{i=1}^{\infty}(1 + r)^{-i}$$

$$= qB + \delta qB\frac{1}{r} \qquad (10a)$$

Equation 10a can now be substituted into equation 9:

$$(1-u)\,p\frac{\partial Q}{\partial K}\frac{1}{r} - \delta q\frac{1}{r} + uqB + u\delta qB\frac{1}{r} > q$$

$$(1-u)\,p\frac{\partial Q}{\partial K} - \delta q + ruqB + u\delta qB > rq$$

$$(1-u)\,p\,\frac{\partial Q}{\partial K} > rq + \delta q - ruqB - u\delta qB$$

$$> q\,(r + \delta - ruB - u\delta B)$$

$$> q\,[(r + \delta) - uB\,(r + \delta)]$$

$$> q\,(r + \delta)\,(1 - uB)$$

$$\therefore p\,\frac{\partial Q}{\partial K} > q\,(r + \delta)(1 - uB) / (1 - u) = c \qquad (11)$$

Appendix C. The discounted value of depreciation allowances

$$B = \int_0^\theta d(t)\, e^{-rt}\, dt$$ [1]

If the straight-line depreciation method is assumed, the depreciation is constant over a period of θ, the lifetime for tax purposes. The depreciation per period is $\frac{1}{\theta}$ and

$$d(t) = \frac{1}{\theta}$$

$$\text{thus } B = \int_0^\theta \frac{1}{\theta}\, e^{-rt}\, dt$$

$$= -\frac{1}{\theta}\,\frac{1}{r}\, e^{-rt} \Big/_0^\theta$$

$$= -\frac{1}{r\theta}\, e^{-r\theta} - \left(-\frac{1}{r\theta}\right)$$

$$= \frac{1}{r\theta}\,(1 - e^{-r\theta})$$

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1 Hall, R.E. and Jorgenson, D.W. Tax policy and investment behaviour. *American Economic Review*, June 1967, pp. 394–395.