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A Stylised Semi-structural Model of the Equilibrium REER

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Abstract

We present a model of the equilibrium exchange rate that builds on the foundation of the theory of purchasing power parity and uncovered interest parity. The model highlights the importance of productivity differentials, real commodity price trends, and more recently the structural budget balance, in determining equilibrium outcomes. This simplistic model, when paired with the more agnostic VECM model of the exchange rate, expands our toolset for producing a consistent narrative and forecast of the exchange rate gap – a key driver of inflation outcomes over the policy horizon.

1 Introduction

The exchange rate is a fundamental driver of much of the economic outcomes in a small open economy like South Africa. Recently the SARB adopted the Quarterly Projection Model (QPM) as its forecasting model. In the QPM, the real exchange rate gap is one of four main gaps used to understand the dynamics of the South African economy and make policy recommendations. The prominence of the exchange rate in our economic developments and model for policymaking, however, creates the impression that we fully understand its dynamics. We do not.

The "true" model of the exchange rate does not exist in the theories we have (which also disagree about the appropriate way to determine the exchange rate) and the econometrics we do. Unable to reach the holy grail, this note describes a stylised model of the real exchange rate and its fundamental or equilibrium values. In using such a model, we attempt to provide an easy explanation for why the real exchange rate deviates from its fundamental value and why this fundamental value might move. In so doing, we aim to provide a narrative for what we believe to be the key drivers of the real exchange rate.

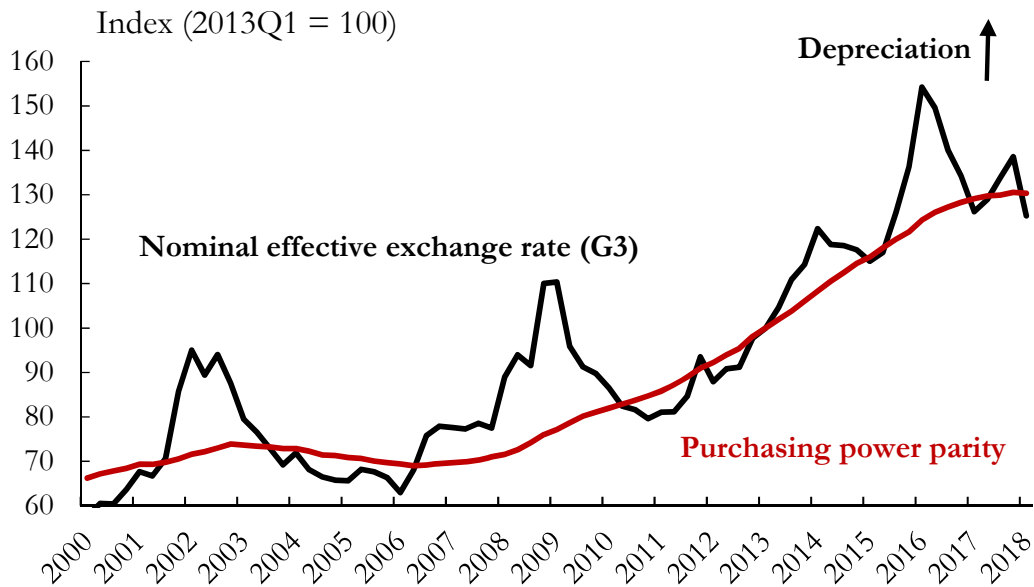## 2 From 16th century Spain to 21st century South Africa

The origins of Purchasing Power Parity (PPP) come from the writings of scholars at a monastery in 16th century Spain (see Dornbusch (1985) for a detailed history of PPP). The theory of PPP states that the exchange rate between two countries should be equal to the difference in price levels for the same basket of goods (and services). The rationale was simple, if goods were cheaper somewhere else, then they would just be imported (bar other costs of getting the goods there). And so the exchange rate would act as the equaliser to ensure that once converted into a common currency, goods prices would be equal accross borders. The convenience of this theory, although roundly rejected by economists, means that in 21st century South Africa we use this as a starting point to think about the nominal and real exchange rate, and to build models of our economy.

PPP makes a strong claim, however, that the **only** thing explaining movements in the nominal exchange rate is the price differentials between two countries. Figure 1 graphs the nominal effective exchange rate of the Rand[1] against the ratio of consumer prices domestically and in the G3 economies (US, euro area, and Japan)[2]. If PPP held, then the red line should explain all movements in the black line. This is clearly not the case, with the nominal exchange rate deviating for long periods from PPP, up to four years at a time. In economics jargon, economists will call the fact that the two lines are not on top of one another "a rejection of the theory of absolute PPP". Despite absolute PPP's failure, there is clearly a longer-run trend between where South Africa's nominal exchange rate goes and PPP. On average, the Rand has depreciated by the difference between prices in South Africa and the G3.

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[1]This is the effective exchange rate against the US dollar, Euro, and Japanese Yen.

[2]The G3 economies are used for convenience. This relationship remains true for just the US and also a more broad effective exchange rate measure.

Figure 1: The nominal effective exchange rate and PPP

Index (2013Q1 = 100)

Nominal effective exchange rate (G3)

Depreciation

Purchasing power parity

The failure of PPP means that there are other things that matter for explaining exchange rate developments in South Africa. So what is left to be explained after we account for PPP? The difference between the red and black lines in Figure 1 is the real effective exchange rate. Therefore we need a theory for the drivers of the real exchange rate. Figure 2 plots the real effective exchange rate for South Africa, deflated using consumer prices.

Figure 2: The real effective exchange rate

Index (2013Q1 = 100)

We divide the real exchange rate into two parts: A 'fundamental' (or equilibrium) concept which describes longer-run movements in the real exchange rate; and a 'gap' concept (deviations from

this trend) which describes shorter-run developments. We exploit two theories to be able to explain the equilibrium and gap.

The first theory is uncovered interest parity (UIP) and is used to explain shorter-run movements in the real exchange rate gap. This theory states that expected movements in the exchange rate will be a function of the difference in interest rates between two countries. It is based on the idea that the return on a domestic asset should be equal to the return on a foreign asset once we have accounted for the expected change in the exchange rate. If this was not true then investors could make money off the price difference (so-called arbitrage).

The second theory is not really one theory, but rather a combination of theories from economic literature of what can cause longer-run deviations from PPP. The part that explains the equilibrium. We focus on two main economic concepts: productivity differentials (in economics this is referred to as the Balassa-Samuelson theory) and commodity prices. We believe movements in these two variables explain almost all of the movements in the equilibrium exchange rate. When commodity prices persistently rise for a commodity producer like South Africa, real incomes will also rise and this should appreciate the currency. When our productivity rises faster than our trading partners we expect that wages and prices in the overall economy will rise. The increase in prices in the services sector (or non-tradables more broadly) will make us less competitive and lead to an appreciation of our real exchange rate. The opposite is also true. We also use two other drivers of the equilibrium exchange rate including the fiscal stance and net international investment position (NIIP).

This note proposes a simple model of the determination of the real effective exchange rate's trend and deviations from this trend. Simplicity allows for a clear interpretation of the drivers of the trend.

3   Exposition of the model

Nominal purchasing power parity states that the exchange rate must reflect the difference between the domestic and foreign price level (assuming competitiveness remains unchanged). That is, if foreign goods go up in price, the exchange rate must appreciate such that the price of foreign goods relative to that of domestic goods remains unchanged (in the home economy). Here we define the nominal exchange rate as the number of domestic currency units that can be bought by one unit of foreign currency. This implies that the home currency appreciates if the level of the exchange rate decreases.

Equation 1 gives a less strict version of this relationship by adding that there may be a wedge between the level of the exchange rate and the price level differential, given by $\mathbf{b}'\mathbf{x}_t$. This wedge reflects permanent deviations from absolute PPP due to economic fundamentals affecting the structure of the economy and trade.

$$s_t = p_t - p_t^* + \mathbf{b}'\mathbf{x}_t \tag{1}$$

In real terms, where the real level of the exchange rate is given by the identity $z_t = s_t + p_t^* - p_t$, this relationship can be re-expressed by the dynamic version of PPP given by Equation 2. Where we add a lagged term, to capture the observed deviations from PPP as highlighted in figure 1.

$$\Delta z_t = \alpha \Delta z_{t-1} + (1 - \alpha)\mathbf{b}'\Delta \mathbf{x}_t \tag{2}$$

This implies that the real exchange rate can deviate from the relative PPP relationship, but converges back to PPP at a rate determined by α. Furthermore the rate of appreciation/depreciation of the real exchange rate at PPP equilibrium equals the steady state growth rate of the variables that represent the fundamentals that create the wedge discussed earlier.

So to recap, we have posited that currency market equilibrium is determined by the PPP relationship, with two caveats. One, there are possibly fundamentals that allow for a permanent difference between the purchasing power of the home currency when compared to the foreign currency. Two, changes in these fundamentals are persistent, but decay over time.

It is an empirical fact that the observed exchange rate is more volatile than what can be consistently be explained by PPP. This squares with the idea that it takes time for the economy to adjust to its PPP equilibrium, with interest rates, speculative price movements, etc. temporarily driving the exchange rate away from equilibrium (Stephens et al., 2004).

To explain deviations from PPP, we add the uncovered interest parity relationship given by Equation 3 to our discussion. UIP states that exchange rate expectations of the domestic currency depend on the interest rate differential between the home (i_t) and foreign country (i_t^*) as well as the risk of holding domestic country bonds (u_t).

$$E_t(s_{t+1}) - s_t = i_t - i_t^* - u_t \tag{3}$$

To make explicit that the PPP relationship denotes an equilibrium value and not the spot value we denote it as $\bar{z}_t$ while we keep referring to the spot value as z_t. Getting to the real UIP relationship involves a little algebraic wrangling:

$$E_t(s_{t+1}) - s_t - E_t(\pi_{t+1}) + E_t(\pi_{t+1}^*) =$$
$$(i_t - E_t(\pi_{t+1})) - (i_t^* - E_t(\pi_{t+1}^*)) - u$$
$$(E_t(s_{t+1}) - E_t(p_{t+1}) + E_t(p_{t+1}^*)) - (s_t - p_t + p_t^*) =$$
$$r_t - r_t^* - u$$

The real UIP relationship:

$$E_t(z_{t+1}) - z_t = r_t - r_t^* - u \tag{4}$$

In order to marry these two relationships together we assume that investors know about PPP and that future exchange rates will conform, at least on average, to this relationship (Stephens et al., 2004). This implies that:

$$z_t = \bar{z}_t - r_t + r_t^* + u + v_t \tag{5}$$
$$\Delta\bar{z}_{t+1} = \alpha\Delta\bar{z}_t + (1-\alpha)\mathbf{b}'\Delta\mathbf{x}_{t+1} + \varepsilon_t \tag{6}$$

4 Results

The equilibrium real exchange rate is obtained by estimating the model from 2000Q1 to 2018Q1 using Bayesian techniques. See appendix A for a discussion of the data used, the priors, and the

estimation results.

4.1 The equilibrium real exchange rate

The equilibrium real exchange rate provides us with information of the competitiveness of the currency, highlighting the role of terms of trade and productivity differential shocks in changing export competitiveness. Figure 3 plots the equilibrium exchange rate against the volatile spot value. Deviations beyond the economic structure and trade wedge are determined by UIP and unmodelled factors.

Figure 3: Real exchange rate and its equilibrium
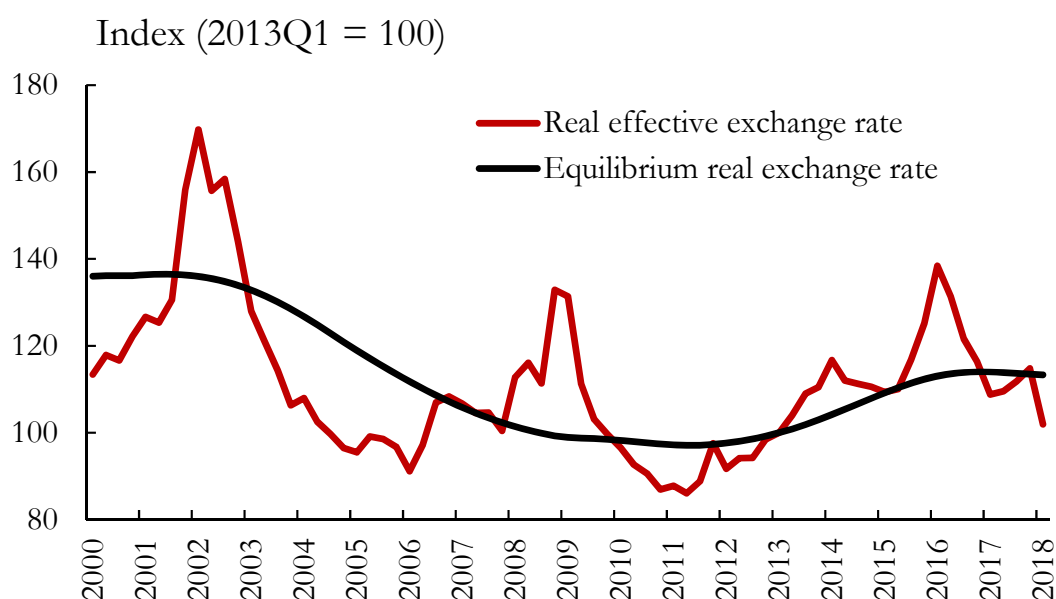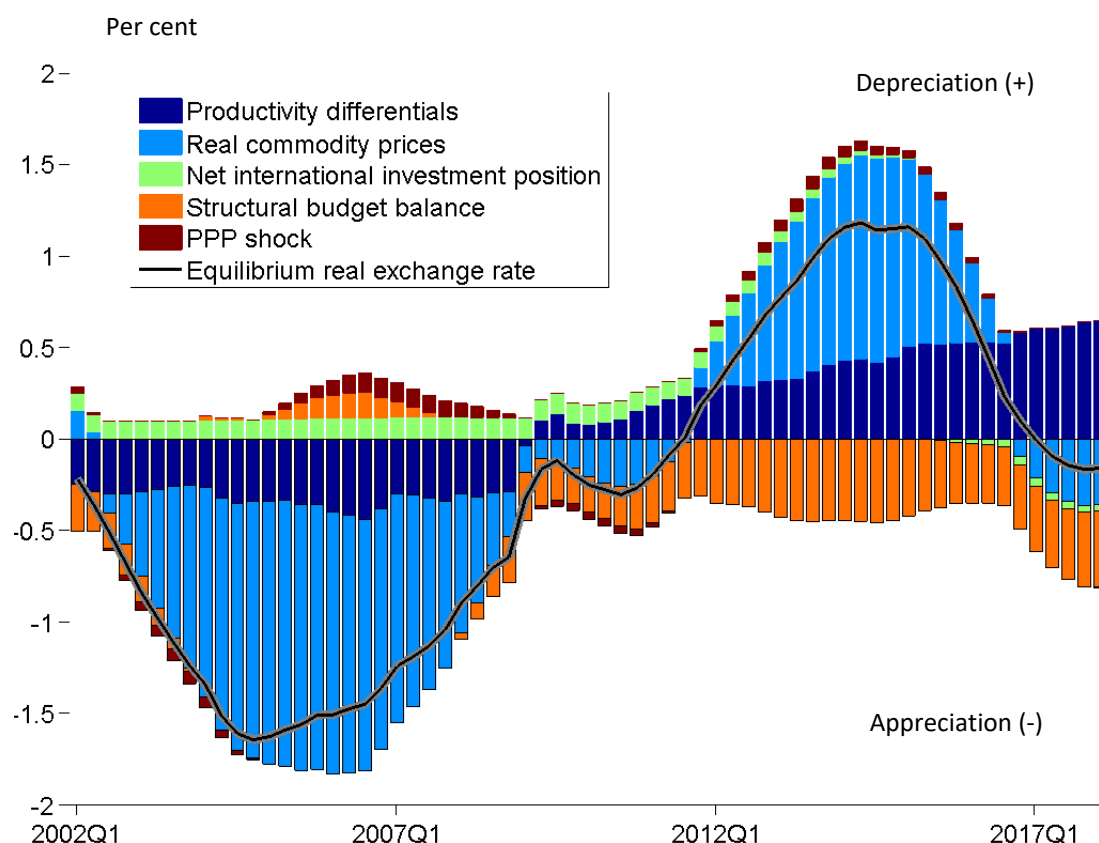
Index (2013Q1 = 100)

Figure 4 decomposes the growth in the equilibrium real exchange rate, showing the contribution of each driver to the overall movement in the long-term trend. The largest contributors to the trend are the productivity gap between SA and SA's major trading partners, real commodity prices, and the structural budget balance. *Real commodity prices*[3] is a proxy South Africa's terms of trade. Rising commodity prices improve the terms of trade and result in an appreciated equilibrium real exchange rate, while falling commodity prices drive the equilibrium in the opposite direction. Over much of the early- and mid-2000s South Africa experienced rapidly growing commodity prices, and improving terms of trade, which raised the level of real incomes. As real incomes rose in both the services (non-tradables) and goods (tradables) sectors, the equilibrium real exchange rate appreciated. The current account, consequently, deteriorated to a peak of 6.9 per cent of GDP in 2007Q3 as imports accelerated. This trend reversed following the global financial crisis (GFC) and the slowdown in the Chinese economy. From 2011 onwards, the deterioration in commodity prices led to slowing real income growth and imports, and depreciated the equilibrium real exchange rate.

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[3]The percentage change in an HP-filter of South Africa's real export commodities prices.

Figure 4: Growth decomposition of the equilibrium real exchange rate

Productivity differentials[4] has a similar impact as the trend in commodity prices. From 2000 to 2007, South Africa achieved a faster growth rate than the average of its trading partners, particularly advanced economies. Higher productivity growth (through the Balassa-Samuelson effect) saw an appreciation of the equilibrium real exchange rate. Following the GFC, global and domestic factors saw South Africa's potential growth slow precipitously to around 1 per cent, significantly lower than that of its trading partners. At its peak, the deteriorated gap between domestic and foreign productivity depreciated the currency's equilibrium by 0.7 percentage points in 2018Q1.

The *structural fiscal budget balance* provides a gauge of the relative support of fiscal policy to the economy[5]. Its impact on the equilibrium exchange rate is through two channels. First, during periods of accommodative fiscal policy, domestic savings will dwindle, and domestic demand should rise, leading to an appreciation in the exchange rate. Second, from the perspective of Balassa-Samuelson, accommodative fiscal policy would lead to a rise in the demand for non-tradables (whether through high public sector wages or higher demand for non-tradable goods), increasing its price and leading to an appreciation of the real exchange rate. Figure 4 shows that since 2008, fiscal policy has helped to appreciate the equilibrium real exchange rate. This impact, however, is likely to be counterintuitive to what some may see as the role of accommodative fiscal policy in depreciating the currency more recently. In this model we use the conventional relationship between fiscal policy and the equilibrium exchange rate. We do not take account of the potential non-linearities of high debt levels on the exchange rate through years of persistent fiscal deficits, the crowding out of private sector investment, or the impacts on confidence.

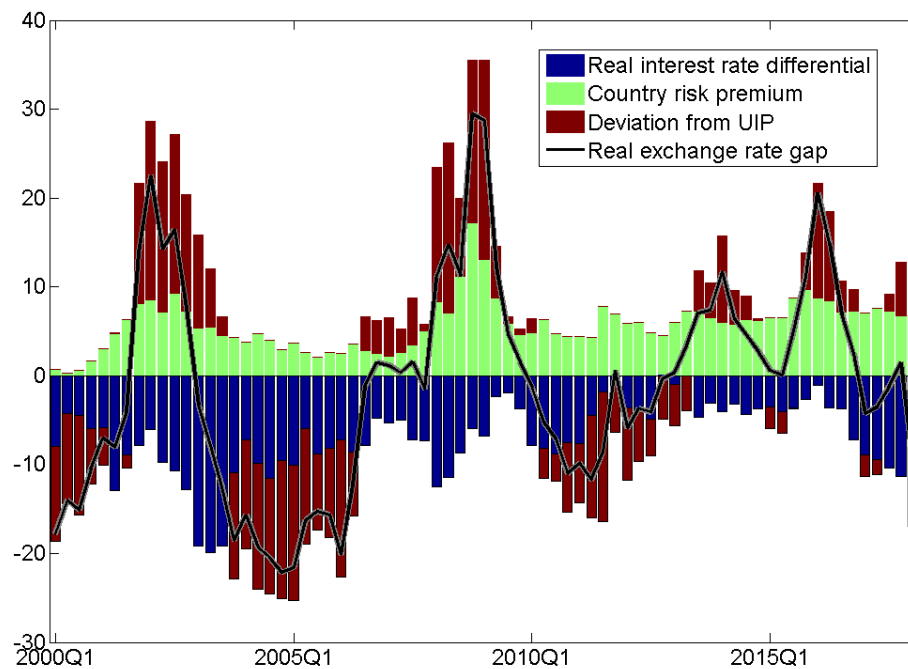[4]Calculated as the trend gap between domestic GDP growth and that of SA's major trading partners.

[5]Both the structural and overall budget balance was used in the estimation step with generally similar results.

The last driver of the equilibrium real exchange rate is SA's *net international investment position* defined as foreign assets less foreign liabilities as a per cent of GDP. From the perspective of the price of tradables and non-tradables, transfers from South Africa to foreign countries lead to a rise in wages and the price of non-tradables in the foreign country, and an appreciation of the foreign countries real exchange rate. This leads to a depreciation of our real exchange rate. Figure 4 shows that over most of the 2000s South Africa ran a negative NIIP which depreciated the real exchange rate. More recently, however, the NIIP has turned positive and has lead to an appreciation of the real exchange rate.

4.2 The real exchange rate gap

Next we look at what drives exchange rate deviations from the equilibrium (the so-called gap). In UIP theory, expectations of the exchange rate are driven by the difference between interest rates in South Africa and abroad as well as some risk premium with which the market will be compensated for holding local assets as opposed to those of a foreign country. In figure 5, the gap is decomposed into the country risk premium (proxied using the EMBI+ spread for SA); real interest rate differentials; and the deviations from UIP (that which is unexplained). A rising country risk premium depreciates the real exchange rate gap while a wider real interest rate gap appreciates the real exchange rate. With a constant risk premium, and assuming that UIP holds, higher local rates relative to foreign rates should push up net foreign assets and appreciate the value of the local currency (De Jager, 2012). While interest rate differentials do explain some of the movement in the gap, the majority of its movements are unexplained. This may not be surprising given that UIP was considered, by Moosa and Bhatti (1997), to be rejected by empirical studies that have tested the hypothesis.
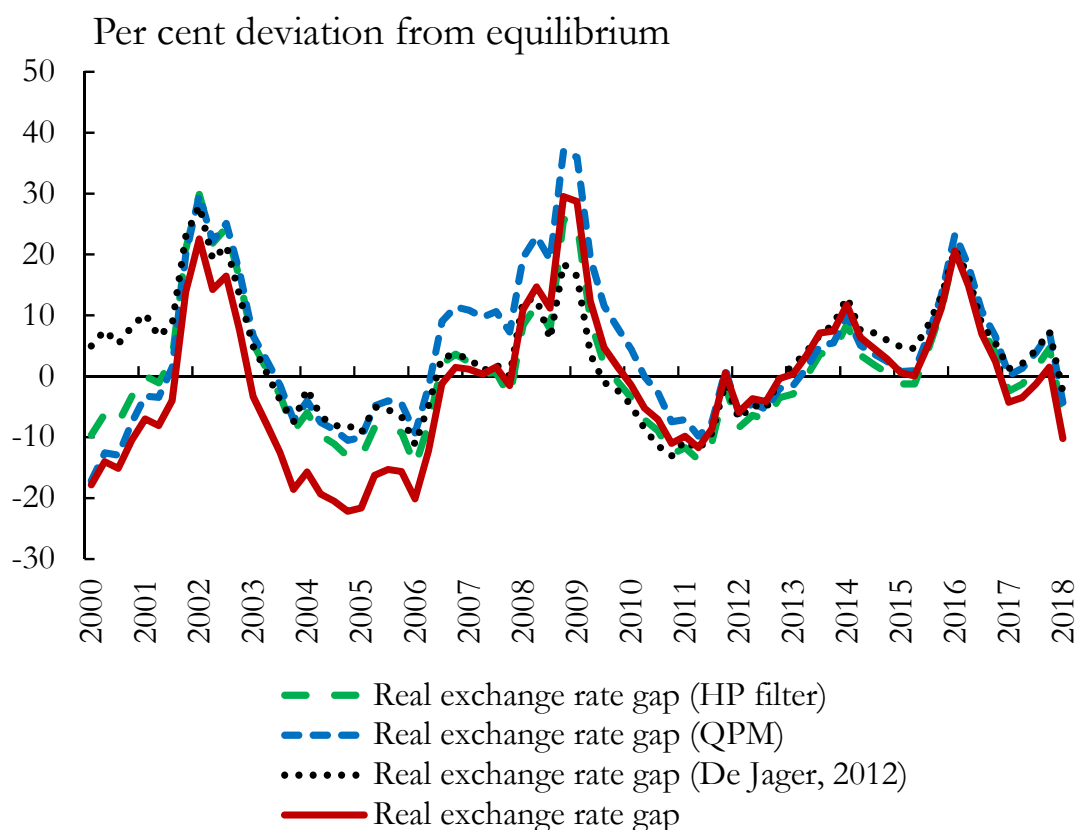
Figure 5: Real exchange rate gaps

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4.3 Comparison to our other models

Our stylised model is intended to be part of a set of models that inform the real exchange rate equilibrium and gap for the QPM. Figure 6 plots the real exchange rate gaps as determined in the QPM, an HP filter, and by De Jager (2012). All measures are well correlated and give a similar description as to the rate of over- (negative) and undervaluation (positive) of the real exchange rate. In 2018Q1, all models saw the real exchange rate as overvalued by between 10 and 2.6%. It should be noted, however, that the current set of models rely on the same drivers of the exchange rate and would therefore give similar results.

Figure 6: Decomposition of real exchange rate gap

The drivers of this stylised model differ from the results of the equilibrium ER model by De Jager (2012) where the largest contributors are openness and capital flows as a ratio of GDP. In fact in that model, openness significantly dominates the equilibrium ER movement. Real interest rate and productivity differentials, terms of trade as well as the government deficit as a ratio of GDP are similarly included in that model but make a muted contribution when compared to openness and capital flows.

5 Conclusion

Using the relationship of PPP as a foundation for the equilibrium exchange rate, and building on that idea we have been able to construct a model of the effective exchange rate of the rand. The exposition of the model focused on augmenting the idea of PPP equilibrium to incorporate fundamental factors that can drive a wedge between the equilibrium value and the value that PPP

would suggest. The equilibrium value of the exchange rate is, however, not observed. To bring the model to the observed data, we further incorporated a UIP relationship in order to explain the deviation of the observed data from its equilibrium. The UIP relationship provides a stylised description of the behaviour of financial markets, expanding the narrative to include the impact of interest rates and country risk on exchange rate outcomes.

In fitting the model to the data using a combination of calibration and estimation, we find that the structure of the model assigns meaningful contributions to productivity differentials, the real commodity price trend, and the structural budget balance. This fits well with the theory and intuition. Namely that in an economy with a relatively large mining sector, with many downstream value-adding processes, commodity prices would have a material impact on the equilibrium exchange rate. Furthermore, in the recent South African context it is intuitive that the structural budget balance would have an increasing role in determining exchange rate pressures.

While we note, in line with the literature, that the UIP relationship has a generally poor fit the relationship does hold with the right signs. It allows us to interpret how domestic and foreign monetary policy as well as more speculative factors that impact sovereign risk affect the exchange rate.

Taken as a tool in conjunction with our more agnostic econometric model of the equilibrium exchange rate we hope that this model will help in building a consistent narrative and forecast of the equilibrium exchange rate and its gap which is so important in explaining future inflation outcomes.

References

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Dornbusch, R. (1985). Purchasing power parity.

Moosa, I. A. and R. H. Bhatti (1997). Does speculation play any role in determining the forward exchange rate? *Applied Financial Economics 7*(6), 611–617.

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A Appendix

A.1 Data

The model uses data from a variety of sources. The Quarterly Projection Model (QPM) and Global Projection Model (GPM) - both new-Keynesian monetary models - were used to generate the inflation expectations used to calculate real interest rates for the domestic and foreign economy. The Hodrick-Prescott filter was heavily relied on to estimate equilibriums and do interpolation where necessary. All the drivers of the equilibrium exchange rate are estimated trends of the underlying observed variables. Table 1 contains the data sources with short explanatory notes concerning the data construction.

Table 1: Data sources

Variable	Source	Notes
Real effective exchange rate	QPM	CPI deflated nominal effective exchange rate of the rand, compiled from the (trade-weighted) bilateral exchange rates of the US, Euro Area, and Japan (G3 economies).
Foreign real interest rate	GPM	G3 economies' policy rate (short term interest rate), adjusted for expected inflation (weighted on a trade weight basis).
Domestic real interest rate	QPM	Repurchase rate adjusted for expected inflation one-quarter-ahead.
Country risk premium	JPMorgan	Country risk is proxied by the EMBI+ strip spread for South Africa.
Productivity growth differential	QPM/GPM	Difference between the trend growth rate of output of the domestic and SA's major trading partners (calculated in the QPM and GPM models).
Real commodity price equilibrium	TSA	Dollar priced commodity prices adjusted for foreign inflation. The trend is estimated using the Hodrick-Prescott Filter with a lambda of 400.
Net international investment	TSA	Total foreign assets minus foreign liabilities as a ratio to GDP. Annual data is interpolated and smoothed using the Hodrick-Prescott Filter with a lambda of 1600.
Structural budget balance	TSA	The structural budget balance is calculated by adjusting the components of total revenue for the output gap and terms of trade. This structural revenue component is then used to calculate the public sector borrowing requirement as a ratio of GDP to give the structural budget balance.

A.2 Estimation

The model is estimated using a quasi-Bayesian approach. Estimates for the parameters of the PPP equation were first calculated using ordinary least squares (OLS). For each parameter a prior distribution is specified with its OLS estimate taken as the mean.

Each exogenous variable in the model also has its own autoregressive equation to describe its dynamics, the mean for each autoregressive parameter was set to 0.5 a priori. Likewise the standard deviations for these equations were all set to unity.

The standard deviations of the UIP and PPP equations were calibrated not estimated. The UIP equation's mean on the prior standard deviation (ε_t in equation 5) was set equal to the sample standard deviation of the REER gap from the QPM. The prior mean of the standard deviation (ν_t in equation 6) in the PPP equation is set equal to $\frac{1}{50} * \varepsilon_t$ i.e. the ratio of the UIP to equilibrium standard deviation was set to 50. Lastly, a parameter was added to the UIP equation to allow the risk adjusted real interest rate differential to have a more than one-to-one impact on expected real depreciations, this parameter was set to one a priori.

Once the prior distributions have been set up, the posterior point estimate of the parameters is found using a maximum likelihood type procedure.

Table 2: Estimation results

Variable	Prior distribution	Prior mean	Std. deviation	Max. posterior
PPP equation				
Lagged REER equilibrium growth	Beta	0.700	0.100	0.478
Productivity growth differential	Beta	0.251	0.100	0.179
Real commodity price equilibrium	Beta	0.303	0.100	0.194
Net foreign assets	Normal	0.236	0.100	0.317
Structural budget balance	Normal	5.921	1.000	4.532
Autoregressive equations for exogenous variables				
Foreign real interest rate	Beta	0.500	0.150	0.861
Domestic real interest rate	Beta	0.500	0.150	0.846
Country risk premium	Beta	0.500	0.150	0.780
Productivity growth differential	Beta	0.500	0.150	0.969
Real commodity price equilibrium	Beta	0.500	0.150	0.960
NIIP	Beta	0.500	0.150	0.877
Structural budget balance	Calibrated	0.500	0.150	0.637
UIP multiplier				
	Normal	1.000	0.500	3.064
Standard deviations				
Foreign real interest rate	Inverse gamma	1.000	0.300	1.063
Domestic real interest rate	Inverse gamma	1.000	0.300	1.603
Country risk premium	Inverse gamma	1.000	0.300	1.107
Productivity growth differential	Inverse gamma	1.000	0.300	0.551
Real commodity price equilibrium	Inverse gamma	1.000	0.300	0.655
Structural budget balance	Inverse gamma	1.000	0.300	0.143
NIIP	Inverse gamma	1.000	0.300	0.156