Market conventions for ZARONIA-linked money market instruments [*Working Draft*]

prepared by

The Market Practitioners Group's Cash Market Workstream

SOUTH AFRICAN RESERVE BANK

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1. Background

The highly publicised irregularities relating to the production of interbank offered rates (IBORs) in 2012 — see, for example, [Hou and Skeie, 2014] — initiated a global regulatory response to reform major interest rate benchmarks. The use of IBORs in financial markets has subsequently reduced substantially in favour of more robust alternative reference rates (ARRs), namely overnight reference rates (ONRRs) that are *near risk-free*.

South Africa has also embarked on the transition journey with the release of the consultative paper [SARB, 2018] prepared by the South African Reserve Bank (SARB), which detailed its initial proposal to reform domestic benchmark and reference rates. The SARB subsequently formed the Market Practitioners Group (MPG) in 2019 to manage the process of adoption and transition to the new interest rate dispensation. The MPG is a joint public and private sector body, comprising representatives from the SARB, the Financial Sector Conduct Authority (FSCA) and senior professionals from a variety of market interest groups active in the domestic capital market.

2. Existing and alternative reference rates

The traditional suite of benchmark rates in South Africa consists of a set of Johannesburg Interbank Average Rates (Jibars), quoted on the following tenors 1-, 3-, 6-, 9- and 12-month, see [SARB-MMRR, 2021]. Currently, the 3-month Jibar rate is the most commonly used benchmark rate for financial instruments with floating interest rate exposure that are denominated in South African rand (ZAR). Like IBORs, Jibar lacks the primary market activity that puts it at risk of being not representative of the underlying market it is meant to measure. In 2020, the Risk-Free Reference Rate Workstream (RFRWS) published its recommended properties for a viable replacement reference rate for Jibar — see [SARB-RFRWS, 2020]. The RFRWS's recommendations are summarised as follows:

Table 1: RFRWS's recommended properties for a viable replacement reference rate.

Theme	General feature	Definition	Importance
Integrity	Reliability	*Proper governance and administration to safeguard against manipulation or error.*	*Market integrity and functioning.*
	Robustness	*Clear rules for reference rate production, including transparent and well-known fallbacks in periods of market stress.*	*Availability and usability in times of market stress.*
	Representative	*Rate drawn from a representative sample of transactions from the market in question.*	*Correct pricing basis.*
Efficacy	Frequency	*Rates calculated daily to facilitate market functioning.*	*Pricing of new contracts, mark-to-market valuation.*
	Availability	*Published on dedicated sites.*	*Verification of contracts.*
	Transmission	*Market operations ensure functioning markets, liquidity and price transmission.*	*Monetary policy and financial stability objectives.*
Appropriate	Choice	*Provide a suite of reference rates for different applications.*	*Clear distinction between risk-free and risky secured or unsecured transactions.*

Taking into account the above recommendations, the MPG has designated the South African Overnight Index Average (ZARONIA) rate as the preferred successor rate to replace the set of Jibars and the South African Futures Exchange overnight rate (SAFEX ON). The conceptual design of ZARONIA was rigorously tested, using bona fide transactions data to ensure that it is reliable, robust and sufficiently stable — see [SARB, 2020] and [SARB, 2021] for more information. The designation of ZARONIA as the preferred successor rate forms part of a larger transition roadmap which includes establishing a successor rate, adoption of the successor rate in both derivatives and cash markets, transitioning legacy contracts and eventual cessation of Jibar. See the SARB MPG's webpage for a snapshot of the transition roadmap. A detailed technical specification of the construction of ZARONIA is provided in [SARB, 2020].

It should be noted that provision is made in the above-mentioned document for error-handling — under normal circumstances, ZARONIA will be published at 10:00 South African Standard Time (SAST). Should an error in the calculation of ZARONIA be discovered after publication, and if such an error pertains to the calculation process or to the transaction data supplied to the SARB, the ZARONIA rate may be republished at any time before 12:00 SAST. For the purposes of this document and recommendations herein, references to ZARONIA refer to the final republished rate if applicable.

The key difference between the set of Jibars and ZARONIA is that the former are *forward-looking* term-based reference rates (TBRRs) derived from indicative quote data, while the latter is a *backward-looking* ONRR since it is derived from actual transaction data and therefore only available *after-the-fact* or *in-arrears*. Under the ONRR regime, a term rate may be constructed by suitably averaging[1] the respective ONRR over the term rate's specific accrual period. This results in a backward-looking term rate that will, also at best, only be known at the end of the respective term rate's accrual period. This feature is directly attributable to the time required to collate the underlying transaction data that is used in the computation of the ONRR. These nuances are best demonstrated via a practical example — this is shown in Appendix A.

Since ZARONIA is a backward-looking overnight rate with a one business day publication lag, and cash instruments pay periodic interest cash flows, the new and key market conventions that are proposed in this paper are focused on standard methods to use an ONRR to compute term rates that define *in-arrears* interest cash flows.

3. Cash Market Workstream mandate

The MPG relies on dedicated workstreams to carry out its objectives. The workstreams provide technical input and recommendations to the MPG on specific issues that are relevant to the transition from Jibar. Members of these workstreams are drawn from a diverse set of market practitioners with requisite knowledge and expertise to give effect to the mandate of the MPG as well as shape industry opinions on the reform agenda.[2]

The Cash Market Workstream (CMW), constituted in 2021, with a mandate to:

- *Overcome barriers that the cash market may face using ZARONIA for money market securities, most notably ZARONIA-linked floating rate money market instruments (FRMMIs) that will replace the analogous set of Jibar-linked FRMMIs.*
- *Apply current market conventions and methods to inform a target product set of conventions in line with market standards.*
- *Highlight the operational and implementation requirements to adopt a risk-free rate (RFR).*
- *Determine a set of conventions with which to meet industry needs post transition.*

Furthermore, the CMW is tasked with making recommendations on the development of a forward-looking term rate. In line with global developments, it is envisaged that such a term rate will likely be determined using derivatives market activity that will be based on ZARONIA, and hence, it may take some time before a viable term rate is proposed. Consequently, this paper does not cover forward-looking term ZARONIA. Market participants are strongly encouraged to consider the application of backward-looking, compounded ZARONIA rates as proposed in this document to the greatest extent possible. Market developments towards the establishment

[1] Either *arithmetic*, which implies simple interest, or *geometric*, which implies compound interest — see subsection 7.4. for more detail.
[2] For more information, please refer to: SARB Market Practitioners Group.

of a forward-looking term ZARONIA will be published in due course. This should not delay transitioning in the market.

4. Problem statement

FRMMIs are primarily issued by banks and corporates and can be linked to both Jibar and the prime rate. Since these FRMMIs are similar to negotiable certificates of deposit (NCDs) apart from the floating rate feature, the generic version of this particular instrument is referred to as a floating rate negotiable certificate of deposit (FRNCD) in all that follows. These issuances represent a large source of deposits for the banking system and a source of investment return for money market and income funds. Based on the information of active Strate International Securities Identification Numbers (ISINs) as at June 2023, there are R222 billion and R119 billion of outstanding Jibar- and prime-linked issuances respectively.

For the cash market to function efficiently, market participants need a standard set of conventions that underpin the market, including the funding, investing, administering and settling of securities. The CMW embarked upon a process to formulate a set of recommended conventions, the results of which are presented in this white paper together with reasons for the selections. These recommendations should serve as a guideline for the specification of all floating rate money market securities, most notably FRNCDs, that reference the new ARR as well as inform the conventions for on-screen quotes or those received via interbank broking agents.

It should be noted that these recommendations are not intended to prescribe, mandate, or limit the ways in which market participants may transact to satisfy their bespoke needs and clients' requirements.

Please note that this paper does not cover a forward-looking term ZARONIA.

5. Design principles

The CMW was guided by a set of principles that helped determine conventions under which the money market could function using the successor rate. These principles ensure that conventions support market functioning while aligning with the benchmark's characteristics. These principles may be described as follows:

Price transmission

- *Reflect relevant interest rate conditions for respective interest accrual periods.*
- *Avoid non-representative samples.*
- *Accurately reflect the time value of money.*

Market functioning

- *Align with established market conventions and practices (both local and international) where possible, while catering for specific nuances (e.g. business day, day-count, payment lags, etc.).*
- *Ensure that the requirements of major exchanges and associations are observed and accommodated, where possible (e.g. settlement requirements).*
- *Enable comparison in transactions through standardisation, particularly margins are like-for-like.*
- *Aid in the functioning of primary and secondary markets, particularly known settlements.*

Operational

- *Alleviate administrative burdens.*

- *Enable sufficient time for confirmations and payment instructions.*
- *Ensure interest accrual periods and payment dated are cognisant of non-business days.*
- *Accommodate users over different time zones.*

Alignment

- *Avoid unnecessary fragmentation.*
- *Economic benefits should be consistent with deposits, repurchase agreements (repos) and derivative markets.*
- *Easily align derivative market hedges, to avoid potential basis risk and hedging costs.*
- *Hedge effectiveness could be impacted if not closely coupled.*

6. Convention recommendations

The main objective of this section is to recommend conventions for FRMMIs that reference ZARONIA. Unlike the current suite of Jibars, it should be understood that ZARONIA and its implied term rate are near risk-free (i.e. they will contain minimal credit and funding spreads, if any.).

It must be emphasised that the recommendations made within Table 2 should not preclude any money market participant from negotiating a more bespoke instrument to suit individual requirements.

Table 2: Floating rate money market instrument conventions.

Feature	Recommended convention	Comment	Reference
Coupon period	$\leq$ 12M	*Suggested standard coupon or accrual period tenors: 1M, 3M, 6M and 12M.*	7.1.
Business day calendar	ZAJO	*As published by the relevant providers, in accordance with the Public Holidays Act 36 [RSA, of 1994].*	7.2.
Settlement lag	0 bd	*A primary market feature, which defines the issue and settlement date for a newly issued FRNCD.*	7.2.
Business day convention	Modified Following	*Applied in accrual period date generation.*	7.2.
Coupon period date generation	Backward (EOM)	*Unadjusted backward generation from roll-day plus EOM, then adjusted by Modified Following.*	7.2.
Non-standard first period	Short-stub	*In line with international conventions.*	7.2.
Accrual day count convention	ACT/365 Fixed	*Used for the calculation of floating coupons.*	7.2.
Floating reference rate	ZARONIA	*As recommended by the SARB's MPG.*	7.3.
Publication/ Calculation lag	1 bd	*As indicated by the SARB. Calculated with the sub-accrual period start date as the anchor date.*	7.3.
Publication time	10:00	*If no errors, then ZARONIA will be published at 10:00 else publication is at 12:00, as indicated by the SARB.*	7.3.
ACFR calculation	Compounded, 1 bd lookback without obs shift,	*Backward-looking using a 1-business day lookback without observation shift period fixing adjustment to resolve ZARONIA publication lag and settlement issues. The resultant rate is commonly referred to as the cumulative compounded rate (CCR).*	7.4., 7.5.
Payment lag	0 bd	*Calculated with the last publication/calculation date within the respective coupon period as the anchor date.*	7.5.
ACFR convention	Simple, 6 decimal places	*Or 4 decimal places in % format.*	7.5.
Spread	Simple, additive post compounding	*Fixed simple rate added to the CCR for general floating coupon calculation, if necessary.*	7.5.
Coupon rounding	2 decimal places	*Or to the nearest ZAc.*	7.5.

7. Floating rate money market instrument definition and conventions

A ZARONIA-linked FRMMI is theoretically and structurally equivalent to a Jibar-linked FRMMI, except that the series of floating coupons are determined by a *forward-looking* TBRR in the latter and by an averaged *backward-looking* ONRR in the former. The computational mechanisms beneath the floating coupons are therefore fundamentally different between ZARONIA- and Jibar-linked FRMMIs. In this section, the contractual definition of a general FRMMI is first presented using mathematical notation. A general FRMMI may be theoretically defined via the specification of the following contractual features:

i. nominal, tenor and coupon period;

ii. settlement, coupon, accrual period and year fractions;

iii. floating reference rates, publication and calculation lags;

iv. averaging, lockout and lookback with/without observation shift periods; and

v. floating coupon calculations and payment lags.

Specifications are then discussed for each contractual feature and a specific convention is recommended.

7.1. Nominal, tenor and coupon period

This category constitutes standard features, which are specified as follows:

- **FRMMI nominal**: The nominal or notional value of the FRMMI is denoted by N.
- **FRMMI tenor**: The tenor of the FRMMI may be x_m-months or x_y-years, and is denoted by x_mM or x_yY respectively, with $x_m = 12x_y$. The parameter x_y may be a whole number or a fraction, for example, using standard naming conventions, a 5.5Y FRMMI is equivalent to a 66M FRMMI.
- **Coupon period**: The length of each coupon period may be z_m-months or z_y-years, and is denoted by z_mM or z_yY respectively, with $z_m = 12z_y$. The parameter z_y may be a whole number or a fraction.
- **Number of coupon periods**: The number of coupon periods is denoted by n, with $n = x_m/z_m$, assuming here that n is a whole number, or equivalently, that there are n coupon periods of equal length. The case where n is not a whole number will be considered in the next subsection.

Market convention considerations

There are no material market convention considerations for this category. Rather, parameter choices are made that align with international standards, best practices and market participants' practical requirements.

Recommendations

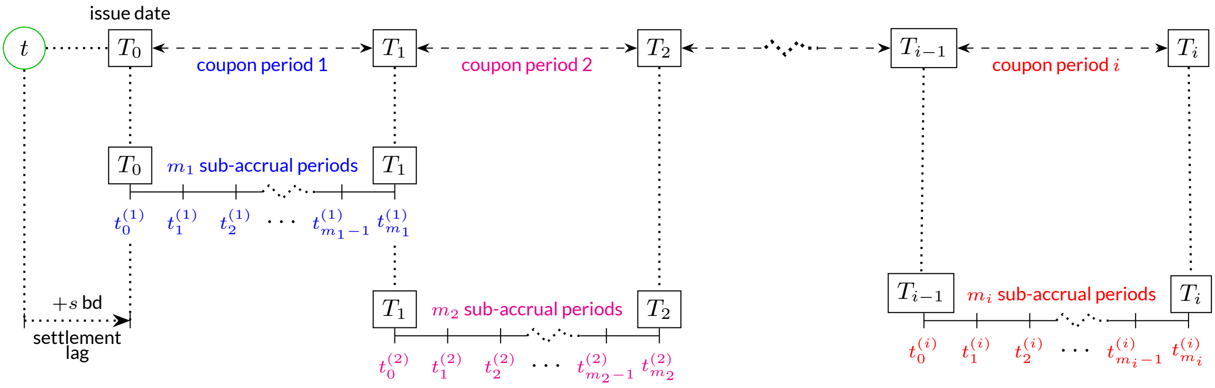
The only parameter that requires a recommendation here is the length of the coupon period for the FRMMI, with the following suggestion:

- **Coupon period**: z_mM, for all $z_m \in \{1,3,6,12\}$.

7.2. Settlement, coupon, accrual period and year fractions

This category details the specification of all the key contractual dates that relate to interest accrual and the calculations thereof, ultimately enabling the calculation of floating coupons. Figure 1 below depicts all of the dates and variables that are defined in this subsection.

Figure 1: Settlement, coupon and accrual period dates.



The following are the key features that constitute this category:

- **Transaction date**: The transaction date for a newly issued FRMMI, in the primary market, is denoted by t.
- **Settlement lag and date**: In the primary market, the face value, nominal or initial consideration of the FRMMI is paid by the buyer to the issuer on date

$$T_0 = t + s \text{ bd} ,$$

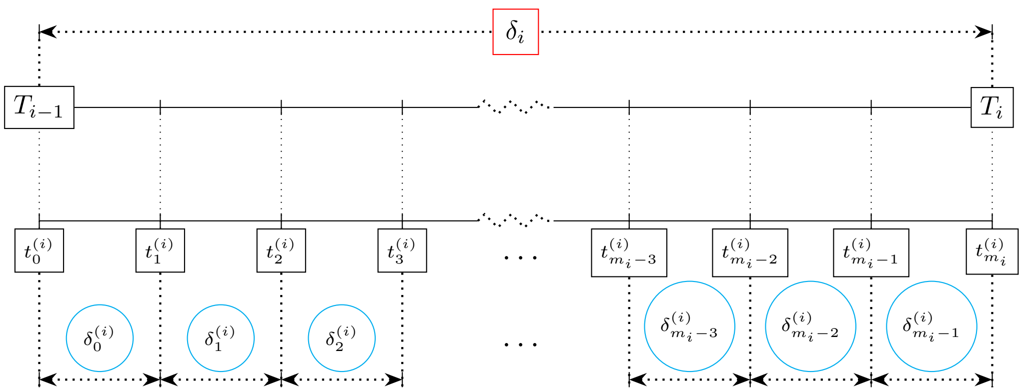
  which is referred to as the settlement or issue date, and is also equal to $t_0^{(1)}$, defined below. The settlement lag is denoted here by s and is quantified in valid business days (bd).
- **Coupon dates**: The set of dates denoted by $\{T_1, T_2, \ldots, T_n\}$ are assumed to denote the full set of coupon dates associated with the general FRMMI under consideration.
- **Interest accrual period dates**: Valid business dates within the i-th coupon period $[T_{i-1}, T_i]$ is denoted by the set:

$$\left\{ t_0^{(i)}, t_1^{(i)}, \ldots, t_{m_i-1}^{(i)}, t_{m_i}^{(i)} \right\} ,$$

  for $i \in \{1, 2, \ldots, n\}$ (i.e. it is assumed that $T_{i-1} = t_0^{(i)}$, $T_i = t_{m_i}^{(i)}$ and that the i-th interest accrual period constitutes $m_i + 1$ valid bd, or m_i overnight interest accrual sub-periods).

Given the key contractual dates defined above, the next feature that requires definition is the method for calculating the interest accrual year fractions. This, in turn, enables the ultimate computation of floating coupons. Figure 2 below depicts all the notations that are used to define the key interest accrual period year fractions.

Figure 2: The interest accrual year fraction for the i-th coupon period $[T_{i-1}, T_i]$.



The relevant accrual and sub-accrual period year fractions are defined as follows:

- **Coupon and interest accrual period year fractions**: The interest accrual year fraction for the i-th coupon period $[T_{i-1}, T_i]$ is denoted by

$$\delta_i := \sum_{j=0}^{m_i-1} \delta_j^{(i)} ,$$

where $\delta_j^{(i)}$ denotes the interest accrual year fraction for the overnight interest sub-accrual period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$, for $j \in \{0, 1, \ldots, m_i - 1\}$ and $i \in \{1, 2, \ldots, n\}$.

Market convention considerations

The practical generation of the dates defined above requires:

i. an official and internationally utilised *South African calendar*;

ii. a choice of *business day convention*; and

iii. a *coupon period date generation algorithm*.

The appropriate calendar is sourced or referenced from a relevant government authority or an internationally recognised publisher of financial trading and settlement calendars.

A business day convention specifies an algorithm to adjust a date when that date is not a valid business day (i.e. the date is a weekend or public holiday), subject to a specific reference calendar. There are six business day convention algorithms that are prominent in international interest rate derivative markets, namely, *Following*, *Preceding*, *Modified Following*, *Modified Preceding*, *Modified Following Bi-Monthly* and *End-of-Month* (EOM).

Given a FRMMI with tenor x_yY, coupon period z_mM, n coupon periods and issue date T_0, the standard interest rate swap (IRS) accrual period date *backward* generation algorithm may be utilised as follows:

- **Step 1** – calculate the *roll-day*, denoted here by U_n, by adding the FRMMI tenor to the issue date:

$$U_n := T_0 + x_y\text{Y} ,$$

which is the unadjusted (i.e. not adjusted by a business day convention) FRMMI maturity date.

- **Step 2** – sequentially subtract the coupon period from the roll-day:

$$U_i := U_n - (n - i)z_m\text{M} ,$$

for $i \in \{n-1, n-2, \ldots, 1\}$, which creates the set of unadjusted coupon period end dates.

- **Step 3** – adjust all the dates from step 1 and 2 as follows:

$$T_i := \beta(U_i) ,$$

for $i \in \{1, 2, \ldots, n\}$, where $\beta(\cdot)$ denotes a function that implements one of the aforementioned business day convention algorithms, which yields all of the required coupon period dates.

The *backward (EOM)* algorithm is identical to the above, except that the EOM algorithm is applied to each unadjusted date in steps 1 and 2. If n is not a whole number, then an additional step before step 1 is required:

- **Step 0** — if n is not a whole number, then set

$$n = \lceil x_m/z_m \rceil = \lceil 12x_y/z_m \rceil ,$$

(i.e. round-up, for an initial period with tenor shorter than zM, referred to as a *short-stub*), or set

$$n = \lfloor x_m/z_m \rfloor = \lfloor 12x_y/z_m \rfloor ,$$

(i.e. round-down, for an initial period with tenor longer than zM, referred to as a *long-stub*).

This date generation algorithm ensures that the non-standard coupon period is the first one. This ensures that once that period has passed, the FRMMI will have the same coupon dates as a directly comparable newly issued standard FRMMI. If the non-standard period was the last period, these FRMMIs will remain non-standard.

The practical calculation of the length of interest accrual periods requires a choice of day-count convention.

A day-count convention is a standardised methodology for calculating the number of days between two dates and then converting this count into a standardised year fraction. There are seven day-count convention methodologies that are prominent in international interest rate financial markets, viz., *30/360*, *30/360 US*, *30E/360*, *ACT/360*, *ACT/365 Fixed*, *ACT/ACT ISDA* and *Business/252*.

Recommendations

The following conventions are recommended:

- **Calendar**: ZAJO as published by a relevant provider that adheres to the Public Holidays Act 36 [RSA, of 1994].
- **Settlement lag**: $s = 0$ is the suggested lag in business days.
- **Business day convention**: Modified Following.
- **Coupon period date generation**: Backward (EOM) algorithm to determine start and end dates. The sub-accrual periods may then be identified using the ZAJO calendar.
- **Number of coupon periods**: If n is not a whole number then round-up (i.e. set $n = \lceil x_m/z_m \rceil$, which will create an initial period that is a short-stub).
- **Day-count convention**: ACT/365 Fixed.

7.3. Floating reference rates, publication and calculation lags

The fundamental difference between ZARONIA- and Jibar-linked NCDs arises from the specification of the floating coupons, with the latter using TBRRs and the former using ONRRs. This subsection introduces the new floating reference rates, based on ONRRs, in a fairly general manner, highlighting nuances in the publication of such a rate and its use in calculations, all of which have a material impact on the eventual computation of floating coupons. Figure 3 below depicts all the notations that are utilised in this subsection.

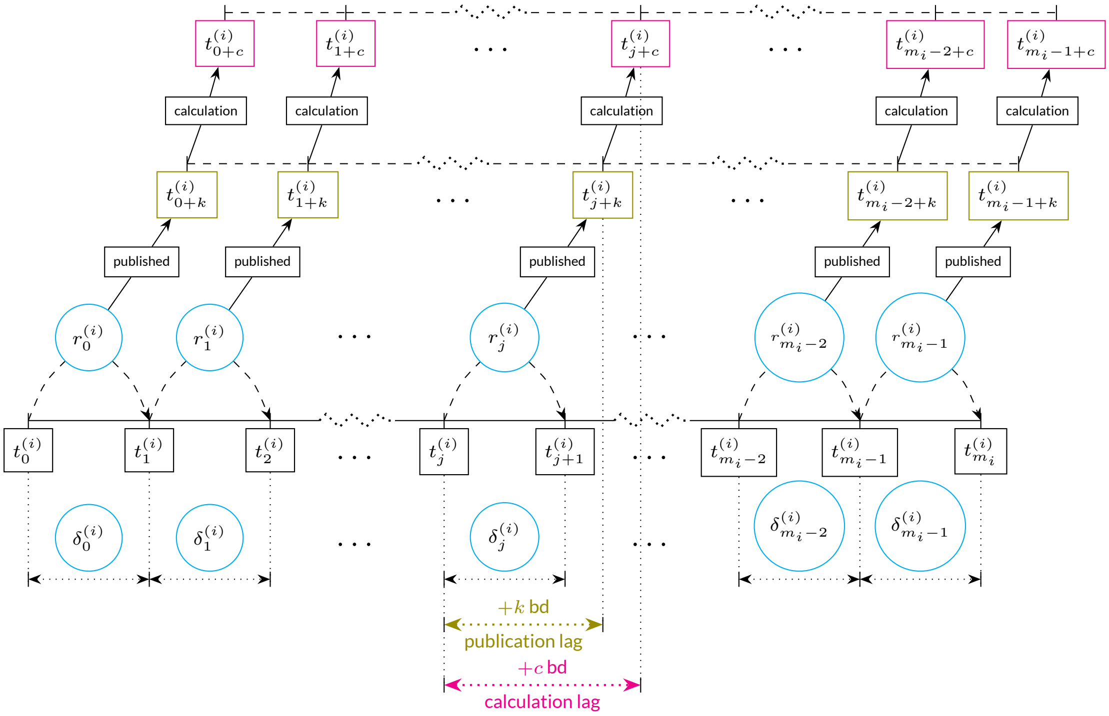
The key feature here is the ONRR, which is the floating reference rate. The definition of the ONRR enables the description of the publication and calculation lag features. These definitions and descriptions are provided below:

- **Floating reference rate**: The floating reference rate is an ONRR that is applicable over $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$, the arbitrary interest accrual sub-period which will always have a tenor of one business day. This ONRR is denoted by the annualised simple rate $r_j^{(i)}$, for $j \in \{1, 2, \ldots, m_i - 1\}$ and $i \in \{1, 2, \ldots, n\}$. The associated overnight capitalisation factor associated with this arbitrary ONRR is denoted and defined as

  $$C_j^{(i)} := 1 + r_j^{(i)} \delta_j^{(i)} ,$$

  again for $j \in \{1, 2, \ldots, m_i - 1\}$ and $i \in \{1, 2, \ldots, n\}$. At the surface, the ONRR defined above is fairly straightforward. However, one of the main differences between the TBRR and ONRR market microstructures is that the former is based on *quoted rates*, while the latter is based on *transacted rates*. Reference rates derived from quoted rates may therefore be calculated and observed *in-advance*, while those derived from transacted rates may only be calculated and observed *in-arrears*, at best. This nuance necessitates the definition of a feature called a *publication lag*, which is explained next.

Figure 3: The floating ONRRs within $[T_{i-1}, T_i]$, with the lags depicted for the j-th ONRR $r_j^{(i)}$.



- **Publication lag**: While the arbitrary ONRR, $r_j^{(i)}$, has an accrual period that starts on $t_j^{(i)}$, the *calculation agent* will only be able to observe relevant transactions during day $t_j^{(i)}$, or mathematically over the period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right)$ and therefore, the earliest that the agent could calculate the relevant ONRR will be at the end of day $t_j^{(i)}$. This means that the rate will be available for use on day $t_{j+1}^{(i)}$. However, operational issues and inefficiencies (potential or otherwise) may preclude the calculation agent from publishing the ONRR on day $t_{j+1}^{(i)}$ consistently. The calculation agent may therefore choose to be prudent and specify a publication lag that is greater than one business day after $t_j^{(i)}$. This publication lag feature is captured here via the date

$$t_{j+k}^{(i)} \geq t_{j+1}^{(i)} ,$$

where k denotes the publication lag and is quantified in valid business days (bd).

- **Calculation lag**: A user of the arbitrary ONRR, $r_j^{(i)}$, may prefer to be more prudent than the calculation agent, for their own operational reasons and add a lag of their own when using the ONRR for interest accrual calculation purposes. This calculation lag feature is captured here via the date $t_{j+c}^{(i)} \geq t_{j+k}^{(i)}$, where c denotes the calculation lag and is quantified in valid bd.

Market convention considerations

Recommendations

The following conventions are recommended:

- **Floating reference rate**: ZARONIA.
- **Publication/Calculation lag**: The SARB's MPG recommends that the calculation lag be set equal to the SARB's chosen publication lag (i.e. $c = k$). Furthermore, the SARB has indicated that the publication lag will be one business day (i.e. $k = 1$), with the standard publication time being 10:00 SAST if there are no errors, or 12:00 SAST if there are errors that warrant republication. For more detailed information on the computation and publication processes, please refer to [SARB, 2020].

7.4. Averaging, lockout and lookback with/without observation shift periods

Another nuance that arises due to the use of floating ONRRs, as opposed to TBRRs, in the specification of FRMMIs, is the definition of the *annualised cumulative floating rate* (ACFR) for a given full accrual period and based on the floating ONRRs. This requires the notion of *averaging* the ONRRs, for which there are two alternatives, the use of which result in the following types of ACFRs:

- **Simple ACFR**: Based on an *arithmetic average* that is weighted by the length of each sub-accrual period, the simple ACFR for the arbitrary i-th accrual period is denoted and calculated as $F_{(i)} := \frac{1}{\delta_i}\sum_{j=0}^{m_i-1} r_j^{(i)}\delta_j^{(i)}$, where $\delta_j^{(i)}$ and δ_i is defined in subsection 7.2., and $r_j^{(i)}$ is defined in subsection 7.3..

- **Compounded ACFR**: Splitting the i-th accrual period into m_i sub-accrual periods of equal length Δ_i, it is possible to define the *nominal annual compounded* m_i-times rate

$$f_{(i)} := \frac{1}{\Delta_i}\left[\left(\prod_{j=0}^{m_i-1}\left[1 + r_j^{(i)}\delta_j^{(i)}\right]\right)^{1/m_i} - 1\right] ,$$

which is based on a *geometric average*, with $\Delta_i := \delta_i/m_i$. Then, the following equation

$$F_{(i)} := \frac{1}{\delta_i}\left(\left[1 + f_{(i)}\Delta_i\right]^{m_i} - 1\right) = \frac{1}{\delta_i}\left(\prod_{j=0}^{m_i-1}\left[1 + r_j^{(i)}\delta_j^{(i)}\right] - 1\right) ,$$

yields the compounded ACFR that is applicable for the i-th accrual period.

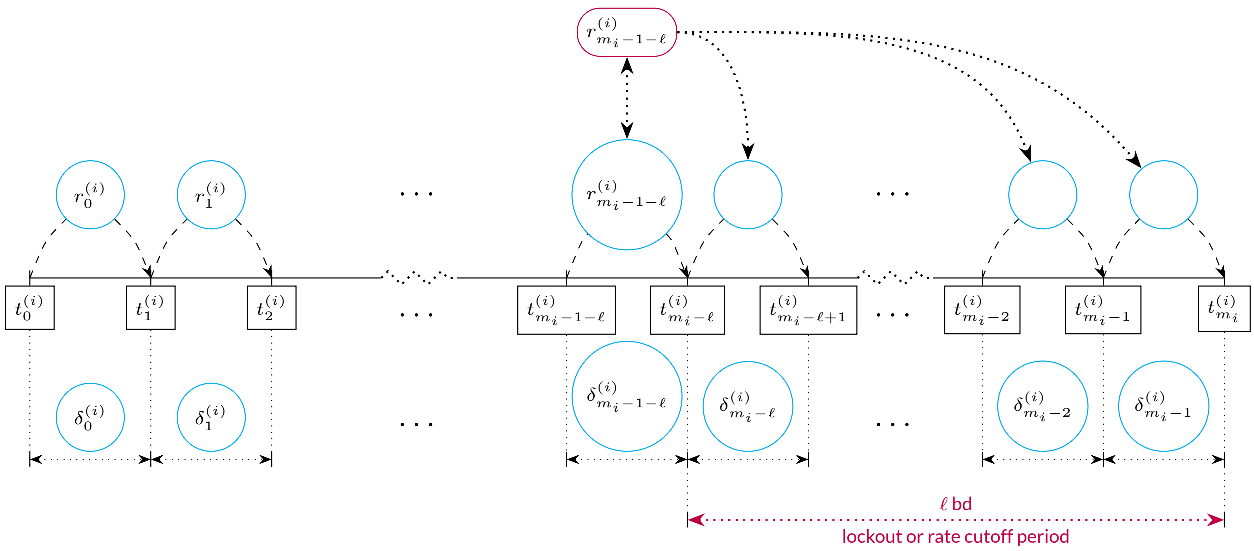
The publication and calculation lags, defined in subsection 7.3., create non-intuitive payment or settlement issues, since users of the floating reference rate may only know interest cash flows *post in-arrears*. Even in-arrears knowledge of the interest cash flow may be problematic, as same-day settlement may not be possible. In order to resolve these practical timing issues, market practitioners have proposed the following fixing adjustments:

- **Lockout/Rate cutoff period**: Given a *lockout* or *rate cutoff period* value equal to ℓ business days means that

$$r_j^{(i)} = r_{m_i-1-\ell}^{(i)} ,$$

for all $j \in \{m_i-\ell, m_i-\ell+1, \ldots, m_i-1\}$. Figure 4 below depicts the practical implications of this adjustment.

Figure 4: A depiction of the lockout or rate cutoff period adjustment.

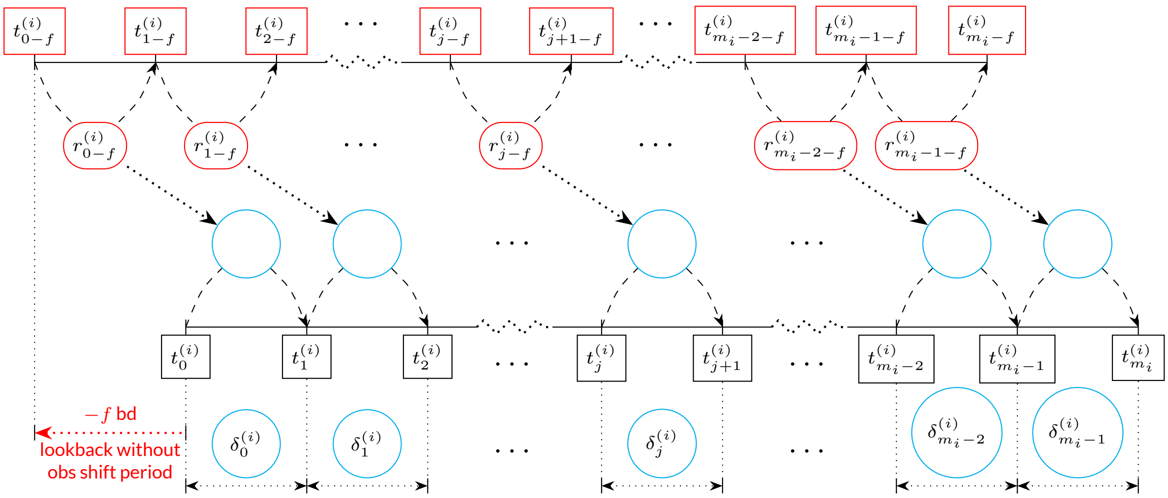


- **Lookback without observation shift period**: Consider the sub-accrual period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$ and a *lookback without observation shift period* value equal to f business days. Then, the capitalisation factor for this period is calculated as

$$C_j^{(i)} = 1 + r_{j-f}^{(i)}\, \delta_j^{(i)} \;,$$

which will be calculable at date $t_{j-f+k}^{(i)}$, where k is the publication lag. Therefore, one can compute interest cash flows in-advance if $f = k$. Figure 5 below depicts the practical implications of this adjustment.

Figure 5: A depiction of the lookback without observation shift period adjustment.

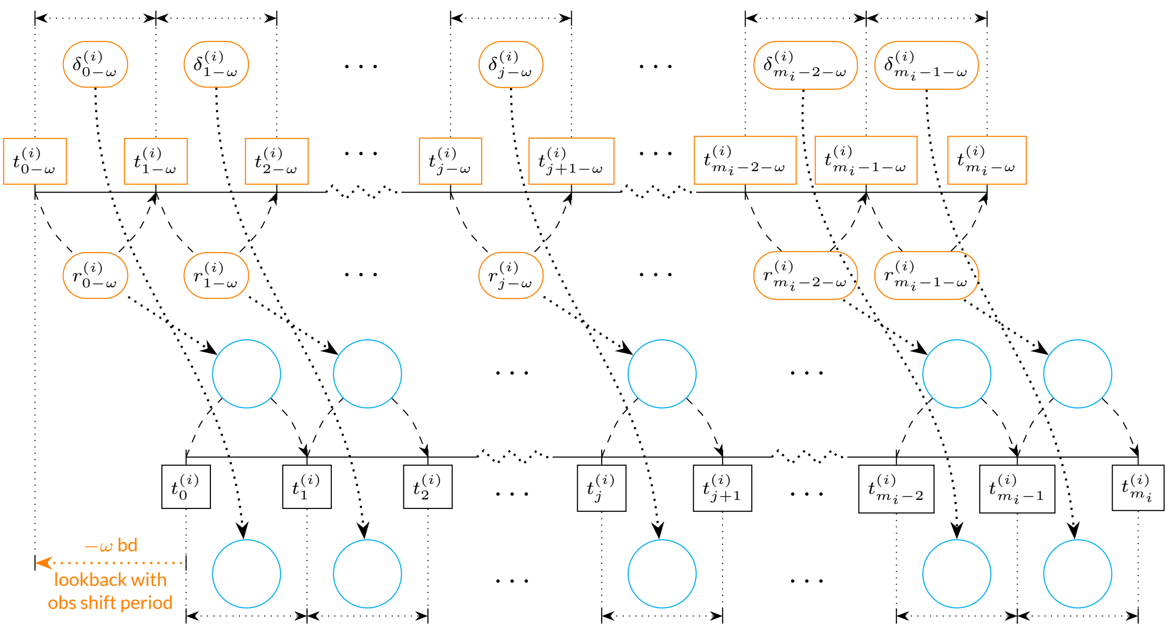


- **Lookback with observation shift period**: Consider the sub-accrual period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$ and a *lookback with observation shift period* value equal to ω business days, then the capitalisation factor for this period is calculated as

$$C_j^{(i)} = 1 + r_{j-\omega}^{(i)}\, \delta_{j-\omega}^{(i)} = C_{j-\omega}^{(i)} \;,$$

which will be calculable at date $t_{j-\omega+k}^{(i)}$, where k is the publication lag. Therefore, with this adjustment, the capitalisation factor for the past sub-accrual period $\left[t_{j-\omega}^{(i)}, t_{j+1-\omega}^{(i)}\right]$ is used for the actual period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$. Figure 6 below depicts the practical implications of this adjustment.

Figure 6: A depiction of the lookback with observation shift period adjustment.



Market convention considerations

Recommendations

The following conventions are recommended:

- **Averaging**: Geometric averaging, resulting in a compounded ACFR, which enables the correct representation of the time value of money.
- **Lockout/rate cutoff period**: $\ell = 0$.
- **Lookback without observation shift period**: $f = 1$.
- **Lookback with observation shift period**: $\omega = 0$.

In other words, or in summary, the recommendation here is to utilise a compounded ACFR based on a lookback without observation shift period fixing adjustment equal to 1 business day. Take note that this does not enable one to make use of an index for interest cash flow calculations. This means that floating coupons will only be known 1 business day before the end of coupon periods, since the calculation lag is recommended to be set equal to the publication lag, which is 1 business day.

7.5. Floating coupon calculations and payment lags

Having defined the ACFR in the previous subsection, this subsection details the calculation of the floating coupon. All the necessary parameters, variables and equations are depicted in Figure 7 below.

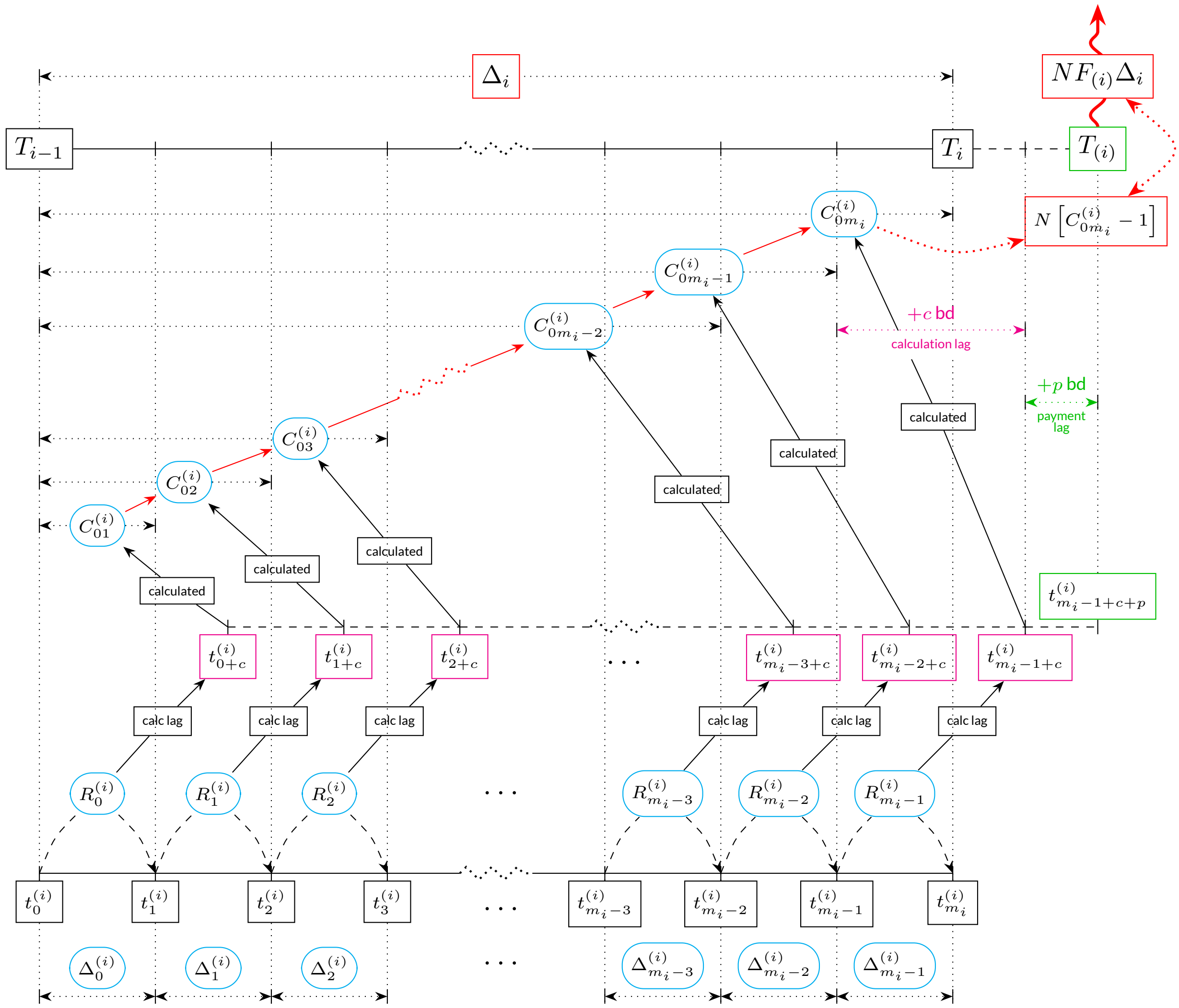
The key quantity here is the computation of the realised capitalisation factor over the coupon period. This enables the computation of the compounded ACFR, the eventual floating coupon and its specific nuances. All these quantities are defined as follows:

- **Adjusted floating reference rate and accrual year fractions**: If one of the fixing adjustments, which have been described in subsection 7.4., has been utilised, then the respective floating ONRRs and their associated accrual year fractions may not align with their natural accrual periods. To cater for this, an *adjusted* ONRR and accrual year fraction has been used in Figure 7 — namely, $R_j^{(i)}$ and $\Delta_j^{(i)}$ with both these quantities applying to the period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$, for $j \in \{0, 1, 2, \ldots, m_i - 1\}$. For example, if the lookback adjustment is applied, then
$$R_j^{(i)} = r_{j-f}^{(i)} \text{ and } \Delta_j^{(i)} = \delta_j^{(i)} ,$$
is applicable to the period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$, for each $j \in \{0, 1, 2, \ldots, m_i - 1\}$ and $i \in \{1, 2, \ldots, n\}$.

- **Realised capitalisation factors**: Given the i-th coupon period and based on the *adjusted* ONRRs within this period, the capitalisation factor that is realised over the sub-accrual period $[t_0^{(i)}, t_h^{(i)}]$ is denoted and defined by
$$C_{0h}^{(i)} := \prod_{j=0}^{h-1} C_j^{(i)} = \prod_{j=0}^{h-1} \left[1 + \Delta_j^{(i)} R_j^{(i)}\right] ,$$
which will be calculable on or before the last calculation date $t_{h-1+c}^{(i)}$, (i.e. taking into account the calculation lag of c bd), for $h \in \{1, 2, \ldots, m_i\}$, depending on the use and choice of the fixing adjustment.

- **Floating rate**: The floating rate for the i-th coupon period may then be denoted and defined as
$$F_{(i)} := \frac{1}{\Delta_i} \left[C_{0m_i}^{(i)} - 1\right] ,$$
where $\Delta_i = \sum_{j=0}^{m_i - 1} \Delta_j^{(i)}$. This rate is the compounded ACFR, and is a *backward-looking* or *realised* term rate implied from the corresponding realised capitalisation factor. It is therefore also only calculable on or before $t_{m_i-1+c}^{(i)}$, for $i \in \{1, 2, \ldots, n\}$, depending on the use and choice of the fixing adjustment.

Figure 7: Floating coupon calculation, without spread, at the arbitrary i-th payment date $T_{(i)}$.



- **Floating coupons (excluding spread):** Given the compounded ACFR, the floating coupon may then be calculated as

$$NF_{(i)}\Delta_i \ ,$$

  which is also only calculable on or before $t^{(i)}_{m_i-1+c}$, depending on the use and choice of the fixing adjustment. Recall that N denotes the swap nominal and $i \in \{1, 2, \ldots, n\}$ denotes the respective coupon period.

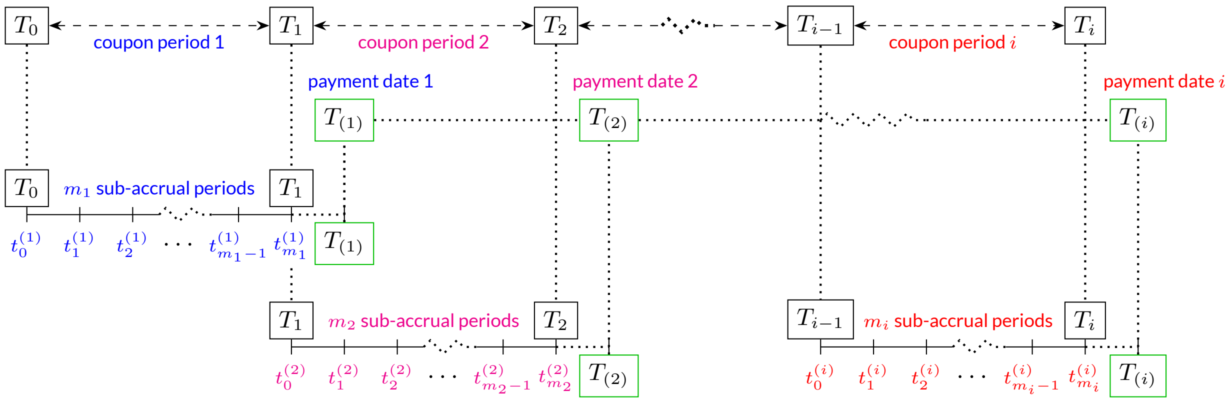
- **Payment lag**: Consider again the arbitrary i-th coupon period. If no fixing adjustment is applied, then the floating coupon is only calculable at date $t^{(i)}_{m_i-1+c}$, which would be the end of the i-th coupon period at best if $c = 1$. This would therefore require same-day settlement, which may not be practically possible. One method to solve this issue is the definition of a *payment lag*, which is captured here via the payment date

$$t^{(i)}_{m_i-1+c+p} := T_{(i)} \ ,$$

  where p denotes the payment lag and is quantified in valid bd. In other words, the payment lag is p bd after the last calculation date. This is the solution that has been applied in the derivatives markets associated with ZARONIA – see [SARB-DWS, 2023] for more detailed information.

These payment dates are depicted in Figure 8 below, which is essentially an update of Figure 1 above excluding the transaction date.

Figure 8: Coupon, accrual period and payment dates.



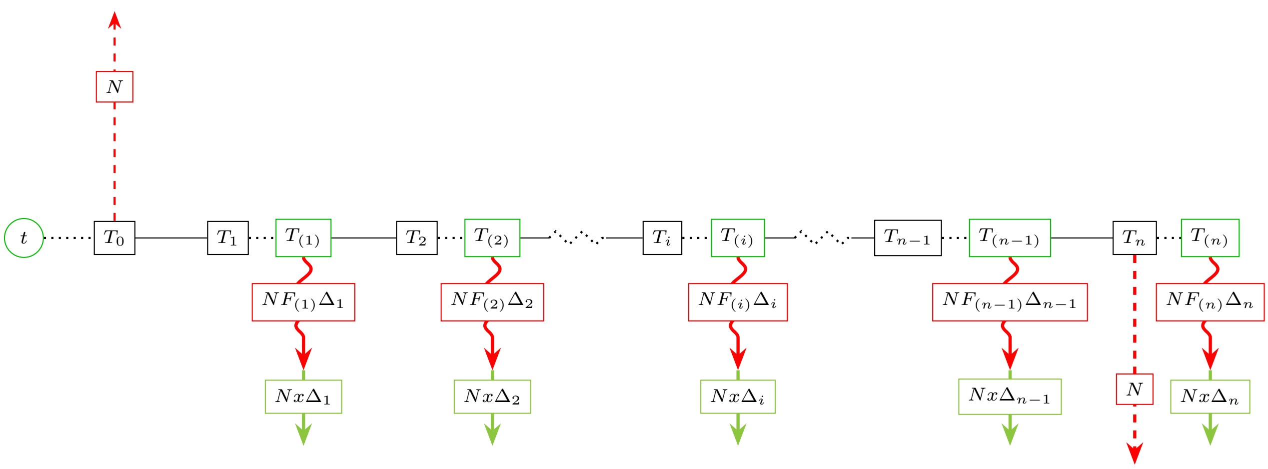
- **Spread**: FRMMIs that are issued by risky entities will require the inclusion of a *fixed spread rate* above the ACFR in the specification of the floating coupon. This is to compensate the buyer for the credit risk exposure of the issuer. Denoting this spread by x, the i-th floating coupon, including the spread, is then calculated as

$$N\left[F_{(i)} + x\right]\Delta_i ,$$

(i.e. the fixed spread rate is an annualised simple rate that is added to the compounded ACFR).

Figure 9 depicts the full set of cash flows associated with a general FRMMI, from the perspective of the issuer.

Figure 9: FRMMI cash flows from the perspective of the issuer.



It is assumed that the issuer receives the nominal on the issue/settlement date, T_0, and pays back the nominal to the holder of the FRMMI at the maturity date, T_n. The set of floating coupons are paid on the corresponding set of payment dates $\{T_{(1)}, T_{(2)}, \ldots, T_{(n)}\}$, as defined above. Observe that if the payment lag feature is not used, then the set of payment dates coincides with the set of coupon dates $\{T_1, T_2, \ldots, T_n\}$.

Market convention considerations

Recommendations

The following conventions are recommended:

- **Payment lag**: $p = 0$ (i.e. each coupon should be settled at the end of its respective coupon period).
- **ACFR convention**: $F_{(i)}$ is an annualised simple interest rate, as defined above, and should be rounded to six decimal places in numerical format, or four decimal places in percentage format. For example, if $F_{(i)} = 0.07123456$, then round to 0.071235 or 7.1235%.
- **Spread**: An annualised simple interest rate that is added to the ACFR defined above (i.e. after geometrically averaging the relevant ONRRs).
- **Coupon rounding**: The floating coupon, $N\left[F_{(i)} + x\right]\Delta_i$, which is denominated in ZAR should be rounded to two decimal places, or to the nearest ZAc.

8. Use of formulae and examples

To demonstrate the recommended conventions for ZARONIA-linked FRMMIs practically, a Microsoft Excel workbook has been created with specific examples, scenarios, actual market data and supporting calculations. A compressed file, ZARONIA_Model_FRMMI.zip, can be found as an accompanying document.

The Excel workbook '*ZARONIA_Model_FRMMI.xlsx*' contains a working model that demonstrates the calculation methodology.

Please note that, in general, the provided model and scenarios highlight features and nuances of the specific conventions that are recommended rather than an exhaustive presentation of all possible conventions.

A further practical guide is included as an accompanying document: ZARONIA_FRMMI_Practical_Guide.zip.

Appendix A Forward- versus backward-looking reference rates

All of the content in this appendix is for descriptive purposes only, and is therefore not directly related to any of the conventions that are suggested or recommended in the rest of the paper.

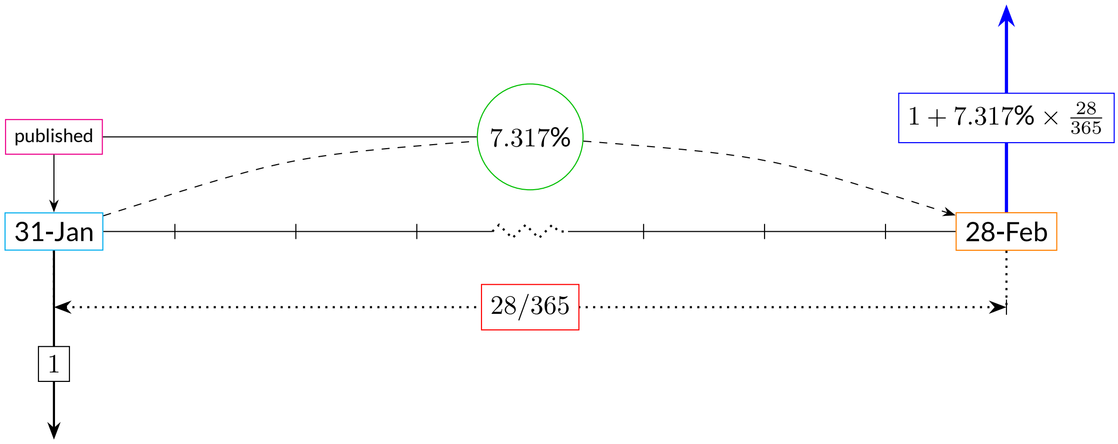
Consider a theoretical 1-month deposit with unit nominal that is initiated on 31-Jan-2023. The maturity date will therefore be 28-Feb-2023. Under the TBRR regime, we assume that 1-month Jibar is the deposit rate, and that the appropriate day-count convention is ACT/365 Fixed.[3] Then, Table 3 and Figure 10 below depict all of the relevant and pertinent features of such a deposit.

Table 3: Relevant data for a 1-month deposit with unit nominal based on 1-month Jibar.

Ref rate	Rate	Accrual period		Pub date	Year frac	1M cap factor
		Start date	End date			
1M Jibar	7.317%	31-Jan	28-Feb	31-Jan	$\frac{28}{365}$	$1 + 7.317\% \times \frac{28}{365}$

Source for 1M Jibar: JSE Client Portal - Downloadable Files

Figure 10: A 1-month deposit with unit nominal based on 1-month Jibar.



Under the ONRR regime, we assume that ZARONIA is the reference rate for the deposit and that the (ACFR)[4] or the *backward-looking term rate* within this context, is determined by compounding the respective set of ZARONIA rates over the relevant 1-month tenor of the deposit. Again, we assume that the appropriate day-count convention is ACT/365. Table 4 and Figure 11 below depict all the relevant and pertinent features of such a deposit.

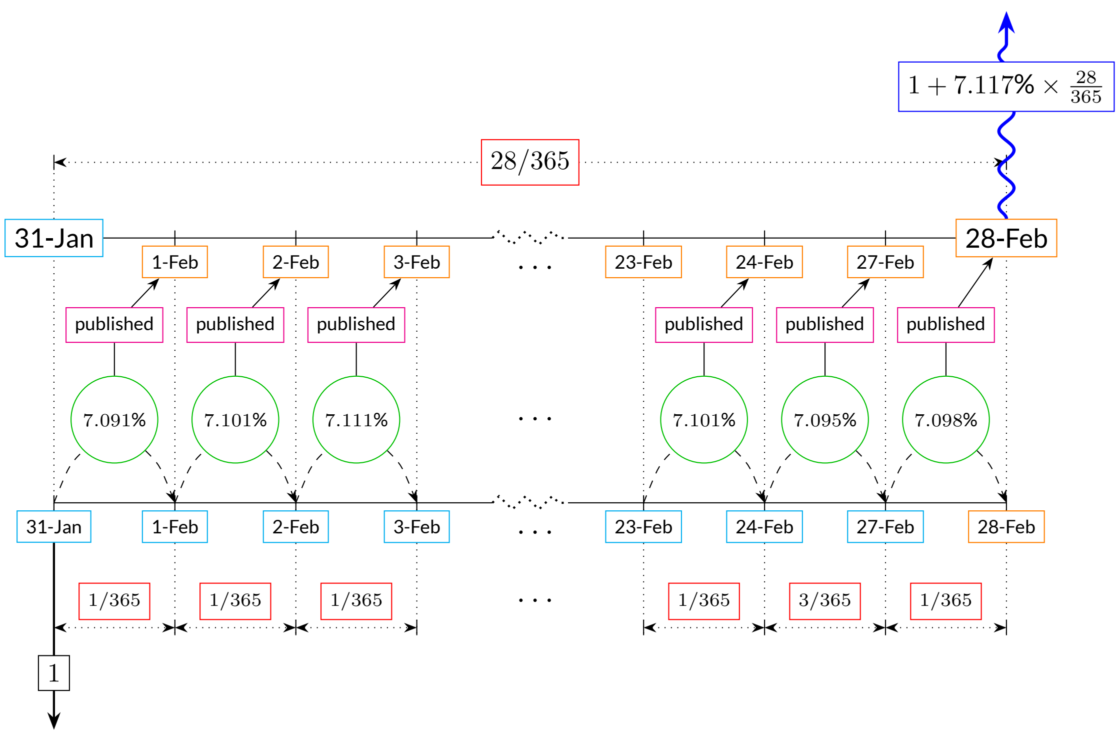
[3] As discussed in subsection 7.2.
[4] As defined in subsection 7.4.

Table 4: Relevant data for a 1-month deposit with unit nominal based on compounded ZARONIA.

Ref rate	Rate	Accrual period		Pub date	Year frac	ON cap factor
		Start date	End date			
ZARONIA	7.092%	31-Jan	1-Feb	1-Feb	$1/365$	$1 + 7.092\% \times \frac{1}{365}$
ZARONIA	7.091%	1-Feb	2-Feb	2-Feb	$1/365$	$1 + 7.091\% \times \frac{1}{365}$
ZARONIA	7.101%	2-Feb	3-Feb	3-Feb	$1/365$	$1 + 7.101\% \times \frac{1}{365}$
ZARONIA	7.111%	3-Feb	6-Feb	6-Feb	$3/365$	$1 + 7.111\% \times \frac{3}{365}$
ZARONIA	7.099%	6-Feb	7-Feb	7-Feb	$1/365$	$1 + 7.099\% \times \frac{1}{365}$
ZARONIA	7.091%	7-Feb	8-Feb	8-Feb	$1/365$	$1 + 7.091\% \times \frac{1}{365}$
ZARONIA	7.087%	8-Feb	9-Feb	9-Feb	$1/365$	$1 + 7.087\% \times \frac{1}{365}$
ZARONIA	7.096%	9-Feb	10-Feb	10-Feb	$1/365$	$1 + 7.096\% \times \frac{1}{365}$
ZARONIA	7.092%	10-Feb	13-Feb	13-Feb	$3/365$	$1 + 7.092\% \times \frac{3}{365}$
ZARONIA	7.098%	13-Feb	14-Feb	14-Feb	$1/365$	$1 + 7.098\% \times \frac{1}{365}$
ZARONIA	7.100%	14-Feb	15-Feb	15-Feb	$1/365$	$1 + 7.100\% \times \frac{1}{365}$
ZARONIA	7.104%	15-Feb	16-Feb	16-Feb	$1/365$	$1 + 7.104\% \times \frac{1}{365}$
ZARONIA	7.097%	16-Feb	17-Feb	17-Feb	$1/365$	$1 + 7.097\% \times \frac{1}{365}$
ZARONIA	7.098%	17-Feb	20-Feb	20-Feb	$3/365$	$1 + 7.098\% \times \frac{3}{365}$
ZARONIA	7.106%	20-Feb	21-Feb	21-Feb	$1/365$	$1 + 7.106\% \times \frac{1}{365}$
ZARONIA	7.096%	21-Feb	22-Feb	22-Feb	$1/365$	$1 + 7.096\% \times \frac{1}{365}$
ZARONIA	7.094%	22-Feb	23-Feb	23-Feb	$1/365$	$1 + 7.094\% \times \frac{1}{365}$
ZARONIA	7.108%	23-Feb	24-Feb	24-Feb	$1/365$	$1 + 7.108\% \times \frac{1}{365}$
ZARONIA	7.101%	24-Feb	27-Feb	27-Feb	$3/365$	$1 + 7.101\% \times \frac{3}{365}$
ZARONIA	7.095%	27-Feb	28-Feb	28-Feb	$1/365$	$1 + 7.095\% \times \frac{1}{365}$
					1M cap factor	$1 + 7.117\% \times \frac{28}{365}$

Source for ZARONIA: SARB - ZARONIA interest rate benchmark webpage

Figure 11: A 1-month deposit with unit nominal based on compounded ZARONIA.



There are two material differences between the two regimes — the first is the calculation of the 1-month capitalisation factor. Under the TBRR regime, the 1-month capitalisation factor is calculated as

$$\left(1 + 7.317\% \times \frac{28}{365}\right) ,$$

while under the ONRR regime, it is calculated as follows:

$$\left(1 + 7.092\% \times \frac{1}{365}\right) \times \left(1 + 7.091\% \times \frac{1}{365}\right) \times \ldots \times \left(1 + 7.101\% \times \frac{3}{365}\right) \times \left(1 + 7.095\% \times \frac{1}{365}\right) , \quad (1)$$

(i.e. by taking the product of the respective set of overnight capitalisation factors). Then the ACFR may be calculated by setting equation (1) equal to $(1 + F \times \frac{28}{365})$, solving for F and rounding to 3 decimal places in the percentage format, which yields $F = 7.117\%$, as shown in Table 4. Economically, the 0.2% difference between 1-month Jibar and the implied ACFR may be attributed to 1-month term credit and liquidity spreads.

The second difference is the dates on which the respective capitalisation factors are calculable and thereby observable. According to current (and prospective[5]) publishing procedures, 1-month Jibar is published at the beginning of its respective accrual period while ZARONIA is and will be published at the end of its respective accrual period. This means that the ACFR will only be calculable and known at the end of its respective accrual period, as is the case in this theoretical example and shown in Table 4 and Figure 11.

[5] As detailed in section 2.

Glossary

List of acronyms

ACFR annualised cumulative floating rate. 8, 14, 16, 17, 18, 19, 20, 22
ARR alternative reference rate. 4, 6

bd business day(s). 8, 10, 13, 16, 17

CCR cumulative compounded rate. 8
CMW Cash Market Workstream. 5, 6

EOM end-of-month (business day convention). 8, 11, 12

FSCA Financial Sector Conduct Authority. 4
FRNCD floating rate negotiable certificate of deposit. 6, 8
FRMMI floating rate money market. 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 18, 19

IBOR interbank offered rates. 4
IRS interest rate swap. 11
ISIN International Securities Identification Number. 6

Jibar Johannesburg Interbank Average Rate. 3, 4, 5, 6, 7, 8, 12, 20, 22

MPG Market Practitioners Group. 4, 5, 8, 13

NCD negotiable certificate of deposit. 6, 12

ONRR overnight reference rate. 4, 5, 8, 12, 13, 14, 16, 19, 20, 22

RFR risk-free rate. 5
RFRWS Risk-Free Reference Rate Workstream. 3, 4
repos repurchase agreement. 7

SAFEX ON South African Futures Exchange overnight rate. 4
SARB South African Reserve Bank. 4, 5, 8, 13
SAST South African Standard Time. 5, 13

TBRR term-based reference rate. 5, 8, 12, 14, 20, 22

ZA Republic of South Africa. 19
ZAJO South African calendar - Johannesburg Financial Center. 8, 12
ZAR South African rand. 4, 19
ZARONIA South African Overnight Index Average. 3, 4, 5, 6, 7, 8, 12, 13, 17, 19, 20, 21, 22

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