Historical estimation of the ZARONIA OIS curve

prepared by
The Market Practitioners Group's Derivatives Workstream

Contents

Figures and tables

List of figures

List of tables

1. Overview

The Derivatives Workstream (DWS), a component of the South African Reserve Bank's (SARB's) Market Practitioners Group (MPG), was tasked with developing historical swap curves based on interest rate swap contracts that reference the South African Rand Overnight Index Average (ZARONIA).[1] Such data have many practical uses, with one of the primary applications being the generation of historical scenarios for Value-at-Risk (VaR) and Expected Shortfall (ES) models. These models are critical for determining the market risk regulatory capital charge for banks authorised to use the "*Internal Model Approach*" to quantify interest rate-centric market risk emanating from their trading activities.

In fulfilling this mandate, DWS members considered several potential methodologies and ultimately settled on a final approach that is both financially intuitive and straightforward to implement, provided the necessary data are available. This document outlines the chosen methodology and presents a sample of the results in a clear, concise format. Section 2. describes the required data, Section 3. summarises the methodology using a pseudo-algorithm and Section 4. explains the structure of the results along with relevant visualisations. Appendix A provides a sample of the estimation result for the first date in the historical dataset, while Appendix B offers a visual overview of how the spread between the 3-month Jibar and ZARONIA evolved over the period considered.

It should be noted that the MPG has chosen to provide the resultant estimated dataset to market participants as a convenient, but optional, risk management resource.

2. Data requirements

The most important data required for this task consist of the set of bootstrapped zero-coupon yield curves, which are published daily by the Johannesburg Stock Exchange's (JSE's) market data division. The available historical data span from early 2005 through to the end of 2024.[2]. This dataset includes three zero-coupon yield curves:

(i) The *nominal swap zero curve*

(ii) The *nominal bond zero curve*

(iii) The *real bond zero curve*

For the purposes of this exercise, only the nominal swap zero curve is relevant. This curve is constructed using short-term deposits (overnight, 1-, and 3-month), forward rate agreements (FRAs), and interest rate swaps (IRSs). The aforementioned derivatives reference the 3-month Johannesburg Interbank Average Rate (Jibar). The JSE's market data division employs a standard bootstrapping algorithm and *monotone preserving interpolation* to derive this curve daily. Further details on the interpolation method can be found in [du Preez, 2011].

Since the estimation methodology hinges on adjusting 3-month Jibar forward rates to estimate corresponding overnight ZARONIA forward rates, it is also necessary to obtain historical data for both the spot (or realised) Jibar and ZARONIA rates. The historical spread between these rates, observed over various time horizons, is used for this adjustment. As a result, a longer time series of rate data is needed compared to the zero curve data, with the relevant period extending from the beginning of 2000 to the end of 2024. While the Jibar rate is readily available for this entire period, the ZARONIA rate is only available from the beginning of 2016 (which can be accessed via the SARB website). For the period from 2000 to 2015, the South African Futures and Options Exchange (SAFEX) overnight rate, as chosen by the DWS, serves as a suitable proxy for the ZARONIA rate.

[1] These interest rate swap contracts are commonly referred to as overnight indexed swap OIS contracts.

[2] It should be noted that the JSE's zero curve dataset is missing data for four historical dates, viz. 24-Dec-07, 02-Jan-09, 24-Nov-11, and 21-May-13. Consequently, the final set of results associated with this document is also missing data for these dates.

3. Methodology

In this section a theoretical overview of the methodology is presented. The following section will illustrate each step with visual results to further enhance the reader's understanding. For any given historical date, denoted as t, the methodology may be summarised as follows:

Step 1: Use the JSE nominal swap zero curve published for date t. The format of this data is shown below.

Table 1: Symbolic representation of the JSE's nominal swap zero curve for a given date t

Tenor date	NACC rate
$t+1$	$r(t, t+1)$
$t+2$	$r(t, t+2)$
$\vdots$	$\vdots$
$t+15000$	$r(t, t+15000)$

In other words, the JSE publishes a bootstrapped zero curve in the form of tenor dates and nominal annual compounded continuously (NACC) rates, with tenors spanning from 1-calendar day to 15000-calendar days. The notation $r(t, t+x)$ refers to the corresponding NACC rate for the accrual period $[t, t+x]$, where x is measured in calendar days.

Step 2: Using the curve established in Step 1, calculate the 3-month Jibar forward rates on a daily basis. The resultant data are structured as depicted below, where bd represents business day and N is the number of business days added to date t to determine the final payment date of the 30-year ZARONIA OIS, following the conventions outlined in [SARB-DWS, 2023].

Table 2: Symbolic representation of calculated 3-month Jibar forward rates at a daily frequency at date t

Reset date	Simple forward rate
t	$f_{3\mathsf{M}}(t)$
$t+1\,\mathsf{bd}$	$f_{3\mathsf{M}}(t+1\,\mathsf{bd})$
$t+2\,\mathsf{bd}$	$f_{3\mathsf{M}}(t+2\,\mathsf{bd})$
$\vdots$	$\vdots$
$t+(N-1)\,\mathsf{bd}$	$f_{3\mathsf{M}}(t+(N-1)\,\mathsf{bd})$

Note that if the final payment date of the 30-year OIS is $t+N$ bd, then the last reset date of the overnight forward rate required is $t+(N-1)$ bd. For any $x \in \{0, 1, 2, \ldots, N-1\}$, the simple 3-month Jibar forward rate is calculated using this formula:

$$f_{3\mathsf{M}}(t+x\,\mathsf{bd}) := \frac{1}{\tau_{3\mathsf{M}}(t,x) - \tau(t,x)} \left(\frac{\exp\left[r\left(t, t_{3\mathsf{M}}(x)\right) \tau_{3\mathsf{M}}(t,x)\right]}{\exp\left[r\left(t, t(x)\right) \tau(t,x)\right]} - 1 \right) ,$$

where we have defined the following new functions:

- $t(x) := t + x\,\mathsf{bd}$, which denotes the reset date; and
- $t_{3\mathsf{M}}(x) := t(x) + 3\mathsf{M}$, which denotes the expiry date of the forward rate.

This means the forward rate covers the accrual period from $t(x)$ to $t_{3\mathsf{M}}(x)$. The accrual year fractions are expressed as $\tau(t,x) := [t(x) - t]/365$ and $\tau_{3\mathsf{M}}(t,x) := [t_{3\mathsf{M}}(x) - t]/365$.

Step 3: Calculate the Jibar-ZARONIA spread on each date u. First, for each date u, compute the spread between the 3-month Jibar and ZARONIA spot rates as follows:

$$s_0(u) := J(u) - Z(u) \ , \tag{1}$$

where $J(u)$ and $Z(u)$ represent the 3-month Jibar and ZARONIA spot rates, respectively. The index u refers to business dates in the set

$$u \in \{t, t-1\,\text{bd}, \ldots, t-Q_y\text{bd}\} \ ,$$

where $y \in \{0\,\text{bd}, 1\text{M}, 3\text{M}, 6\text{M}, 9\text{M}, 1\text{Y}, 2\text{Y}, 3\text{Y}, 4\text{Y}, 5\text{Y}\}$ and Q_y is the exact number of business days that must be subtracted from date t to generate the first business date that is y (months/years) in the past from date t.

Next, calculate the following quantities at date t:

$$s_y(t) := \text{median}\left\{s_0(t-Q_y\text{bd}), s_0(t-(Q_y-1)\text{bd}), \ldots, s_0(t-1\,\text{bd}), s_0(t)\right\} \ , \tag{2}$$

which is simply the median[3] of the Jibar-ZARONIA spreads over the period determined by y. We assume that $s_y(t)$ is the appropriate spread that may be utilised to adjust a 3-month Jibar forward rate (with reset date $t+y$) to obtain an overnight ZARONIA forward rate, using the formula:

$$f_{1\text{bd}}(t+y) := f_{3\text{M}}(t+y) - s_y(t) \ , \tag{3}$$

where $f_{1\text{bd}}(t+y)$ is the overnight ZARONIA forward rate with reset date equal to $t+y$. Note that if $y = 0$bd, then $f_{3\text{M}}(t) = J(t)$ and $f_{1\text{bd}}(t) = Z(t)$ and equations (3) and (1) are consistent. The table below outlines the structure of this forward spread data.

Table 3: Symbolic representation of forward spreads that adjust Jibar into ZARONIA forward rates at date t

Tenor	Tenor (years)	Forward spread
0bd	0	$s_0(t)$
1M	1/12	$s_{1\text{M}}(t)$
3M	1/4	$s_{3\text{M}}(t)$
6M	1/2	$s_{6\text{M}}(t)$
9M	3/4	$s_{9\text{M}}(t)$
1Y	1	$s_{1\text{Y}}(t)$
2Y	2	$s_{2\text{Y}}(t)$
3Y	3	$s_{3\text{Y}}(t)$
4Y	4	$s_{4\text{Y}}(t)$
5Y	5	$s_{5\text{Y}}(t)$

A tenor-based spread ensures that the very short-end of the resultant estimated ZARONIA OIS curve recovers the observed or proxied ZARONIA rate exactly, while also producing non-trivial dynamics for other tenors. As demonstrated in Appendix B, the variation of these spreads reduces with tenor. This is commensurate with academic research, such as the evidence provided in chapter 4 of [Kenyon and Stamm, 2012].

Step 4: To construct an overnight ZARONIA forward rate curve with reset dates covering the range specified in Step 2, viz. $\{t, t+1\,\text{bd}, \ldots, t+(N-1)\,\text{bd}\}$, it is necessary to extend the forward spread data calculated in Step 3 into a continuous function over the domain $[0, ((t+(N-1)\,\text{bd})-t)/365] \approx [0, 30+\epsilon]$, where the quantity ϵ reflects adjustments for business days and day count convention. This extension is achieved as follows:

- Apply linear interpolation over the domain $[0, 5]$.

[3]The median statistic is chosen for the same reasons provided by the International Swaps and Derivatives Association (ISDA) in their specification of fallback rates — see Final Parameters for Benchmark Fallback Adjustments and references therein for further information.

- For values greater than 5-years (i.e. over the interval $(5, 30 + \epsilon]$), use flat extrapolation.

This means that we now have a forward spread function $s_y(t)$ defined over the domain $y \in [0, 30 + \epsilon]$.

Step 5: Apply the relevant forward spreads derived in Step 4 to the forward rates establised in Step 2, using the following approach. Define the overnight ZARONIA forward rate at each reset date as:

$$f_{1\text{bd}}(t + x\,\text{bd}) := f_{3\text{M}}(t + x\,\text{bd}) - s_y(t)\ , \tag{4}$$

where $x \in \{0, 1, 2, \ldots, N-1\}$ and $y = ((t + x\,\text{bd}) - t)/365$. Equation (4) yields a proxy or estimate for the overnight ZARONIA forward rate curve.

Step 6: Using the overnight ZARONIA forward rate curve obtained from Step 5, you can compute the associated discount factors as follows:

$$Z(t, t + x\,\text{bd}) := \prod_{i=0}^{x-1} \left[1 + f_{1\text{bd}}(t + i\,\text{bd})\tau_{1\text{bd}}(t, i)\right]^{-1}\ , \tag{5}$$

for $x \in \{1, 2, \ldots, N\}$, where $\tau_{1\text{bd}}(t, i) := \left[(t + (i+1)\,\text{bd}) - (t + i\,\text{bd})\right]/365$.

Step 7: Using the discount factor curve obtained in Step 6, we can now compute the fair, or par, rates for any linear benchmark instrument. Following the conventions outlined in [SARB-DWS, 2023], we determine the fair rates for the following benchmark instruments:

- spot-starting single-period (SSSP) OIS contracts, with maturities from 1 month (1M) to 12 months (12M) in 1-month increments; and
- spot-starting multi-period (SSMP) OIS contracts, with maturities from 15 months (15M) to 30 years (30Y) in 3-month increments.

The final set of results includes:

i. the fair rates of the benchmark instruments; and

ii. the discount factors associated with their respective expiry/tenor and payment dates.

Although the discount factors referenced in ii are all derived from Step 6, we distinguish between the term structure of discount factors as follows:

a. discount factors at expiry/tenor dates are called the "*Forecast Curve*", since these are used to forecast floating cash flows for the benchmark OIS instruments; and

b. discount factors at payment dates are called the "*Discount Curve*, as these are used to discount cash flows for the benchmark OIS instruments.

Appendix A contains the detailed outputs for each step described above, using the first date in the dataset (i.e. 03-Jan-2005) as an example. The next section explains the structure of these results and provides visualisations.

4. Results

The accompanying results for this document are organised into the following components:

(i) **Benchmark Rates:** Provided in an Excel workbook titled "*Benchmark_Rates.xlsb*", this workbook contains the estimated fair or par rates for all ZARONIA-based benchmark instruments previously specified, covering the entire historical period under consideration.

(ii) **Forecast Curves:** Supplied as two Excel workbooks, the first titled "*Forecast_Curve_Expiry_Dates.xlsb*" (listing all relevant expiry and tenor dates) and "*Forecast_Curve_Discount_Factors.xlsb*" (containing the associated discount factors). Together, these files present the full set of expiry/tenor dates and corresponding discount factors for each forecast curve throughout the historical period.

(iii) **Discount Curves:** Also provided via two Excel workbooks: "*Discount_Curve_Payment_Dates.xlsb*", which includes all payment dates, and "*Discount_Curve_Discount_Factors.xlsb*", which details the related discount factors. Combined, these files supply the complete set of payment dates and discount factors for each discount curve over the entire historical period.

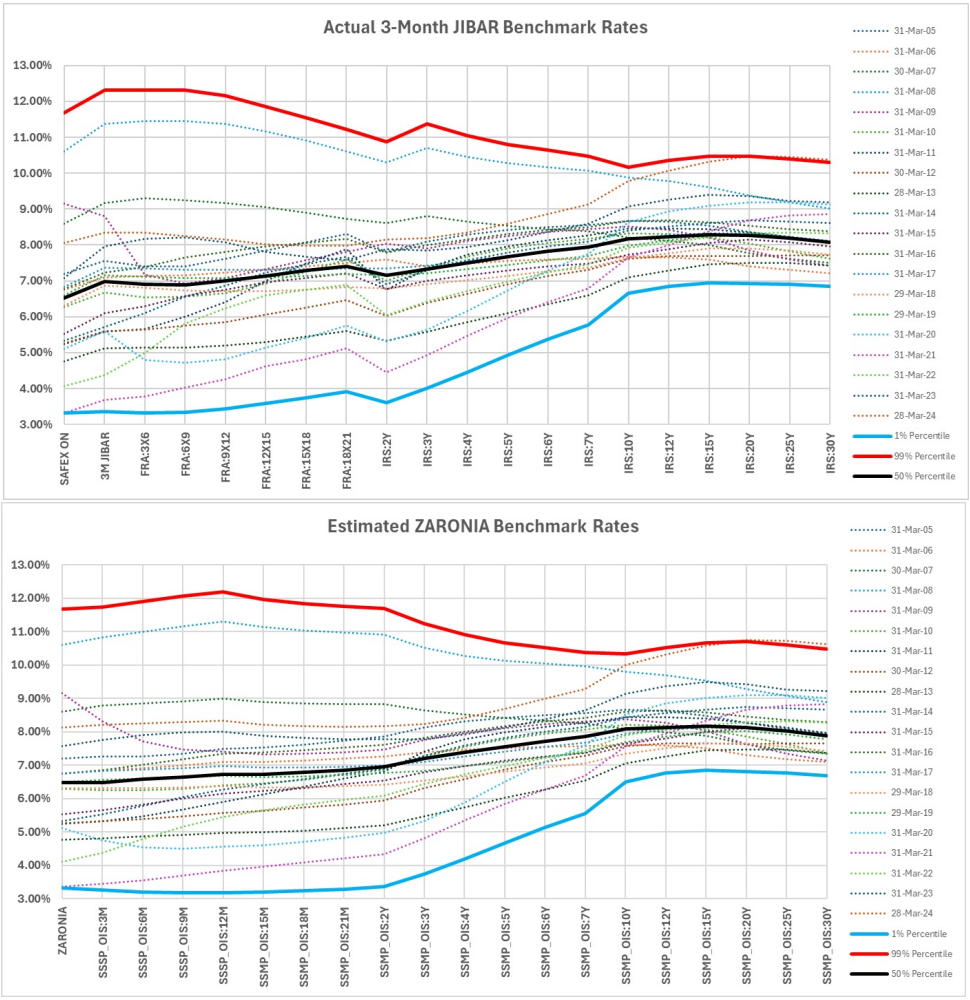
As detailed in Step 7 of Section 3., the discount factors in both the forecast and discount curve components (items ii and iii above) are all drawn from the same estimated ZARONIA OIS curve for each date in the historical period. The main reason for providing discount factor information in this manner is to enable the user to recover the benchmark rates without needing to make any interpolation assumptions, or have access to complete ZARONIA OIS term structures.

Although it is challenging to fully assess the accuracy of the results, we provide visual comparisons with corresponding Jibar data. These comparisons reveal that the historical estimation process displays behaviour that is reasonable and satisfactory for practical applications.

The first set of figures below shows comparable fair/par rates for benchmark instruments representing both the actual 3-month Jibar swap and the estimated ZARONIA OIS curves at selected time points — the end of March for each year in the historical period under consideration. In addition, these graphs also plot the 1st, 50th, and 99th percentiles for each of these benchmark rates across the entire historical period.

It is important to note that, in general, these benchmark rates are not directly comparable due to differences in accrual period across the various instruments. All derivative benchmarks forming the 3-month Jibar swap zero curve have a 3-month accrual period. By contrast, for the estimated ZARONIA OIS curve, the SSSP OISs have accrual period ranging from 1 month to 12 months, while all SSMP OISs have an accrual period of 12 months.

Figure 1: A sample of 3-month Jibar and ZARONIA benchmark rates as well as percentiles for the entire period



While Figure 1 offers a cross-sectional comparison between the results and actual 3-month Jibar data, Figures 2 and 3 provide the longitudinal trends. To maintain clarity in the visuals, only a subset of short- and long-term benchmark instrument rates are depicted. Overall, these figures show that the estimated ZARONIA results behave consistently with the actual 3-month Jibar data. Moreover, both cross-sectional and longitudinal variations observed in the estimated ZARONIA data align closely with those of the 3-month Jibar data. Therefore, these results are deemed satisfactory for practical risk management applications.

Figure 2: A longitudinal view of short-term 3-month Jibar and ZARONIA benchmark rates

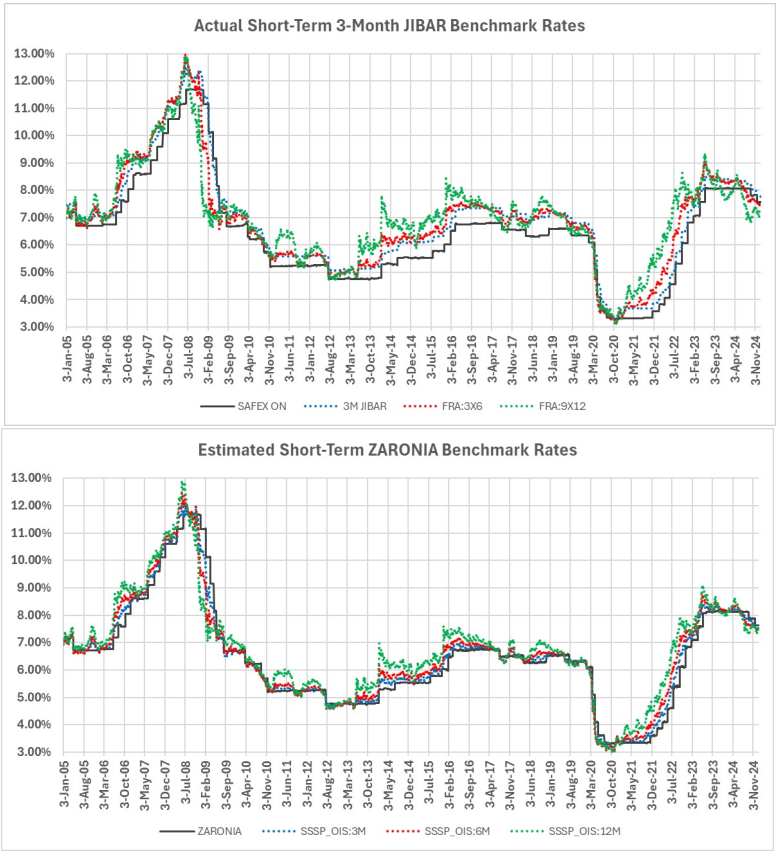
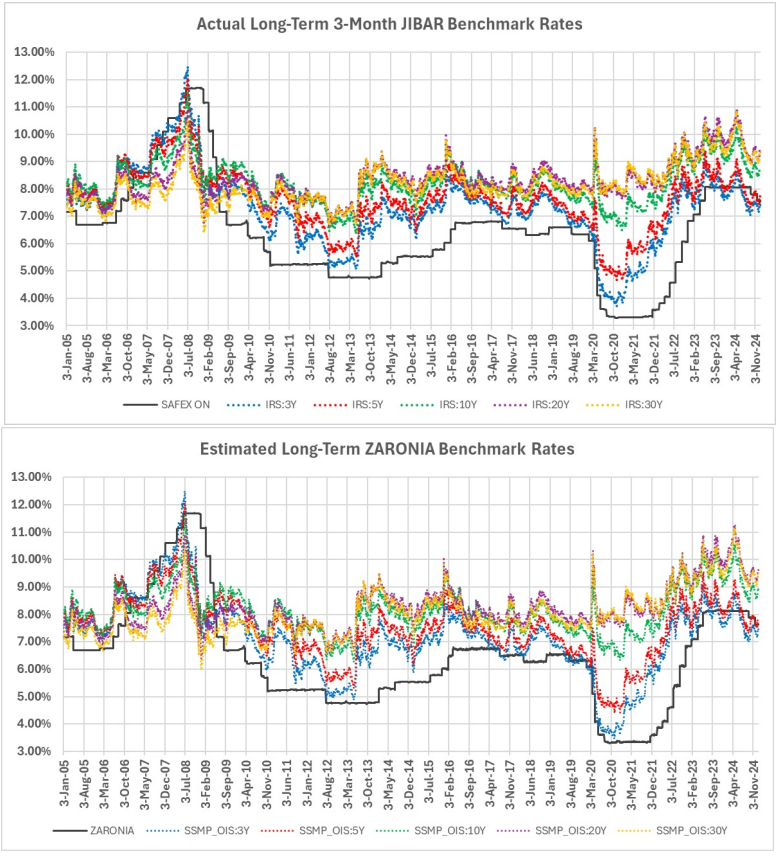


Figure 3: A longitudinal view of long-term 3-month Jibar and ZARONIA benchmark rates



Abbreviations

Abbreviations

DWS Derivatives Workstream. 4

ES Expected Shortfall. 4

FRA forward rate agreement. 4

IRS interest rate swap. 4

ISDA International Swaps and Derivatives Association. 6

Jibar Johannesburg Interbank Average Rate. 2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 14, 16

JSE Johannesburg Stock Exchange. 3, 4, 5, 12

MPG Market Practitioners Group. 4

NACC nominal annual compounded continuously. 5

OIS overnight indexed swap. 1, 4, 5, 6, 7, 8, 15

SAFEX South African Futures & Options Exchange. 4

SARB South African Reserve Bank. 4

SSSP spot-starting single-period. 7, 8, 15

SSMP spot-starting multi-period. 7, 8, 15

VaR Value-at-Risk. 4

ZARONIA South African Overnight Index Average. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16

References

Kenyon, C. and R. Stamm (2012). *Discounting, LIBOR, CVA and funding: Interest rate and credit pricing*. Palgrave Macmillan, London. (cited on page 6).

du Preez, P.F. (2011). "An investigation into popular methods for constructing yield curves". In: *M.Sc. Thesis, Department of Mathematics and Applied Mathematics, University of Pretoria* (cited on page 4).

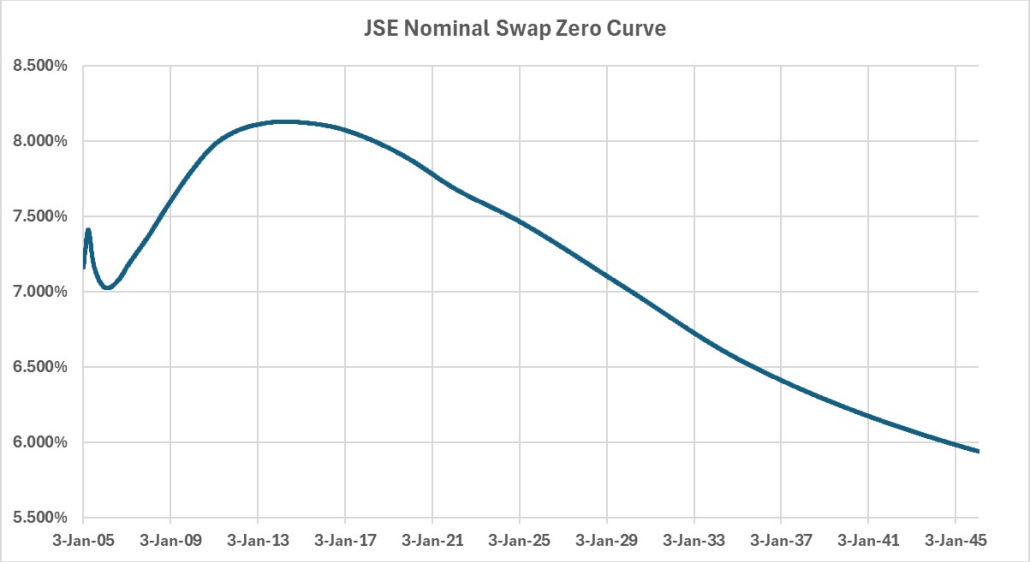
SARB-DWS (2023). "Market conventions for ZARONIA-based derivatives". In: URL: `https://www.resbank.co.za/content/dam/sarb/publications/media-releases/2023/zaronia-based-derivatives/Market%20conventions%20for%20ZARONIA-based%20derivatives.pdf` (cited on pages 5, 7).

A Single result sample

In this section, we present the results for the earliest historical date in our dataset: 03-Jan-2005.

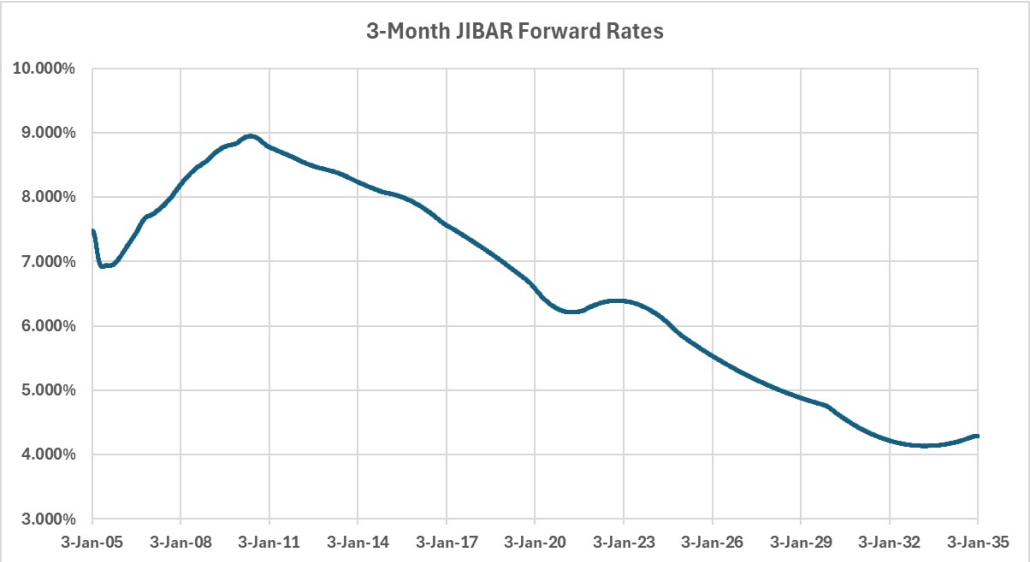
Step 1: The figure below visually represents Table 1, illustrating the JSE's nominal swap zero curve as observed on 03-Jan-2005. This curve serves as the foundation of the historical estimation exercise.

Figure 4: The JSE's nominal swap zero curve on 03-Jan-2005



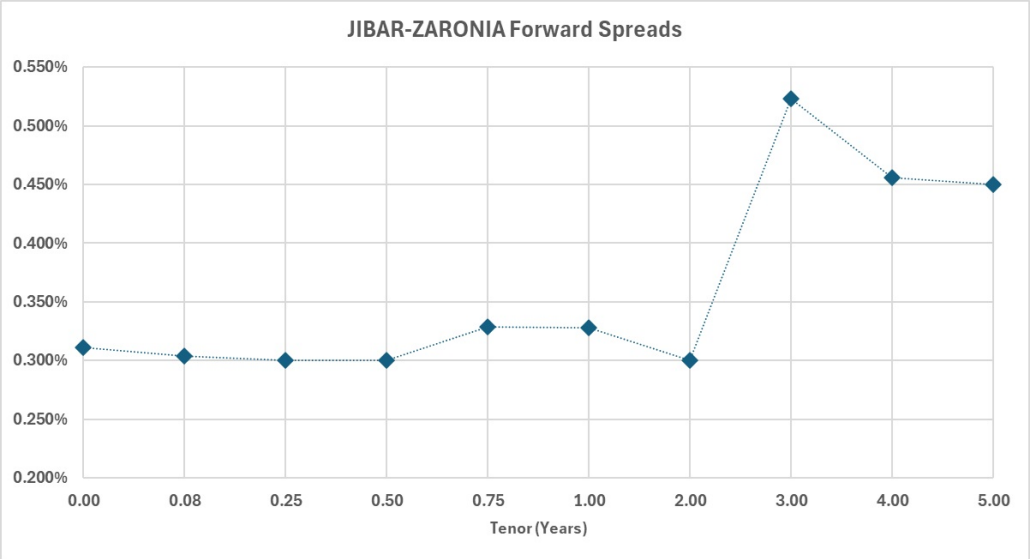
Step 2: The figure below depicts Table 2, showing the 3-month Jibar forward rates derived from the curve in Step 1. Note that while the curve from Step 1 extends beyond 40-years, the set of forward rates has reset dates up to just over 30-years — in this case, 08-Jan-2035.

Figure 5: The set of 3-month Jibar forward rates computed at a daily frequency on 03-Jan-2005



Step 3: The figure below visually represents Table 3 as at 03-Jan-2005. While the graph's shape may appear unusual, each point is calculated using equation (2), which returns a median of a historical spot spreads between Jibar and ZARONIA. The dotted line is included to highlight the short-end shape; however, the actual data are depicted by the markers.

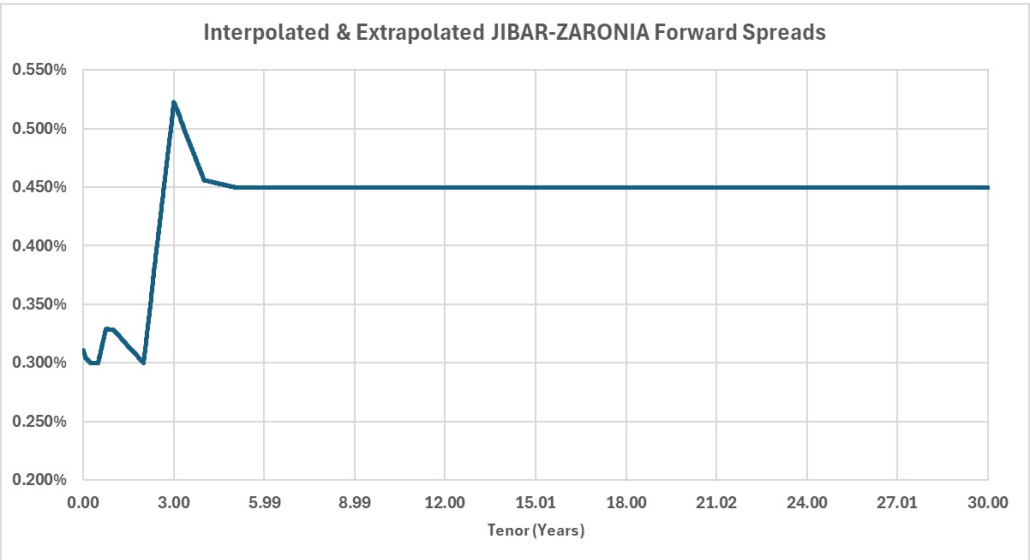
Figure 6: Jibar-ZARONIA forward spreads computed on 03-Jan-2005



For a comprehensive view of the spreads across all tenor points in the dataset, refer to Appendix B, which demonstrates their variability over time.

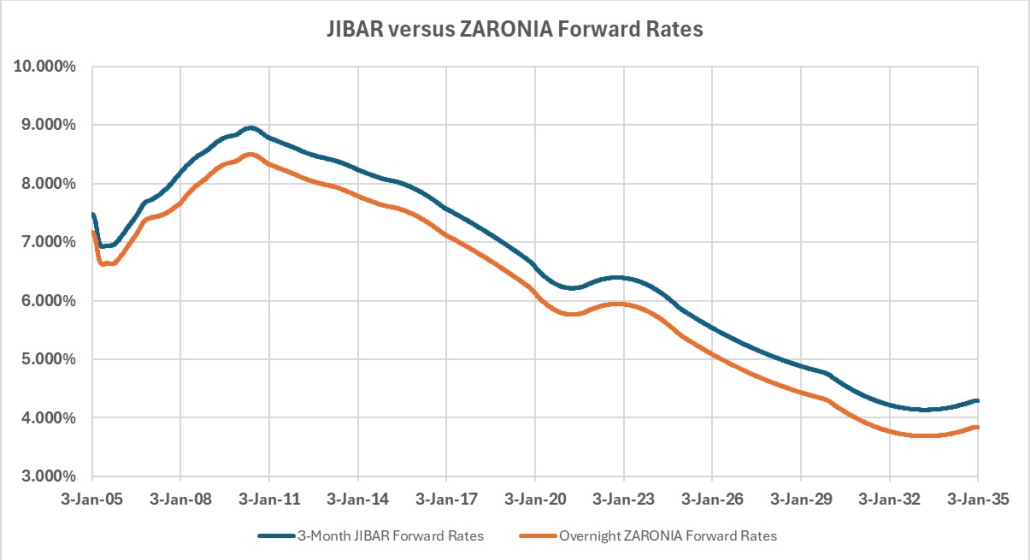
Step 4: The figure below depicts interpolated and extrapolated Jibar-ZARONIA forward spreads on 03-Jan-2005. As noted in Section 3., linear interpolation is applied for tenors between 0 and 5 years, with flat extrapolation used for longer tenors.

Figure 7: Interpolated and extrapolated Jibar-ZARONIA forward spreads computed on 03-Jan-2005



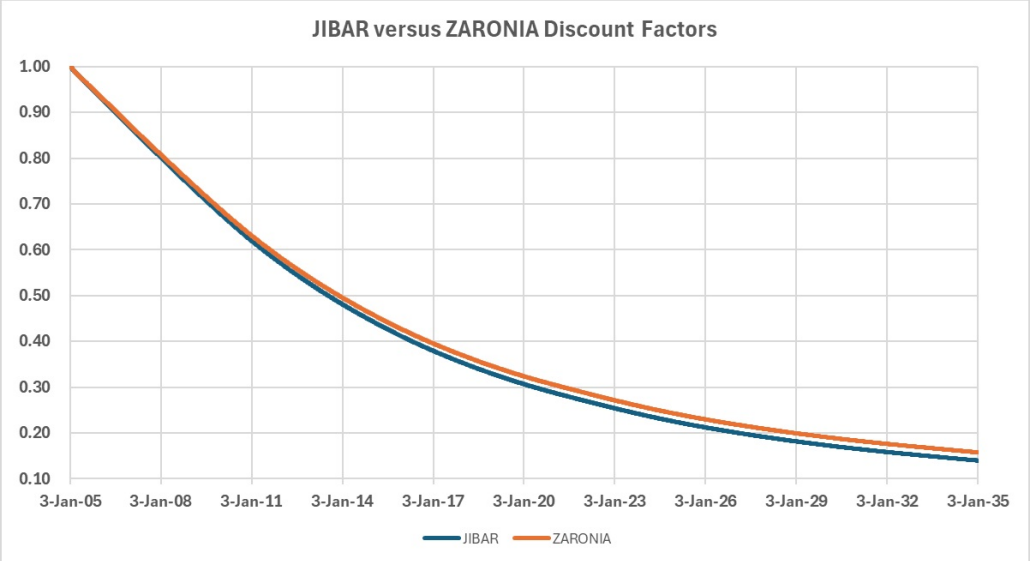
Step 5: By applying the forward spreads from Step 4 to the 3-month Jibar forward rates from Step 2 (i.e. using equation (4)), we obtain the estimated overnight ZARONIA forward rates depicted below.

Figure 8: Actual 3-month Jibar versus estimated overnight ZARONIA forward rates on 03-Jan-2005



Step 6: With equation (5) it is possible to derive the term structure of discount factors associated with the estimated overnight ZARONIA forward rates, which are presented together with the corresponding 3-month Jibar discount factors in the figure below.

Figure 9: Actual 3-month Jibar versus estimated ZARONIA discount factors on 03-Jan-2005



Step 7: The final step is to use the discount factors from Step 6 to compute the fair/par rates for the benchmark instruments listed in Step 7 of Section 3. The figure below shows a subset of these rates as at 03-Jan-2005. For clarity, we have excluded the SSMP OIS contracts that lie between those with annual expiries.

Figure 10: Estimated rates of ZARONIA benchmark instruments on 03-Jan-2005

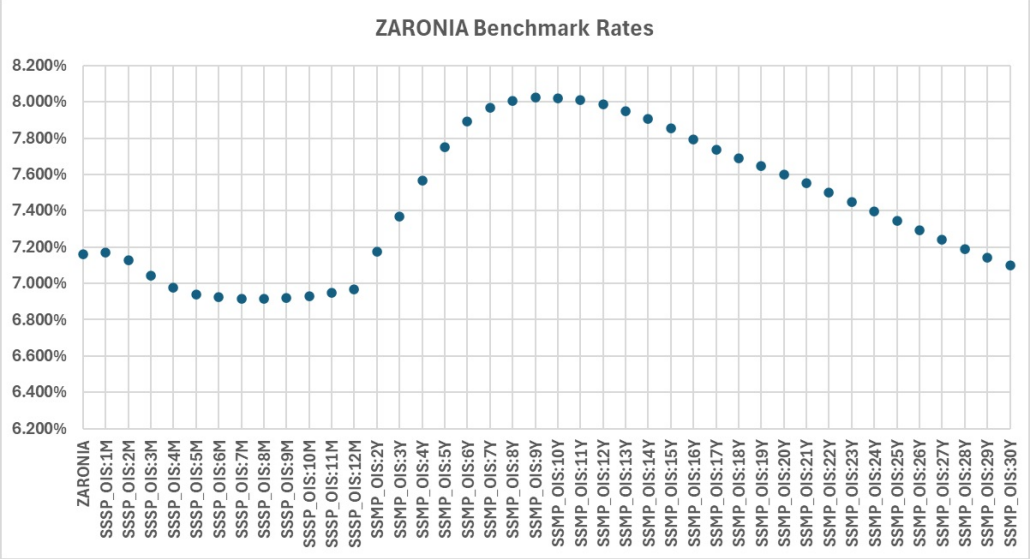


Table 4 below provides an excerpt of the final results, consolidating data from all three components described in Section 4. Furthermore, as described in Sections 3. and 4., the "*Forecast Curve*" can be used to project cash flows for the benchmark instruments, while the "*Discount Curve*" is applied for discounting cash flows. The fair/par rates for SSSP OISs are computed solely from the "*Forecast Curve*", whereas SSMP OISs require both curves for calculation.

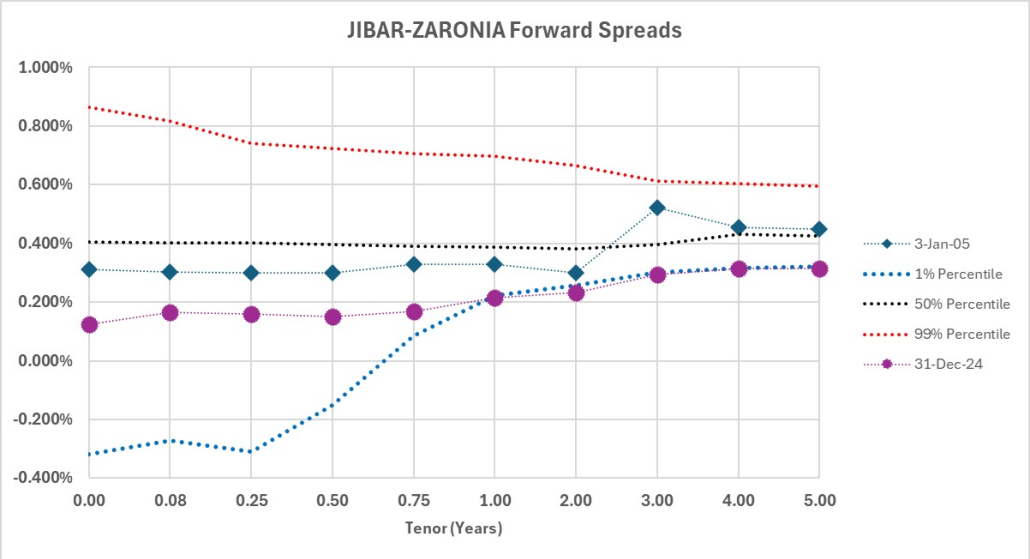
Table 4: A collated excerpt of the final set of results for 03-Jan-2005

Benchmark	Fair rate	Forecast curve		Discount curve	
		Expiry/tenor date	Discount factor	Payment date	Discount factor
ZARONIA	7.160%	04-Jan-05	0.99980	04-Jan-05	0.99980
SSSP_OIS:1M	7.168%	03-Feb-05	0.99395	07-Feb-05	0.99318
⋮	⋮	⋮	⋮	⋮	⋮
SSSP_OIS:12M	6.966%	03-Jan-06	0.93487	05-Jan-06	0.93453
SSMP_OIS:2Y	7.173%	03-Jan-07	0.87050	05-Jan-07	0.87014
⋮	⋮	⋮	⋮	⋮	⋮
SSMP_OIS:30Y	7.098%	03-Jan-35	0.15748	05-Jan-35	0.15744

B Jibar-ZARONIA forward spreads

The figure below depicts Jibar-ZARONIA forward spreads computed on the first and last dates of the historical dataset under consideration: 03-Jan-2005 and 31-Dec-2024. The methodology for calculating these spreads is detailed in Step 3 of Section 3. Along with spread data for these two dates, this figure also shows the 1st, 50th, and 99th percentiles for the Jibar-ZARONIA forward spread at each tenor, summarising the entire historical dataset.

Figure 11: Jibar-ZARONIA forward spreads computed on 03-Jan-2005



As expected, given the calculation methodology, the range of variation is much wider for shorter than longer tenors. It is also interesting to note how the spread has generally decreased through time, as evidenced by the data for 31-Dec-2024 versus that for 03-Jan-2005, as well as the percentile data.