Market conventions for ZARONIA-based non-linear derivatives

prepared by

The Market Practitioners Group's Derivatives Workstream

SOUTH AFRICAN RESERVE BANK

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1. Background

The highly publicised irregularities relating to the production of interbank offered rates (IBORs) in 2012 — see, for example, [Hou and Skeie, 2014] — initiated a global regulatory response to reform major interest rate benchmarks. The use of the IBORs in financial markets has subsequently reduced substantially in favour of more robust alternative reference rates (ARRs), namely overnight reference rates (ONRRs) which are *near risk-free*.

Derivatives are an integral part of financial markets and are critical to reference rate transition plans in major jurisdictions. The latest Bank for International Settlements (BIS) Report, [BIS, 2023], shows that the gross notional of over-the-counter (OTC) derivatives totalled US$715 trillion globally. Interest rate derivatives account for approximately 80% of the outstanding global derivatives notional and the proportion of interest rate derivatives that are centrally cleared remains stable above 75%. Non-linear interest rate derivatives comprise 8% of the overall interest rate derivatives market, with gross notional outstanding last estimated at US$45 trillion.

Given the significant role of interest rate derivatives in financial markets, it is essential that a derivative transition process is managed transparently and consultatively while considering the requirements of clearing houses and other relevant venues. The above will ensure market depth is preserved and potentially enhanced.

2. Derivatives workstream mandate

South Africa has also embarked on the transition journey with the release of the consultative paper [SARB, 2018], prepared by the South African Reserve Bank (SARB), which detailed its initial proposal to reform domestic benchmark and reference rates. The SARB subsequently formed the Market Practitioners Group (MPG) in 2019 to manage the process of adoption and transition to the new interest rate dispensation. The SARB's MPG is a joint public and private sector body, comprising representatives from the SARB, the Financial Sector Conduct Authority (FSCA) and senior professionals from a variety of institutions from different market interest groups active in the domestic money market.

The MPG relies on dedicated workstreams and technical subgroups to carry out its objectives. The workstreams and subgroups have a responsibility of providing technical input and recommendations to the MPG on specific issues that are relevant to the transition from the Johannesburg Interbank Average Rate (Jibar). Members of these workstreams are drawn from a diverse set of market practitioners whose insights and expertise are required to give effect to the mandate of the MPG as well as shape industry opinions on the reform agenda.[1]

The Derivatives Workstream (DWS), constituted in 2021, is mandated with making recommendations on the development of derivative markets and contracts that reference the successor rate. More specifically and as set in its terms of reference, the roles and responsibilities of the DWS are as follows:

- *The DWS shall construct an action plan aligned to the objectives set out by the MPG.*
- *In line with the stated functions of the MPG, the DWS will be responsible for:*
  - *consulting widely and making recommendations on the development of derivative markets and contracts that reference the successor rate;*
  - *formulating and implementing strategies to facilitate the market adoption of derivatives that reference the successor rate;*
  - *formulating and recommending strategies to derive term risk free rates from underlying derivatives activity;*
  - *providing input to the finalisation and refinement of the transition plan and monitor the progress made in the derivative markets; and*

[1] For more information, please refer to: SARB Market Practitioners Group.

– *aligning with progress and recommendations of all workstreams of the MPG.*

The DWS has made steady progress since its constitution. In July 2023, the DWS published a paper which documents market conventions for the South African Overnight Index Average (ZARONIA)-based linear derivatives — see [SARB-DWS, 2023]. This paper specifies the technical conventions that need to be considered when transacting *overnight indexed swaps* (OISs) and *cross-currency basis swaps* (CCBSs) that reference ZARONIA.

3. Problem statement

For a derivatives market to initiate, market participants need to consider (and agree upon) the various conventions that underpin the said derivative. Using the conventions published in [SARB-DWS, 2023] as a base reference, the DWS embarked on a process to deliberate on and summarise a set of recommended conventions for the fundamental set of non-linear derivatives that will reference ZARONIA, namely caps, floors and swaptions. The results of these deliberations are presented in this white paper together with reasons for the articulated selections.

This working paper should serve as a resource for market participants to consider when using ZARONIA as a reference rate in the specification of non-linear derivative contracts. It is not meant to prescribe, mandate, or limit the ways in which they can transact based on their needs and requirements. The specific recommendations herein offer the standard conventions that will form the basis for the on-the-run interbank market, which constitutes caps, floors, and swaptions that reference ZARONIA, and will in future be quoted on screens and/or via interbank broking agents.

4. Derivative design principles

In contemplating an optimal parameter set for non-linear derivatives, the DWS adopted the following design principles:

- Support the deepening of financial markets and ease operational complexity.
- Align with major developed markets unless domestic nuances dictate otherwise.
- Ensure that the requirements of major exchanges and clearing houses are observed and satisfied (e.g. settlement requirements).
- Consider the related ZARONIA quotation, timing and application of the rate to derivatives.

In addition to derivative conventions, the DWS also provided recommendations on certain elements of market micro-structure. It must be emphasised that the recommendations made within this document should not preclude any derivative user from negotiating a more bespoke derivative to suit individual requirements.

5. International market conventions

The table below summarises the key market conventions for on-the-run interbank caps and floors that trade in the United States (US) (USD-denominated) and United Kingdom (UK) (GBP-denominated).

Table 1: Market conventions for USD- and GBP-denominated caps and floors

Feature	US	UK
Accrual period	3M	3M
Include first accrual period	Yes	Yes
Business day calendar[2]	USNY	GBLO
Spot lag	2 bd USGS and USNY	0 bd
Business day convention	Modified Following	Modified Following
Accrual period date generation	Backward (EOM)	Backward (EOM)
Accrual day count convention	ACT/360	ACT/365 Fixed
Floating reference rate	SOFR	SONIA
Publication/ Calculation lag	1 bd	1 bd
Floating rate calculation	Compounded, 0 bd lockout, 0 bd lookback, 0 bd obs shift	Compounded, 0 bd lockout, 0 bd lookback, 0 bd obs shift
Floating rate convention	Simple, 7 decimal places	Simple, 6 decimal places
Fixed strike rate quotation	Simple, 3-4 decimal places	Simple, 3-4 decimal places
Net Cash Flow Rounding	2 decimal places	2 decimal places
Payment lag	2 bd USNY	0 bd
Benchmark CSA	USD	GBP
Premium type	Spot	Spot
Premium lag	2 bd USNY	0 bd before 11:00 1 bd after 11:00
Quote format	Premium or Normal volatility	Premium or Normal volatility

[2] The business day calendar is utilised for all business day and date generation, unless stated otherwise.

The table below summarises the key market conventions for on-the-run interbank swaptions that trade in the US (USD-denominated) and UK (GBP-denominated).

Table 2: Market conventions for USD- and GBP-denominated swaptions

Feature	US	UK
Underlying swap	Forward-starting SOFR OISs	Forward-starting SONIA OISs
Business day calendar	USNY	GBLO
Business day convention	Modified Following	Modified Following
Fixed strike rate quotation	Simple, 3-4 decimal places	Simple, 3-4 decimal places
Benchmark CSA	USD	GBP
Premium type	Forward	Forward
Premium lag	2 bd USNY	0 bd
Quote method	Premium or Normal volatility	Premium or Normal volatility
Exercise procedure	Fallback exercise	Fallback exercise
Exercise time	09:00 to 11:00	09:00 to 11:00
Settlement	Physical and cleared	Physical and cleared

Take note that the underlying swap contracts are forward-starting from the vantage point of the swaption transaction date but evolve into standard *on-the-run* spot-starting OISs at the swaption expiry date.

6. Market and convention recommendations

This section presents recommendations and suggestions for market microstructure, products and associated conventions for ZARONIA-based non-linear derivatives that are considered to be critical to enable similar financial functionality and utility as the current, but soon-to-be ceased, Jibar-based non-linear derivatives market. These fundamental non-linear derivative contracts are: (i) *caplets and floorlets*; (ii) *spot-starting caps and floors*; (iii) *forward-starting caps and floors*; and (iv) *swaptions*.

6.1. Caplets and floorlets

Caplets and floorlets inherit many of the features of corresponding *spot-* and *forward-starting single-period OISs*. These features and conventions are defined in [SARB-DWS, 2023].

Table 3: Market microstructure and conventions for caplets and floorlets

Feature	Recommended convention	Comment	Reference
Accrual period	3M	*The length of the future accrual and reference period over which the ONRR is compounded or averaged.*	7.1., 7.2.
Business day calendar	ZAJO	*As published by the relevant providers, in accordance with the Public Holidays Act 36 of 1994.*	7.2.
Forward period	$\leq$21M	*Given a quoted caplet/floorlet with tenor* xM3M, *then* xM *is the length of the forward period, which precedes the* 3M *accrual period.*	7.2.
Option tenor	$\leq$24M	*Suggested standard quote option tenors:* 0M3M, 1M3M, 2M3M, 3M3M, 6M3M *and* 9M3M; *and* 12M3M, 15M3M, 18M3M *and* 21M3M.	7.2.
Spot lag	0 bd	*There is no lag between the end of the forward period and the start of the accrual period.*	7.2.
Business day convention	Modified Following	*Applied in accrual period date generation.*	7.2.
Accrual period date generation	Backward (EOM)	*Unadjusted backward generation from roll-day plus EOM, then adjusted by Modified Following.*	7.2.
Accrual day count convention	ACT/365 Fixed	*Used for both the calculation of floating and fixed leg interest cash flows.*	7.2.
Floating reference rate	ZARONIA	*The final republished rate, if applicable, as prescribed by the SARB's MPG in [SARB-DWS, 2023].*	7.3.
Publication/ Calculation lag	1 bd	*Calculated with the sub-accrual period start date as the anchor date.*	7.3.
ACFR calculation	Compounded, 0 bd lockout, 0 bd lookback, 0 bd obs shift	*Backward-looking without lookback or lockout period. Payment lag to resolve calculation lag.*	7.4., 7.5.
ACFR convention	Simple, 6 decimal places	*Or 4 decimal places in % format.*	7.5.
Payment lag	2 bd	*Calculated with the last publication/calculation date within the respective accrual period as the anchor date.*	7.5.
Spread	Simple, additive post compounding	*Fixed simple rate added to the compounded ACFR for floating cash flow calculation, if necessary.*	7.5.
Fixed rate strike quotation	Simple, 6 decimal places, [-2% $+K$, $K+$ 2%]	*Suggested quoted strikes for each option tenor:* $K\pm$ 2%, $K\pm$ 1.5%, $K\pm$ 1%, $K\pm$ 0.75%, $K\pm$ 0.5%, $K\pm$ 0.25%, *and* K *which is the respective ATM rate.*	7.6.
Net cash flow rounding	2 decimal places	*Net unrounded floating and fixed cash flows, then round to the nearest ZAc for settlement purposes.*	7.6.
Benchmark CSA	USD	*Assumed to be a zero-threshold CSA, with SOFR being the relevant collateral rate.*	8.2.
Premium type and premium lag	Spot, 2 bd	*The option premium is paid 2 business days after the transaction date, according to the ZAJO calendar.*	8.3.1.
Market quote conventions	Premium, Black or Normal volatility	*Three equivalent formats may be quoted with or without volatility decay, in units specified.*	8.

6.2. Spot-starting caps and floors

Spot-starting caps and floors inherit many of the features of corresponding *spot-starting multi-period OISs*. These features and conventions are defined in [SARB-DWS, 2023].

Table 4: Market microstructure and conventions for spot-starting caps and floors

Feature	Recommended convention	Comment	Reference
Option tenor	$\geq$1Y	*Suggested standard quote option tenors:* 1Y, 2Y, . . ., 9Y, 10Y, 12Y, *and* 15Y.	7.2.
Accrual period	3M	*The length of the future accrual and reference period over which the ONRR is compounded or averaged.*	7.1., 7.2.
Include first accrual period	Yes	*For example, an* xY *cap/floor constitutes the following caplets/floorlets:* 0M3M, 3M3M, . . ., $(12x-3)$M3M.	7.1., 7.2.
Business day calendar	ZAJO	*As published by the relevant providers, in accordance with the Public Holidays Act 36 of 1994.*	7.2.
Spot lag	0 bd	*Trade date coincides with first accrual period start date.*	7.2.
Business day convention	Modified Following	*Applied in accrual period date generation.*	7.2.
Accrual period date generation	Backward (EOM)	*Unadjusted backward generation from roll-day plus EOM, then adjusted by Modified Following.*	7.2.
Accrual day count convention	ACT/365 Fixed	*Used for both the calculation of floating and fixed leg interest cash flows.*	7.2.
Floating reference rate	ZARONIA	*The final republished rate, if applicable, as prescribed by the SARB's MPG in [SARB-DWS, 2023].*	7.3.
ACFR calculation	Compounded, 0 bd lockout, 0 bd lookback, 0 bd obs shift	*Backward-looking without lookback or lockout period. Payment lag to resolve calculation lag.*	7.4., 7.5.
ACFR convention	Simple, 6 decimal places	*Or 4 decimal places in % format.*	7.5.
Payment lag	2 bd	*Calculated with the last publication/calculation date within the respective accrual period as the anchor date.*	7.5.
Spread	Simple, additive post compounding	*Fixed simple rate added to the compounded ACFR for floating cash flow calculation, if necessary.*	7.5.
Fixed rate strike quotation	Simple, 6 decimal places, [-3% $+K$, $K+$ 3%]	*Suggested quoted strikes for each option tenor:* $K\pm$ 3%, $K\pm$ 2.5%, $K\pm$ 2%, $K\pm$ 1.5%, $K\pm$ 1%, $K\pm$ 0.5%, *and* K *which is the respective ATM rate.*	7.6.
Net cash flow rounding	2 decimal places	*Net unrounded floating and fixed cash flows, then round to the nearest ZAc for settlement purposes.*	7.6.
Benchmark CSA	USD	*Assumed to be a zero-threshold CSA, with SOFR being the relevant collateral rate.*	8.2.
Premium type and premium lag	Spot, 2 bd	*The option premium is paid 2 business days after the transaction date, according to the ZAJO calendar.*	8.3.1.
Market quote conventions	Premium, Black or Normal volatility	*Three equivalent formats may be quoted with or without volatility decay, in units specified.*	8.

6.3. Forward-starting caps and floors

Forward-starting caps and floors inherit many of the features of corresponding *forward-starting multi-period OISs*. These features and conventions are defined in [SARB-DWS, 2023].

Table 5: Market microstructure and conventions for forward-starting caps and floors

Feature	Recommended convention	Comment	Reference
Accrual period	3M	*The length of the future accrual and reference period over which the ONRR is compounded or averaged.*	7.1., 7.2.
Business day calendar	ZAJO	*As published by the relevant providers, in accordance with the Public Holidays Act 36 of 1994.*	7.2.
Forward period	$\leq$10Y	*Given a forward-starting cap/floor with tenor aYbY, then aY is the length of the forward period, which precedes the bY cap/floor as defined in section 6.2.*	7.2.
Option tenor	$\geq$1Y	*Suggested quote tenors: aMcY and bYcY where $a \in$ {1,2,3,6,12,18}; $b \in$ {2,3,. . .,10}; $c \in$ {1,2,. . .,10}.*	7.2.
Spot lag	0 bd	*There is no lag between the end of the forward period and the start of the cap's/floor's first accrual period.*	7.2.
Business day convention	Modified Following	*Applied in accrual period date generation.*	7.2.
Accrual period date generation	Backward (EOM)	*Unadjusted backward generation from roll-day plus EOM, then adjusted by Modified Following.*	7.2.
Accrual day count convention	ACT/365 Fixed	*Used for both the calculation of floating and fixed leg interest cash flows.*	7.2.
Floating reference rate	ZARONIA	*The final republished rate, if applicable, as prescribed by the SARB's MPG in [SARB-DWS, 2023].*	7.3.
ACFR calculation	Compounded, 0 bd lockout, 0 bd lookback, 0 bd obs shift	*Backward-looking without lookback or lockout period. Payment lag to resolve calculation lag.*	7.4., 7.5.
ACFR convention	Simple, 6 decimal places	*Or 4 decimal places in % format.*	7.5.
Payment lag	2 bd	*Calculated with the last publication/calculation date within the respective accrual period as the anchor date.*	7.5.
Spread	Simple, additive post compounding	*Fixed simple rate added to the compounded ACFR for floating cash flow calculation, if necessary.*	7.5.
Fixed rate strike quotation	Simple, 6 decimal places, [-3% $+K$, $K+$ 3%]	*Suggested quoted strikes for each option tenor: $K\pm$ 3%, $K\pm$ 2.5%, $K\pm$ 2%, $K\pm$ 1.5%, $K\pm$ 1%, $K\pm$ 0.5%, and K which is the respective ATM rate.*	7.6.
Net cash flow rounding	2 decimal places	*Net unrounded floating and fixed cash flows, then round to the nearest ZAc for settlement purposes.*	7.6.
Benchmark CSA	USD	*Assumed to be a zero-threshold CSA, with SOFR being the relevant collateral rate.*	8.2.
Premium type and premium lag	Spot, 2 bd	*The option premium is paid 2 business days after the transaction date, according to the ZAJO calendar.*	8.3.1.
Market quote conventions	Premium, Black or Normal volatility	*Three equivalent formats may be quoted with or without volatility decay, in units specified.*	8.

6.4. Swaptions

Like forward-starting caps and floors, swaptions also inherit many of the features of corresponding *forward-starting multi-period OISs*. The fundamental difference though is that a physically-settled swaption's exercise event offers one the right but not the obligation to enter into a contemporaneous spot-starting multi-period OIS, as opposed to forward-starting caps and floors which constitute series of forward-starting caplets and floorlets respectively, with physical settlement not applicable. Swaptions therefore inherit most of their conventions from their respective underlying OIS contracts, with the *forward period* now coinciding with the tenor of the optionality.

Table 6: Market microstructure and conventions for swaptions

Feature	Recommended convention	Comment	Reference
Business day calendar	ZAJO	*As published by the relevant providers, in accordance with the Public Holidays Act 36 of 1994.*	7.2.
Business day convention	Modified Following	*Applied in accrual period date generation.*	7.2.
Forward period/option tenor	$\leq$10Y	*Given a swaption with tenor aYbY, then aY is the length of the forward/option tenor, which precedes the spot-starting bY OIS defined in [SARB-DWS, 2023].*	7.2.
Spot lag	0 bd	*There is no lag between the end of the forward period and the start of the OIS's first accrual period.*	7.2.
Underlying swap	Forward-starting ZARONIA OISs	*Suggested quote tenors: aMcY and bYcY where $a \in$ {1,2,3,6,12,18}; $b \in$ {2,3,. . .,10}; $c \in$ {1,2,. . .,10}. All underlying OISs have accrual periods equal to 1Y.*	[SARB-DWS, 2023]
Fixed rate strike quotation	Simple, 6 decimal places, [-4% $+K$, $K+$ 4%]	*Suggested quoted strikes for each option tenor: $K\pm$ 4%, $K\pm$ 3%, $K\pm$ 2%, $K\pm$ 1.5%, $K\pm$ 1%, $K\pm$ 0.5%, and K which is the respective ATM rate.*	7.6.
Exercise business days	ZAJO & GBLO	*As specified by the referenced ISDA document, and impacts the generation of the forward period date, which is also the expiry date, as shown in section 7.2.*	[ISDA, 2024]
Exercise procedure	Fallback exercise	*As specified by the referenced ISDA document.*	[ISDA, 2023] [ISDA, 2024]
Exercise time	09:00 to 11:00, Johannesburg	*As specified by the referenced ISDA document.*	[ISDA, 2024]
Settlement	Physical and cleared	*As specified by the referenced ISDA document.*	[ISDA, 2024]
Benchmark CSA	USD	*Assumed to be a zero-threshold CSA, with SOFR being the relevant collateral rate.*	8.2.
Premium type and premium lag	Forward, 0 bd	*Premium paid at end of forward period, except for: (i) wedge structures[3] which is spot, 2 bd; and (ii) 0 bd and 1 bd expiries which is also spot, 2 bd.*	8.3.3.
Market quote conventions	Premium, Black or Normal volatility	*Three equivalent formats may be quoted in units specified.*	8.

[3] A wedge structure constitutes a forward-starting cap or floor along with a swaption, where the forward period and option tenor of the cap or floor coincides with the option tenor and underlying swap tenor of the swaption, respectively.

7. Option definitions and conventions

As mentioned in the previous section, caplets/floorlets and caps/floors inherit many features from corresponding OISs, while swaptions offer the holder optionality in relation to a position in a forward-starting OIS. Therefore, most of the option definitions and conventions are sourced from the corresponding and associated OIS definitions and conventions — as a result, this section reads very similarly to section 8 in [SARB-DWS, 2023]. The subsections that follow use mathematical notation to define a general underlying OIS, which encapsulates all the potential underlying variants for the options mentioned above, while also including and overlaying relevant option specific features. This is achieved via the specification of the following contractual features:

i. nominal, tenor and accrual period;

ii. forward, settlement and accrual period dates, year fractions and option tenors;

iii. floating reference rates, publication and calculation lags;

iv. averaging, lookback and lockout periods;

v. floating cash flow calculations and payment lags; and

vi. fixed rate strikes, payoff calculations and settlement.

Specifications are then discussed for each contractual feature and a specific convention is recommended.

7.1. Nominal, tenor and accrual period

This category constitutes standard features, which are specified as follows:

- **Nominal**: The nominal or notional value of the underlying swap and option is denoted by N.
- **Swap tenor**: The tenor of the swap may be x_m-months or x_y-years, and is denoted by x_mM or x_yY, respectively, with $x_m = 12x_y$. The parameter x_y may be a whole number or a fraction, for example, using standard spot-starting swap naming conventions, a 0.25Y OIS denotes a 3-month spot-starting OIS.
- **Interest accrual period**: The length of each interest accrual period may be z_m-months or z_y-years, and is denoted by z_mM or z_yY, respectively, with $z_m = 12z_y$. The parameter z_y may be a whole number or a fraction. The length of both the fixed and floating interest accrual periods will be the same.
- **Number of accrual periods**: The number of accrual periods is denoted by n, with $n = x_m/z_m$, assuming here that n is a whole number, or equivalently, that there are n interest accrual periods of equal length. The case where n is not a whole number will be considered in the next sub-section.

Market convention considerations

There are no material market convention considerations for this category. Rather, parameter choices are made that align with international standards, best practices and market participants' practical requirements.

Recommendations

The following parameters are suggested for caplets and floorlets:

- **Swap tenor**: $x_m = 3$ (i.e. all underlying swaps have a tenor equal to 3M).
- **Interest accrual period**: $z_m = x_m$ (i.e. these are all single-period OISs).

The following parameters are suggested for spot-starting caps and floors:

- **Swap tenor**: x_yY, for all $x_y \in \{1,2,3,4,5,6,7,8,9,10,12,15\}$.
- **Interest accrual period**: $z_m = 3$ (i.e. the length of all underlying swaps' accrual periods is 3M).

The following parameters are suggested for forward-starting caps and floors:

- **Swap tenor**: x_yY, for all $x_y \in \{1,2,3,4,5,6,7,8,9,10\}$.
- **Interest accrual period**: $z_m = 3$ (i.e. the length of all underlying swaps' accrual periods is 3M).

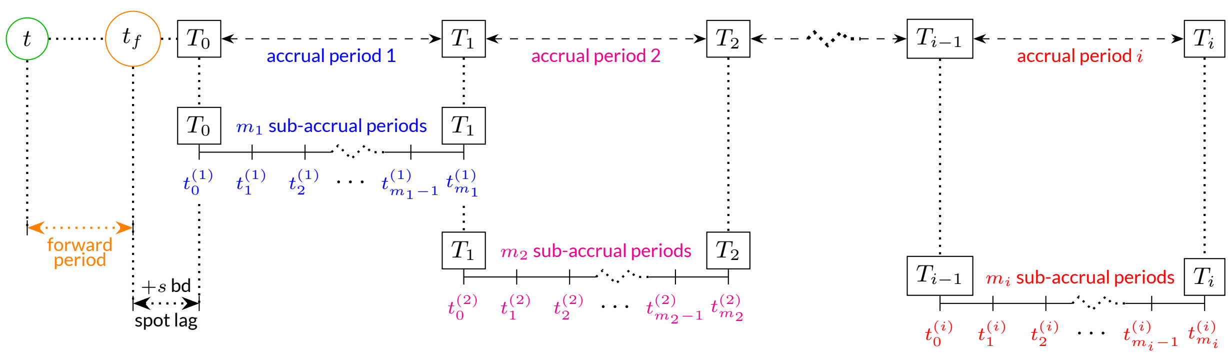
The following parameters are suggested for swaptions:

- **Swap tenor**: x_yY, for all $x_y \in \{1,2,3,4,5,6,7,8,9,10\}$.
- **Interest accrual period**: $z_m = 12$ or $z_y = 1$ (i.e. the length of all underlying swaps' accrual periods is 1Y, which aligns with the conventions recommended for ZARONIA-based multi-period OISs).

7.2. Forward, settlement and accrual period dates, year fractions and option tenors

The first part of this sub-section leads to the specification of all the key contractual dates of the underlying OIS that relate to interest accrual and the calculations thereof. Figure 1 below depicts all the dates and variables that are defined in this sub-section.

Figure 1: Forward, settlement and accrual period dates



The following are the key features that constitute this category:

- **Trade date**: The trade or transaction date is denoted by t.
- **Forward period and date**: The length of the forward period may be a_m-months or a_y-years, and is denoted by a_mM or a_yY, respectively, with $a_m = 12a_y$. The forward date is denoted and calculated as

$$t_f := \beta(t + a_y\text{Y}) = \beta(t + a_m\text{M}) \ ,$$

  where the function $\beta(\cdot)$ implements a suitable business day convention algorithm. The parameter a_y may be a whole number or a fraction.

- **Spot lag and settlement/start/effective date**: The settlement or start or effective date, which also coincides with the first interest accrual period's start date, is denoted and calculated as

$$T_0 := t_f + s\ \text{bd} \ ,$$

  where s denotes the spot lag and is quantified in valid business days (bd).

- **Interest accrual period start and end dates**: The interest accrual period start and end dates are denoted by the sets $\{T_0, T_1, \ldots, T_{n-2}, T_{n-1}\}$ and $\{T_1, T_2, \ldots, T_{n-1}, T_n\}$, respectively, such that $[T_{i-1}, T_i]$ is the i-th interest accrual period with tenor z_mM, for $i \in \{1, 2, \ldots, n\}$, and $[T_0, T_n]$ is the full swap tenor of length x_yY.

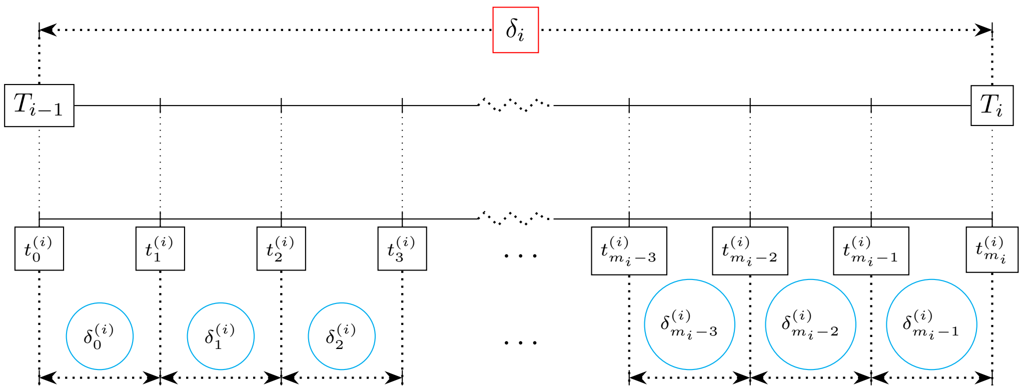
- **Interest accrual period dates**: Valid business dates within the i-th interest accrual period $[T_{i-1}, T_i]$ is denoted by the set:

$$\left\{ t_0^{(i)}, t_1^{(i)}, \ldots, t_{m_i-1}^{(i)}, t_{m_i}^{(i)} \right\} ,$$

  for $i \in \{1, 2, \ldots, n\}$, i.e. it is assumed that $T_{i-1} = t_0^{(i)}$, $T_i = t_{m_i}^{(i)}$ and that the i-th interest accrual period constitutes $m_i + 1$ valid business days, or m_i overnight interest accrual sub-periods.

Given the key contractual dates defined above, the next feature that requires definition is the method to be utilised for the calculation of interest accrual year fractions. This, in turn, enables the ultimate computation of floating and fixed cash flows. Figure 2 below depicts all the notation that is used in this sub-section to define the key interest accrual period year fractions.

Figure 2: The interest accrual year fraction for the i-th interest accrual period $[T_{i-1}, T_i]$



The relevant accrual and sub-accrual period year fractions are defined as follows:

- **Interest accrual period year fractions**: The interest accrual year fraction for the i-th interest accrual period $[T_{i-1}, T_i]$ is denoted by

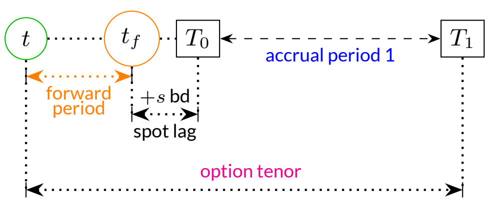
$$\delta_i := \sum_{j=0}^{m_i-1} \delta_j^{(i)} ,$$

  where $\delta_j^{(i)}$ denotes the interest accrual year fraction for the overnight interest sub-accrual period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$, for $j \in \{0, 1, \ldots, m_i - 1\}$ and $i \in \{1, 2, \ldots, n\}$.

The second part of this sub-section, which begins here, describes the manner in which the tenors for each of the options under consideration are defined. In particular, both the *forward period* and *accrual periods* play an important role in defining the option tenor in the new regime of options on ONRRs, whereas the former played the key role in the old regime of TBRRs.

Figures 3, 4, 5 and 6 below depict the option tenors for caplets/floorlets, spot-starting caps/floors, forward-starting caps/floors and swaptions, respectively.

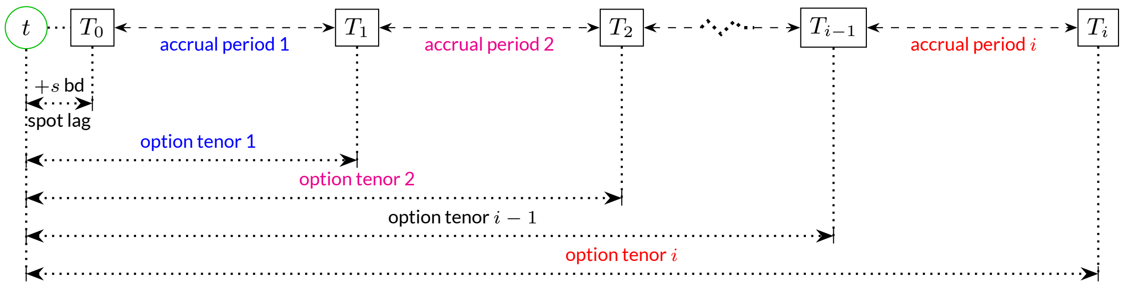
Figure 3: Option tenor for a caplet or floorlet



For a caplet or floorlet, the option tenor is defined as follows:

- **Caplet of floorlet tenor**: Having only a single accrual period, the option tenor associated with a caplet or floorlet is the entire period $[t, T_1]$ (i.e. a combination of the forward, spot lag, and accrual period).

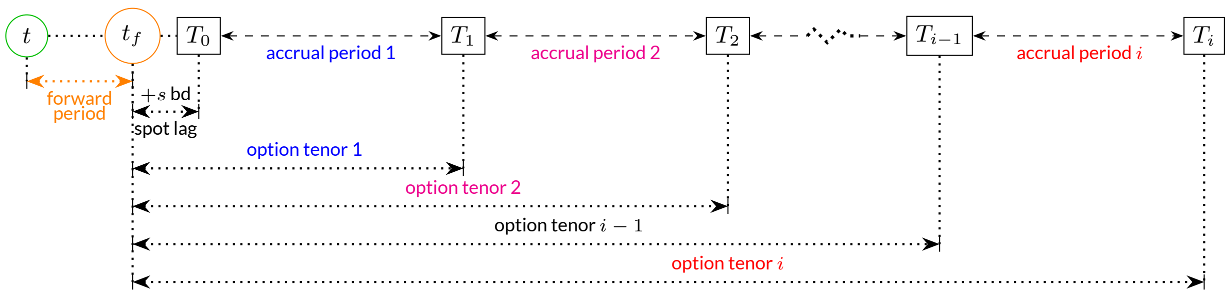
Figure 4: Option tenors for a spot-starting cap or floor



For a spot-starting cap or floor, the option tenor is defined as follows:

- **Spot-starting cap or floor tenor**: The option tenor associated with the i-th caplet or floorlet that constitutes a spot-starting cap or floor is the period $[t, T_i]$, where $[t, T_0]$ is the spot lag period and $[T_0, T_{i-1}]$ covers the set of accrual periods that precedes the main accrual period, $[T_{i-1}, T_i]$.

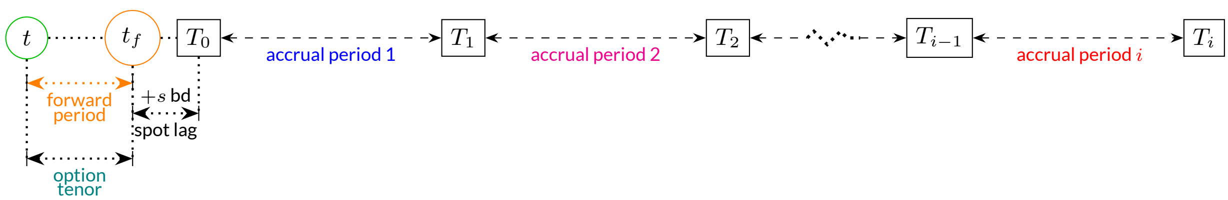
Figure 5: Option tenors for a forward-starting cap or floor



For a forward-starting cap or floor, the option tenor is defined as follows:

- **Forward-starting cap or floor tenor**: The option tenor associated with the i-th caplet or floorlet that constitutes a forward-starting cap or floor is the period $[t_f, T_i]$, where $[t_f, T_0]$ is the spot lag period and $[T_0, T_{i-1}]$ covers the set of accrual periods that precedes the main accrual period, $[T_{i-1}, T_i]$.

Figure 6: Option tenor for a swaption



For a swaption, the option tenor is defined as follows:

- **Swaption tenor**: The option tenor associated with a swaption coincides with the forward period (i.e. the option tenor is $[t, t_f]$, the spot lag period is $[t_f, T_0]$ and the set of accrual periods associated with the underlying OIS covers the period $[T_0, T_i]$).

Market convention considerations

The practical generation of the dates defined above requires:

i. an official and internationally utilised *South African calendar*;

ii. a choice of *business day convention*; and

iii. an *interest rate swap (IRS) interest accrual period date generation algorithm.*

The appropriate calendar is sourced or referenced from a relevant government authority or an internationally recognised publisher of financial trading and settlement calendars.

A business day convention specifies an algorithm to adjust a date when that date is not a valid business day (i.e. the date is a weekend or public holiday), subject to a specific reference calendar. There are six business day convention algorithms that are prominent in international interest rate derivative markets, namely, *Following*, *Preceding*, *Modified Following*, *Modified Preceding*, *Modified Following Bi-Monthly* and *End-of-Month* (EOM).

Given an IRS with tenor x_yY, interest accrual period z_mM, n accrual periods and spot settlement date T_0, the standard IRS interest accrual period date *Backward* generation algorithm may be presented as follows:

- **Step 1** — calculate the *roll-day*, denoted here by U_n, by adding the swap tenor to the spot settlement date:

$$U_n := T_0 + x_y\text{Y} ,$$

  which is the unadjusted (i.e. not adjusted by a business day convention) IRS maturity date.

- **Step 2** — sequentially subtract the interest accrual period from the roll-day:

$$U_i := U_n - (n - i)z_m\text{M} ,$$

  for $i \in \{n-1, n-2, \ldots, 1\}$, which creates the set of unadjusted interest accrual period end dates.

- **Step 3** — adjust all the dates from step 1 and 2 as follows:

$$T_i := \beta(U_i) ,$$

  for $i \in \{1, 2, \ldots, n\}$, where $\beta(\cdot)$ denotes a function which implements one of the aforementioned business day convention algorithms, which yields all the required interest accrual period dates.

The *Backward (EOM)* algorithm is identical to the above, except that the EOM algorithm is applied to each unadjusted date in steps 1 and 2. If n is not a whole number, then an additional step before step 1 is required:

- **Step 0** — if n is not a whole number, then set

$$n = \lceil x_m/z_m \rceil = \lceil 12x_y/z_m \rceil ,$$

  (i.e. round-up, for an initial period with tenor shorter than zM, referred to as a *short-stub*), or set

$$n = \lfloor x_m/z_m \rfloor = \lfloor 12x_y/z_m \rfloor ,$$

  (i.e. round-down, for an initial period with tenor longer than zM, referred to as a *long-stub*).

This Backward date generation algorithm ensures that the non-standard interest accrual period is the first one. In turn, this ensures that once that period has passed, the IRS will have the same accrual period dates as a directly comparable newly issued IRS. If the non-standard period was the last period, these IRSs will never align.

The practical calculation of the length of interest accrual periods requires a choice of day-count convention.

A day-count convention is a standardised methodology for calculating the number of days between two dates, and then converting this count into a standardised year fraction. There are seven day-count convention methodologies that are prominent in international interest rate derivative markets, namely *30/360*, *30/360 US*, *30E/360*, *ACT/360*, *ACT/365 Fixed*, *ACT/ACT ISDA* and *Business/252*.

Recommendations

The following conventions are recommended:

- **Calendar**: ZAJO as published by a relevant provider that adheres to the Republic of South Africa's Public Holidays Act 36 of 1994.
- **Spot lag**: $s = 0$ (i.e. the effective date equals the trade or the forward date).
- **Business day convention**: Modified Following.
- **Accrual period date generation**: Backward (EOM) algorithm to determine start and end dates. The sub-accrual periods may then be identified using the ZAJO calendar.
- **Day-count convention**: ACT/365 Fixed.

The following parameters are suggested for caplets and floorlets:

- **Forward period**: a_mM, for all $a_m \in \{0,1,2,3,6,9,12,15,18,21\}$.
- **Option tenor**: $(a_m + x_m)$M, for all $a_m \in \{0,1,2,3,6,9,12,15,18,21\}$ and $x_m = 3$.

The following parameters are suggested for spot-starting caps and floors:

- **Option tenor**: x_yY, for all $x_y \in \{1,2,3,4,5,6,7,8,9,10,12,15\}$.

The following parameters are suggested for forward-starting caps and floors:

- **Forward period**: a_mM, for all $a_m \in \{1,2,3,6,12,18,24,36,48,60,72,84,96,108,120\}$.
- **Option tenor**: x_yY, for all $x_y \in \{1,2,3,4,5,6,7,8,9,10\}$.

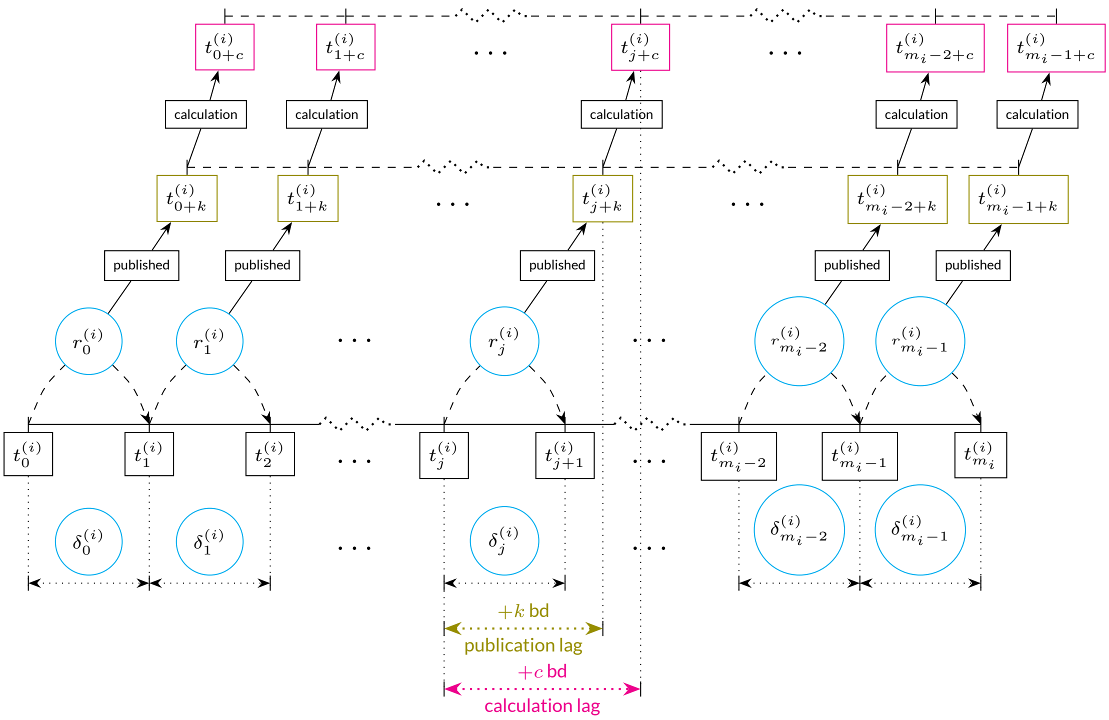
The following parameters are suggested for swaptions:

- **Forward period**: a_mM, for all $a_m \in \{1,2,3,6,12,18,24,36,48,60,72,84,96,108,120\}$.
- **Option tenor**: Equal to the forward period (i.e. $x_m = a_m$).

7.3. Floating reference rates, publication and calculation lags

The fundamental difference between OISs and vanilla IRSs arises from the specification of the floating cash flows, with the latter using TBRRs and the former using ONRRs. This sub-section introduces this new floating reference rate in a fairly general manner, highlighting nuances in the publication of such a rate and its use in calculations, all of which have a material impact on the eventual computation of floating cash flows. Figure 7 below depicts all the notation that is utilised in this sub-section.

Figure 7: The floating ONRRs within $[T_{i-1}, T_i]$, with the lags depicted for the j-th ONRR $r_j^{(i)}$



The key feature here is the ONRR, which is the floating reference rate. The definition of the ONRR enables the description of the publication and calculation lag features. These definitions and descriptions are provided below:

- **Floating reference rate**: The floating reference rate is an ONRR that is applicable over $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$, the arbitrary interest accrual sub-period which will always have a tenor of one business day. This ONRR is denoted by the annualised simple rate $r_j^{(i)}$, for $j \in \{0, 1, 2, \ldots, m_i - 1\}$ and $i \in \{1, 2, \ldots, n\}$. The associated overnight capitalisation factor associated with this arbitrary ONRR is denoted and defined as

$$C_j^{(i)} := 1 + r_j^{(i)} \delta_j^{(i)} ,$$

  again, for $j \in \{0, 1, 2, \ldots, m_i - 1\}$ and $i \in \{1, 2, \ldots, n\}$. At the surface, the ONRR defined above is straightforward. However, one of the main differences between the TBRR and ONRR market micro-structures is that the former is based on *quoted rates*, while the latter is based on *transacted rates*. Reference rates derived from quoted rates may therefore be calculated and observed *in-advance*, while those derived from transacted rates may only be calculated and observed *in-arrears*, at best. This nuance necessitates the definition of a feature called a *publication lag*, which is explained next.

- **Publication lag**: While the arbitrary ONRR, $r_j^{(i)}$, has an interest accrual period that starts on $t_j^{(i)}$, the *calculation agent* will only be able to observe relevant transactions during day $t_j^{(i)}$, or mathematically over the period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right)$ and therefore, the earliest that the agent could calculate the relevant ONRR will be at the end of day $t_j^{(i)}$. This means that the rate will be available for use on day $t_{j+1}^{(i)}$. However, operational issues and inefficiencies (potential or otherwise) may preclude the calculation agent from publishing the ONRR on day $t_{j+1}^{(i)}$ consistently. The calculation agent may therefore choose to be prudent and specify a publication lag that is greater than one business day after $t_j^{(i)}$. This publication lag feature is captured here via the date

$$t_{j+k}^{(i)} \geq t_{j+1}^{(i)} ,$$

where k denotes the publication lag and is quantified in valid business days (bd).

- **Calculation lag**: A user of the arbitrary ONRR, $r_j^{(i)}$, may prefer to be more prudent than the calculation agent, for their own operational reasons, and add a lag of their own when using the ONRR for interest accrual calculation purposes. This calculation lag feature is captured here via the date

$$t_{j+c}^{(i)} \geq t_{j+k}^{(i)} ,$$

where c denotes the calculation lag and is quantified in valid business days (bd).

Market convention considerations

The SARB's MPG has recommended that ZARONIA be used as the main ONRR in the South African interest rate derivatives market and has detailed the technical specification thereof in [SARB, 2020].

Recommendations

The following conventions are recommended:

- **Floating reference rate**: South African Overnight Index Average (ZARONIA).
- **Publication/Calculation lag**: The SARB's MPG recommends that the calculation lag be set equal to the SARB's chosen publication lag (i.e. $c = k$). Furthermore, the SARB has indicated that the publication lag will be one-business day (i.e. $k = 1$), with the standard publication time being 10:00 SAST, or 12:00 SAST if there are errors that warrant republication.

7.4. Averaging, lookback and lockout periods

Another nuance that arises due to the use of floating ONRRs, as opposed to TBRRs, in the specification of an IRS, is the definition of the *annualised cumulative floating rate* (ACFR) for a given full accrual period based on the floating ONRRs. This requires the notion of *averaging* the ONRRs, for which there are two alternatives, the use of which result in the following types of ACFRs:

- **Simple ACFR**: Based on an *arithmetic average* that is weighted by the length of each sub-accrual period, the simple ACFR for the arbitrary i-th accrual period is denoted and calculated as

$$F_{(i)}^{\mathsf{S}} := \frac{1}{\delta_i} \sum_{j=0}^{m_i-1} r_j^{(i)} \delta_j^{(i)} ,$$

where $\delta_j^{(i)}$ and δ_i is defined in sub-section 7.2., and $r_j^{(i)}$ is defined in sub-section 7.3.

- **Compounded ACFR**: Splitting the i-th accrual period into m_i sub-accrual periods of equal length Δ_i, it is possible to define the *nominal annual compounded* m_i-times rate

$$f_{(i)} := \frac{1}{\Delta_i} \left[\left(\prod_{j=0}^{m_i-1} \left[1 + r_j^{(i)} \delta_j^{(i)}\right] \right)^{1/m_i} - 1 \right] ,$$

which is based on a *geometric average*, with $\Delta_i := \delta_i / m_i$. Then, the following

$$F_{(i)} := \frac{1}{\delta_i} \left(\left[1 + f_{(i)} \Delta_i\right]^{m_i} - 1 \right) = \frac{1}{\delta_i} \left(\prod_{j=0}^{m_i-1} \left[1 + r_j^{(i)} \delta_j^{(i)}\right] - 1 \right) ,$$

yields the compounded ACFR that is applicable for the i-th accrual period.

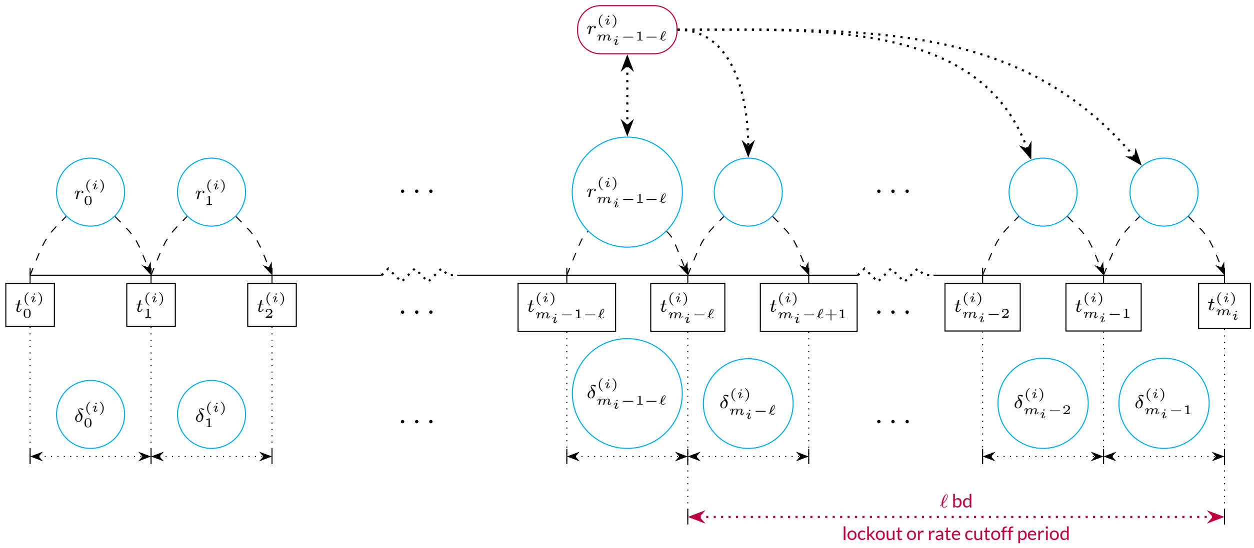
The publication and calculation lags, defined in sub-section 7.3., create non-intuitive payment or settlement issues, since users of the floating reference rate may only know interest cash flows *post in-arrears*. Even in-arrears knowledge of the interest cash flow may be problematic, as same-day settlement may not be possible. To resolve these practical timing issues, market practitioners have proposed the following fixing adjustments:

- **Lockout/Rate cutoff period**: Given a *lockout* or *rate cutoff period* value equal to ℓ business days means that

$$r_j^{(i)} = r_{m_i-1-\ell}^{(i)} \,,$$

for all $j \in \{m_i-\ell, m_i-\ell+1, \ldots, m_i-1\}$. Figure 8 below depicts the practical implications of this adjustment.

Figure 8: A depiction of the lockout or rate cutoff period adjustment

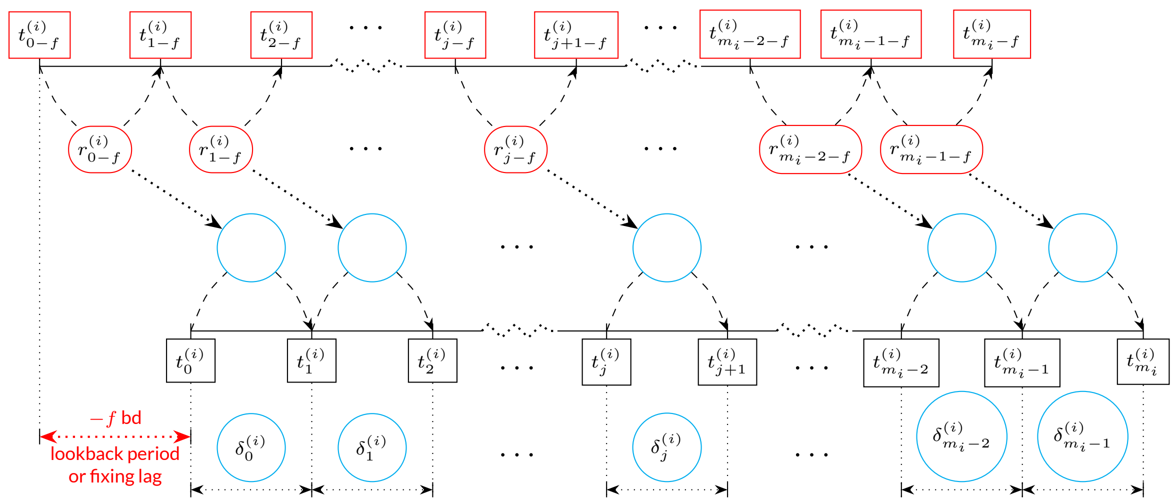


- **Lookback period/Fixing lag**: Consider the sub-accrual period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$ and a *lookback period* or *fixing lag* value equal to f business days. Then, the capitalisation factor for this period is calculated as

$$C_j^{(i)} = 1 + r_{j-f}^{(i)}\, \delta_j^{(i)} \,,$$

which will be calculable at date $t_{j-f+k}^{(i)}$, where k is the publication lag. Therefore, one can compute interest cash flows in-advance if $f = k$. Figure 9 below depicts the practical implications of this adjustment.

Figure 9: A depiction of the lookback period or fixing lag adjustment

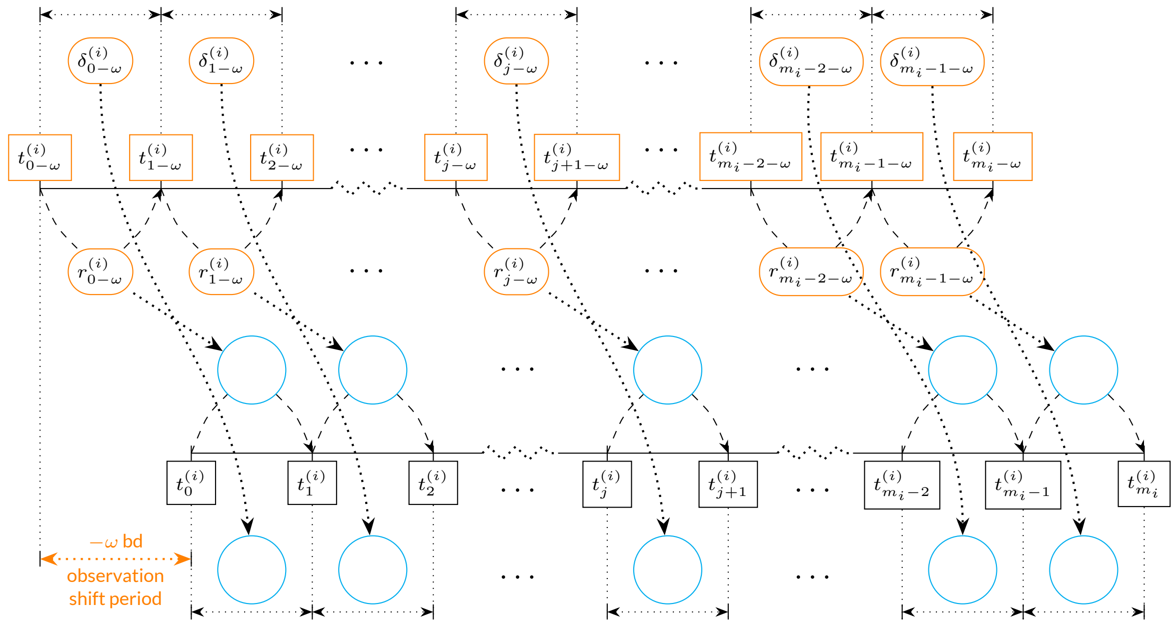


- **Observation shift period**: Consider the sub-accrual period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$ and an *observation shift period* value equal to ω business days, then the capitalisation factor for this period is calculated as

$$C_j^{(i)} = 1 + r_{j-\omega}^{(i)}\,\delta_{j-\omega}^{(i)} = C_{j-\omega}^{(i)}\,,$$

  which will be calculable at date $t_{j-\omega+k}^{(i)}$, where k is the publication lag. Therefore, with this adjustment, the capitalisation factor for the past sub-accrual period $\left[t_{j-\omega}^{(i)}, t_{j+1-\omega}^{(i)}\right]$ is used for the actual period $\left[t_j^{(i)}, t_{j+1}^{(i)}\right]$. Figure 10 below depicts the practical implications of this adjustment.

Figure 10: A depiction of the observation shift period adjustment



Market convention considerations

Recommendations

The following conventions are recommended:

- **Averaging**: Geometric averaging, resulting in a compounded ACFR.

The following adjustments are prominent in cash markets, but have been included here for completeness.

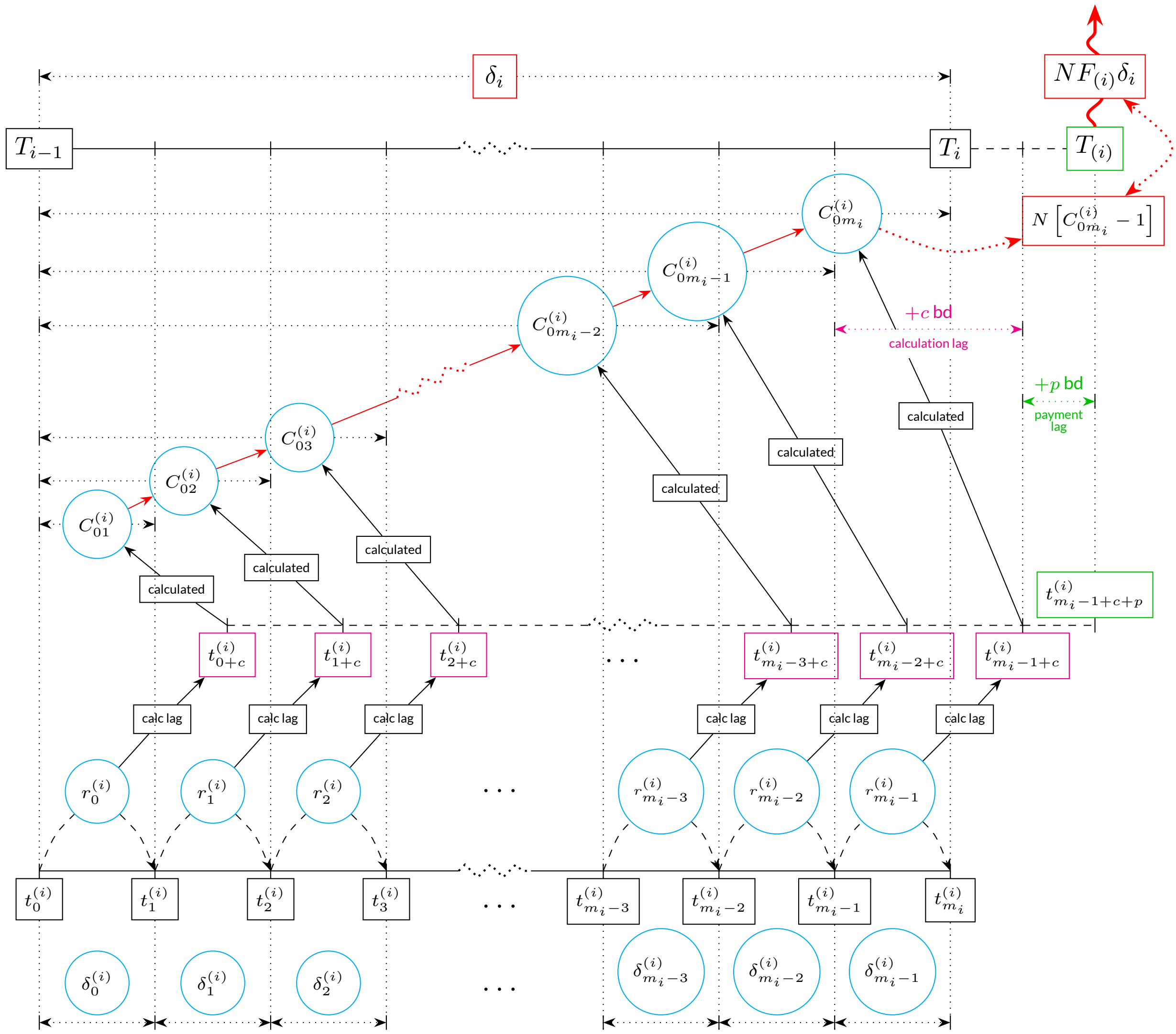
- **Lockout/rate cutoff period**: $\ell = 0$.
- **Lookback period/fixing lag**: $f = 0$.
- **Observation shift period**: $\omega = 0$.

In summary, no fixing adjustments are recommended for the specification of derivative market instruments. Rather, the payment lag feature is recommended to solve for practical settlement issues.

7.5. Floating cash flow calculations and payment lags

Having defined the ACFR in the previous sub-section, this sub-section details the calculation of the floating cash flow. All the necessary parameters, variables and equations are depicted in Figure 11 below.

Figure 11: Floating cash flow calculation at the arbitrary i-th payment date $T_{(i)}$



The key quantity here is the computation of the realised capitalisation factor over the interest accrual period. This enables the computation of the compounded ACFR, the eventual floating cash flow and its specific nuances. All these quantities are defined as follows:

- **Realised capitalisation factors**: Given the i-th interest accrual period and based on the floating ONRRs within this period, the capitalisation factor that is realised over the sub-accrual period $[t_0^{(i)}, t_h^{(i)}]$ is denoted and defined by

$$C_{0h}^{(i)} := \prod_{j=0}^{h-1} C_j^{(i)} = \prod_{j=0}^{h-1} \left[1 + \delta_j^{(i)} r_j^{(i)}\right] ,$$

  and is only calculable at $t_{h-1+c}^{(i)}$ (i.e. taking into account the calculation lag of c bd), for $h \in \{1, 2, \dots, m_i\}$.

- **Floating rate**: The floating rate for the i-th interest accrual period may then be denoted and defined as

$$F_{(i)} := \frac{1}{\delta_i} \left[C_{0m_i}^{(i)} - 1\right] ,$$

  which is the compounded ACFR, and a *backward-looking* or *realised* term rate implied from the corresponding realised capitalisation factor, and is therefore also only calculable at $t_{m_i-1+c}^{(i)}$, for $i \in \{1, 2, \dots, n\}$.

- **Floating cash flows**: Given the compounded ACFR, the floating cash flow may then be calculated as

$$N F_{(i)} \delta_i \ ,$$

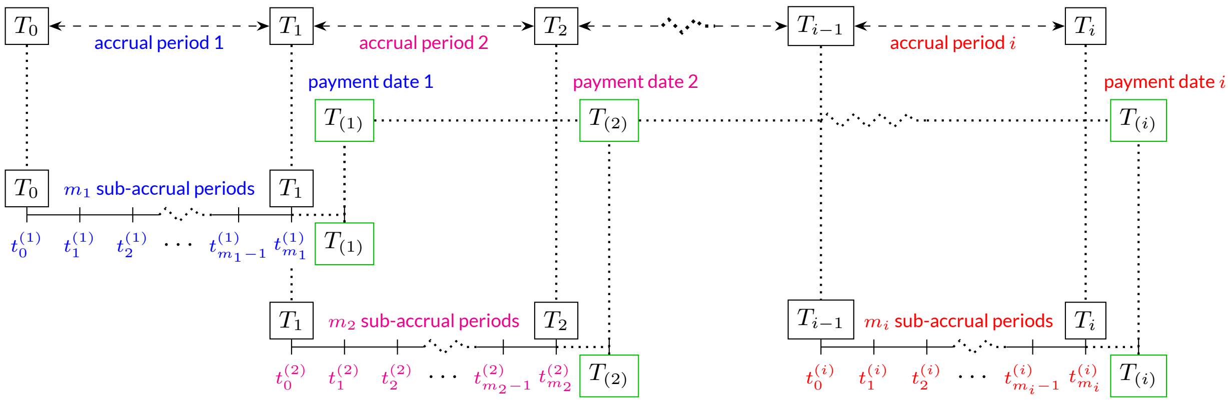
which is also only calculable at $t^{(i)}_{m_i-1+c}$, where N denotes the swap nominal, for $i \in \{1, 2, \ldots, n\}$.

- **Payment lag**: Consider again the arbitrary i-th interest accrual period. Since the floating cash flow is only calculable at date $t^{(i)}_{m_i-1+c}$, which would be the end of the i-th interest accrual period at best if $c = 1$, and the execution or settlement of payments may be subject to delays in practice, it would be naive to assume that same-day settlement is possible consistently. This necessitates the definition of a payment lag, which is captured here via the payment date

$$t^{(i)}_{m_i-1+c+p} := T_{(i)} \ ,$$

where p denotes the payment lag and is quantified in valid business day (bd). In other words, the payment lag is p business days after the last calculation date. These payment dates are depicted in Figure 12 below, which is essentially an update to Figure 1 above.

Figure 12: Accrual period and payment dates



- **Spread**: For some bespoke transactions involving OISs or components thereof, for example, *par-par asset swaps* or *cross-currency basis swaps*, it may be necessary to add a *fixed spread rate*, denoted here by x, to the floating leg. The i-th floating cash flow for such a swap is then calculated as

$$N \left[F_{(i)} + x \right] \delta_i \ ,$$

(i.e. the fixed spread rate is an annualised simple rate that is added to the compounded ACFR).

Market convention considerations

Recommendations

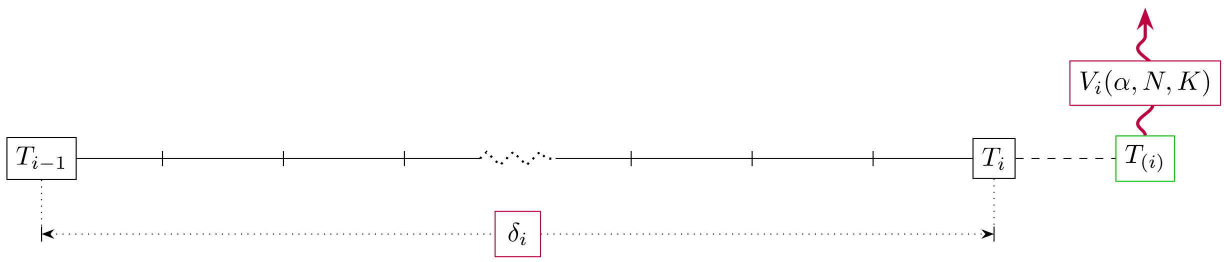
The following conventions are recommended:

- **ACFR convention:** $F_{(i)}$ is an annualised simple interest rate, as defined above, and should be rounded to 6 decimal places in numerical format, or 4 decimal places in percentage format. For example, if $F_{(i)} =$ 0.07123456, then round to 0.071235 or 7.1235%.
- **Payment lag**: $p =$ 2 (i.e. considering that the publication/calculation lag $c = k = 1$, this means that settlement of the net cash flow will occur two-business days after then end of an accrual period).

7.6. Fixed rate strikes, payoff calculations and settlement

The calculation of the fixed cash flow associated with the payoffs for caplets and floorlets, and thereby caps and floors, follows in the same way as that associated with the underlying OIS. Figure 13 depicts all the notation that is utilised in this sub-section, which is required to define the fixed cash flow and payoff associated with a caplet or floorlet with accrual period $[T_{i-1}, T_i]$.

Figure 13: Payoff for a caplet or floorlet with accrual period $[T_{i-1}, T_i]$



The following describes the key features that constitutes a caplet or floorlet's payoff:

- **Fixed rate strike**: The fixed rate strike, which is denoted by K, is an annualised simple rate, as is the case with the fixed rates that determine fixed cash flows for ZARONIA-based linear derivatives.

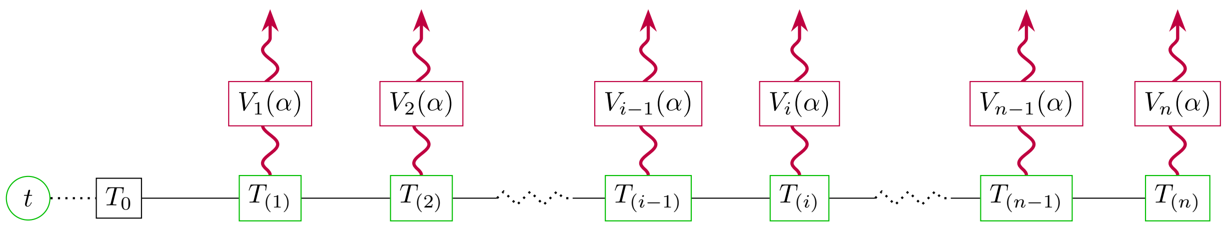
- **Caplet or floorlet payoff**: The terminal payoff of a caplet or floorlet with accrual period $[T_{i-1}, T_i]$ is given by

$$V_i(\alpha, N, K) := N \max \left[\alpha \left(F_{(i)} - K\right), 0\right] \delta_i \ ,$$

  where $\alpha = 1$ for a caplet and $\alpha = -1$ for a floorlet. If this payoff is a positive value, then the writer (short position holder) of the option will pay the (long position) holder this amount at the payment date $T_{(i)}$.

Suppressing the notation for the nominal, N, and the fixed rate strike, K, Figure 14 below depicts the payoff for a spot- or forward-starting cap or floor with accrual periods $[T_{i-1}, T_i]$, for $i \in \{1, 2, \ldots, n\}$. Take note that the forward period associated with forward-starting caps or floors is not depicted in the figure below (see Figure 5 for a reminder of this feature).

Figure 14: Payoff for a cap or floor with n accrual periods.



The following describes the key features that constitutes a cap or floor's payoff:

- **Cap or floor payoff**: The terminal payoff of a cap or floor is given by

$$V_i(\alpha) := N \max \left[\alpha \left(F_{(i)} - K\right), 0\right] \delta_i \ ,$$

  where $\alpha = 1$ for a cap and $\alpha = -1$ for a floor, for $i \in \{1, 2, \ldots, n\}$. As with interest rate swap floating and fixed cash flows, these are also netted or subject to net settlement and the payment lag, as defined in sub-section 7.5., and will occur on the set of payment dates $\{T_{(1)}, T_{(2)}, \ldots, T_{(n)}\}$. Figure 14 depicts the full set of net cash flows, or equivalently the full set of payoffs, for a long position in a general cap or floor that has been defined throughout section 7.

The case of swaptions is a bit more complex, since these options offer the holder the right but not the obligation to enter into an underlying OIS at the expiry of the contract. Therefore, physical settlement is more natural with these option contracts; however, a payoff and settlement value may be defined as follows:

- **Swaption payoff**: The terminal payoff of a swaption may be defined as:

$$V(t_f; \alpha, N, K, T_0, T_n) := \max\left[\alpha\left(S(t_f; T_0, T_n) - K\right), 0\right] \sum_{i=1}^{n} \delta_i P(t_f, T_{(i)}) \, ,$$

  where $\alpha = 1$ for a payer swaption and $\alpha = -1$ for a receiver swaption, with t_f being the option expiry date, $\{T_0, T_1, \ldots, T_{n-1}\}$ and $\{T_1, T_2, \ldots, T_n\}$ being the underlying OIS accrual period start and end dates respectively. The quantity $S(t_f; T_0, T_n)$ denotes the prevailing underlying OIS's fixed rate at time t_f, while $P(t_f, T_{(i)})$ is a discount factor that applies to the period $[t_f, T_{(i)}]$ and is based on the respective collateral remuneration rate that is set by the relevant clearing agent.

Market convention considerations

Recommendations

The following conventions are recommended for caplets and floorlets:

- **Fixed rate strike quotation**: K should be rounded to 6 decimal places in numerical format, or 4 decimal places in percentage format. For example, if K =0.07123456, then round to 0.071235 or 7.1235%. Suggested quoted strikes for each option tenor:

$$\{K \pm 2\%, K \pm 1.5\%, K \pm 1\%, K \pm 0.75\%, K \pm 0.5\%, K \pm 0.25\%, K\} \, ,$$

  where K is the respective at-the-money strike rate for the option tenor under consideration.

- **Net cash flow rounding**: For a caplet or floorlet, compute $V_i(\alpha, N, K)$, which is denominated in ZAR, and if positive then round to 2 decimal places, or to the nearest ZAc.

The following conventions are recommended for spot- or forward-starting caps and floors:

- **Fixed rate strike quotation**: K should be rounded to 6 decimal places in numerical format, or 4 decimal places in percentage format. For example, if K =0.07123456, then round to 0.071235 or 7.1235%. Suggested quoted strikes for each option tenor:

$$\{K \pm 3\%, K \pm 2.5\%, K \pm 2\%, K \pm 1.5\%, K \pm 1\%, K \pm 0.5\%, K\} \, ,$$

  where K is the respective at-the-money strike rate for the option tenor under consideration.

- **Net cash flow rounding**: For each underlying caplet or floorlet, compute $V_i(\alpha, N, K)$, which is denominated in ZAR, and if positive then round to 2 decimal places, or to the nearest ZAc.

The following conventions are recommended for swaptions:

- **Fixed rate strike quotation**: K should be rounded to 6 decimal places in numerical format, or 4 decimal places in percentage format. For example, if K =0.07123456, then round to 0.071235 or 7.1235%. Suggested quoted strikes for each option tenor:

$$\{K \pm 4\%, K \pm 3\%, K \pm 2\%, K \pm 1.5\%, K \pm 1\%, K \pm 0.5\%, K\} \, ,$$

  where K is the respective at-the-money strike rate for the option tenor under consideration.

8. Market quote conventions

In this section we outline the standard market quote conventions for non-linear derivatives on ZARONIA. As before, these are caplets, floorlets, caps, floors, and swaptions. Three quote formats are mandated — premium, *Black* volatility and *Normal* (or *Bachelier*) volatility. We provide the market conventions to convert between these three formats using standard formulas.

Generally, premium is considered the primary quote method with volatilities quoted through a standard convention that recovers premium. Historically, the South African market has used the Black formula (Log-normal model) for converting volatility to price and vice versa. Given the high interest rate environment in South Africa, where the probability of negative rates is negligible, we retain the Black formula as a quote method. However, to ensure compatibility with international markets and allow for the future possibility of lower interest rate regimes, we also include the Normal (or Bachelier) formula.

We emphasise that the pricing methodologies introduced here are not meant to dictate how individual institutions should price and hedge these instruments internally. The methodologies are provided to ensure a standardised market convention so that all participants can consistently convert between price and volatility. We now summarise the key standard features common to the non-linear derivative instruments mentioned above:

- **Quote methods:** Premium, Black (Log-normal) volatility and Normal (Bachelier) volatility.
- **Quote units:** Basis points of notional (premium), basis points per annum (volatility) and percentage per annum (volatility).
- **Benchmark CSA:** USD, zero threshold CSA, with SOFR being the relevant collateral rate. See sub-section 8.2.

Premium type and lag are discussed in sub-sections 8.3.1. and 8.3.3. for the relevant derivatives.

The remainder of this section is structured as follows — we introduce the basic formulas required (Black or Normal) and then discuss the discounting associated with the benchmark Credit Support Annex (CSA), as mentioned above. In sub-section 8.3.1. we provide the formulae for caplets and floorlets, as well as spot- and forward-starting caps and floors. As an addendum to this section we discuss the concept of volatility decay associated with backward-looking rates, which is implemented as an optional feature. The formulas for swaptions are presented in sub-section 8.3.3.

8.1. Basic pricing formulas

In this section we describe the formulas used for pricing options on a rate X_t, which may be a backward-looking ZARONIA ACFR or a forward swap rate.

8.1.1. Black or Log-normal formula

The Black or Log-normal ($\mathcal{LN}$) formula assumes that the underlying rate X_t, with maturity T, follows a geometric Brownian motion with volatility $\sigma_{\mathcal{LN}}$ under its forward measure as

$$\mathrm{d}X_t = \sigma_{\mathcal{LN}} X_t \,\mathrm{d}W_t \;, \tag{1}$$

where W_t is a standard Brownian motion. Then the undiscounted price of a call or a put option written on X_t, with strike price K and time to maturity $\mathcal{T} = T - t$ is defined as the expectation taken under the forward measure with the information available at time $t < T$ given by

$$\mathbb{E}_t\big[\,\max[\alpha(X_T - K), 0]\big] = \mathcal{LN}(X_t, K, \sigma_{\mathcal{LN}}, \mathcal{T}, \alpha) \;, \tag{2}$$

where $\alpha = 1$ for a call option and $\alpha = -1$ for a put option, respectively, in terms of the Black formula

$$\mathcal{LN}(X, K, \sigma, \mathcal{T}, \alpha) := \alpha \left(X\Phi(\alpha d_1) - K\Phi(\alpha d_2) \right) \ , \tag{3}$$

with

$$d_1 = \frac{\log\left(\frac{X}{K}\right) + \frac{\sigma^2}{2}\mathcal{T}}{\sigma\sqrt{\mathcal{T}}} \tag{4}$$

and

$$d_2 = \frac{\log\left(\frac{X}{K}\right) - \frac{\sigma^2}{2}\mathcal{T}}{\sigma\sqrt{\mathcal{T}}} \ , \tag{5}$$

where $\Phi(\cdot)$ is the Normal cumulative distribution function.

8.1.2. Bachelier or Normal formula

The Bachelier or Normal ($\mathcal{N}$) formula assumes that the underlying rate X_t, with maturity T, follows an arithmetic Brownian motion with volatility $\sigma_{\mathcal{N}}$ under its forward measure as

$$\mathrm{d}X_t = \sigma_{\mathcal{N}} \, \mathrm{d}W_t \ , \tag{6}$$

where W_t is a standard Brownian motion. Then the undiscounted price of a call or a put option written on X_t, with strike price K and time to maturity $\mathcal{T} = T - t$ is defined as the expectation taken under the forward measure with the information available at time $t < T$ given by

$$\mathbb{E}_t\big[\max[\alpha(X_T - K), 0]\big] = \mathcal{N}(X_t, K, \sigma_{\mathcal{N}}, \mathcal{T}, \alpha) \ , \tag{7}$$

where $\alpha = 1$ for a call option and $\alpha = -1$ for a put option respectively, in terms of the Bachelier formula

$$\mathcal{N}(X, K, \sigma, \mathcal{T}, \alpha) := \alpha(X - K)\Phi(\alpha d) + \sigma\sqrt{\mathcal{T}}\phi(d) \tag{8}$$

and

$$d = \frac{X - K}{\sigma\sqrt{\mathcal{T}}} \ , \tag{9}$$

where $\Phi(\cdot)$ is the Normal cumulative distribution function and $\phi(\cdot)$ is the Normal density function.

8.2. Discounting

In the previous section we introduced the undiscounted prices of call and put options. When constructing caps, floors and swaptions an appropriate discount curve is required. However, discounting is dependent on the currency in which collateral is posted. If the benchmark CSA assumption was that collateral is posted in domestic currency (ZAR), with ZARONIA as the collateral remuneration rate, then the appropriate discount curve would be based on the ZARONIA swap curve.

Within the ZAR interbank non-linear derivatives market, the vast majority of contracts are traded with offshore counterparties under a benchmark USD-denominated CSA. Consequently, the various implied volatilities indicated on the inter-dealer broker screens make the underlying assumption of an underlying USD-denominated CSA. Under this pricing regime, the discount curve should be the ZAR basis curve (i.e., the curve that defines the cost of funding USD synthetically via the foreign exchange derivatives market, using USDZAR forwards, swaps, and cross-currency basis swaps).

The tenor structure assumed is T $= \{T_0, T_1 \ldots, T_n\}$, with $t < T_0 < T_1 < \ldots < T_n$, associated with the instrument we are pricing, where n is the number of caplets, floorlets or swaplets. The discount curve is represented by a series of zero-coupon bonds, denoted by $P(t, T_i)$ and $P(t, T_{(i)})$, paying a unit of currency at maturities T_i and $T_{(i)} = T_i + p$, where p is a payment lag in units of business days.

8.3. Pricing of non-linear derivatives

Assume the tenor structure T introduced in the previous section with associated accrual year fractions $\delta_i = T_i - T_{i-1}$. The time-t ZARONIA ACFR over accrual period i is denoted $F_{(i)}(t)$. The time-t forward swap rate is computed in terms of the n ACFRs and discount factors as

$$S(t; T_0, T_n) = \frac{\sum_{i=1}^{n} \delta_i P(t, T_{(i)}) F_{(i)}(t)}{\sum_{i=1}^{n} \delta_i P(t, T_{(i)})} . \tag{10}$$

8.3.1. Caps and floors

The recommended conventions for caps and floors are as follows:

- **Premium type:** Spot date plus payment lag.
- **Premium lag:** $p = 2$ business days.
- **Volatility decay:** None (default), Linear (optional). See sub-section 8.3.2.

Since we only consider caplets and floorlets on ACFR that have not yet started accruing, the forward date, t_f, is less than or equal to the start of the first accrual period, T_0, with the difference being the spot lag (if any).

The time $t \leq t_f \leq T_0$ price of a cap with first reset date T_0, payment dates $T_1, \ldots, T_n$, where n is the number of caplets with time to maturities given by $\mathcal{T} := \{\mathcal{T}_1, \ldots, \mathcal{T}_n\}$, strike rate K and notional amount N, is

$$\textbf{Cap}(t; \mathrm{T}, \mathcal{T}, N, K) := \begin{cases} N \sum_{i=1}^{n} \delta_i \frac{P(t,T_{(i)})}{P(t,t^*)} \mathcal{LN}(F_{(i)}(t), K, \sigma_{\mathcal{LN}}, \mathcal{T}_i, 1) , & \text{for the } \mathcal{LN} \text{ formula,} \\ N \sum_{i=1}^{n} \delta_i \frac{P(t,T_{(i)})}{P(t,t^*)} \mathcal{N}(F_{(i)}(t), K, \sigma_{\mathcal{N}}, \mathcal{T}_i, 1) , & \text{for the } \mathcal{N} \text{ formula,} \end{cases} \tag{11}$$

where payment happens at $t^* = t + p$ bd. Similarly, the price of a floor is

$$\textbf{Flr}(t; \mathrm{T}, \mathcal{T}, N, K) := \begin{cases} N \sum_{i=1}^{n} \delta_i \frac{P(t,T_{(i)})}{P(t,t^*)} \mathcal{LN}(F_{(i)}(t), K, \sigma_{\mathcal{LN}}, \mathcal{T}_i, -1) , & \text{for the } \mathcal{LN} \text{ formula,} \\ N \sum_{i=1}^{n} \delta_i \frac{P(t,T_{(i)})}{P(t,t^*)} \mathcal{N}(F_{(i)}(t), K, \sigma_{\mathcal{N}}, \mathcal{T}_i, -1) , & \text{for the } \mathcal{N} \text{ formula.} \end{cases} \tag{12}$$

We remind the reader that the time to maturity for the i^{th} caplet/floorlet is given as

$$\mathcal{T}_i = T_i - t, \tag{13}$$

but may be modified to account for an optional volatility decay as described below.

8.3.2. Volatility decay

Volatility decay is implemented as an optional feature when quoting using a volatility. Aside from specifying whether the Black or Normal volatility is being used, market participants should specify whether or not a volatility decay has been applied.

As seen in the previous section, see (13), the time to maturity of the i^{th} caplet/floorlet is longer than is applicable for an underlying legacy forward-looking rate (e.g., 3-month Jibar). It is longer by the i^{th} accrual period because the underlying ZARONIA ACFR is backward-looking and remains volatile after T_{i-1}. However, during this accrual period (between T_{i-1} and T_i) the volatility decays. The reason for this is that there is a progressive fixing of constituent overnight rates that make up $F_{(i)}(\cdot)$. Consider the constituent overnight reference rates ($r_j^{(i)}$ for $0 \leq j \leq m_i - 1$) associated with $F_{(i)}(T_{i-1})$, being the ACFR underlying the i^{th} caplet, as viewed from time T_{i-1}: the first overnight rate ($r_0^{(i)}$) is only volatile for a single day, the second for two days, and so on. As the individual overnight rates fix, only the remaining rates that have not yet fixed remain volatile. In contrast, all the constituent overnight rates that make up $F_{(j)}(T_{i-1})$ for $j > i$ remain volatile for the full period. This explains why the volatility of the backward-looking rate that is currently accruing is less volatile than the backward-looking rate in a period that has not yet started accruing. [Lyashenko and Mercurio, 2019b] provide empirical evidence for this phenomenon, and make the case that volatility decay is necessary in the current accrual period.

Consider the case of the Black or Log-normal model, where the i^{th} rate has dynamics given by

$$\mathrm{d}F_{(i)}(t) = g(t)\sigma_{\mathcal{LN}}F_{(i)}(t)\,\mathrm{d}W_t, \tag{14}$$

where now we have introduced a decay function $g(t)$ to model the effect of this volatility decay. Assuming the decay function takes the form proposed by [Lyashenko and Mercurio, 2019a; Lyashenko and Mercurio, 2019b]:

$$g(t) = \min\left(\frac{(T_i - t)^+}{\delta_i}, 1\right), \tag{15}$$

which linearly reduces the volatility during the fixing period, the equations for caplet and floorlet prices, (11) and (12), introduced in the previous section may be used, unmodified, with an effective change in the time to maturity parameters given by

$$\mathcal{T}_i = (T_i - \delta_i) - t + \frac{\delta_i}{3}. \tag{16}$$

The same effective change in time to maturity is applicable to the Normal model. Given that a flat volatility rate is used for all caplet/floorlets when quoting caps and floors this adjustment ensures that the price is suitably adjusted downward to compensate for decaying volatility over the accrual periods. While all caplet/floorlet prices are reduced, this effect is most pronounced in the caplet/floorlet with the shortest maturity.

8.3.3. Swaptions

The recommended conventions for swaptions are as follows:

- **Premium type:** Forward date, $t_f = T_0$.
- **Premium lag:** No lag associated with payment of premium.

Using the same tenor structure as before, a swaption gives its holder the option to enter into a swap at T_0, with associated tenor structure $\mathsf{T} := \{T_0, \ldots, T_n\}$. Here K is a fixed strike rate and N is the notional amount. Then, the time-T_0 forward price of a payer swaption, is

$$\textbf{PS}\Big(t; \mathsf{T}, N, K\Big) := \begin{cases} \mathcal{LN}(S(t;T_0,T_n), K, \sigma_{\mathcal{LN}}, T_0 - t, 1)N \sum\limits_{i=1}^{n} \delta_i \frac{P(t,T_{(i)})}{P(t,T_0)}, & \text{for the } \mathcal{LN} \text{ model}, \\ \mathcal{N}(S(t,T_0,T_n), K, \sigma_{\mathcal{N}}, T_0 - t, 1)N \sum\limits_{i=1}^{n} \delta_i \frac{P(t,T_{(i)})}{P(t,T_0)}, & \text{for the } \mathcal{N} \text{ model}. \end{cases} \tag{17}$$

Similarly, the price of a receiver swaption is

$$\textbf{RS}\Big(t; \mathsf{T}, N, K\Big) := \begin{cases} \mathcal{LN}(S(t;T_0,T_n), K, \sigma_{\mathcal{LN}}, T_0 - t, -1)N \sum\limits_{i=1}^{n} \delta_i \frac{P(t,T_{(i)})}{P(t,T_0)}, & \text{for the } \mathcal{LN} \text{ model}, \\ \mathcal{N}(S(t;T_0,T_n), K, \sigma_{\mathcal{N}}, T_0 - t, -1)N \sum\limits_{i=1}^{n} \delta_i \frac{P(t,T_{(i)})}{P(t,T_0)}, & \text{for the } \mathcal{N} \text{ model}. \end{cases} \tag{18}$$

9. Numerical examples

To demonstrate the recommended conventions for non-linear derivatives on ZARONIA ACFRs, a Microsoft Excel workbook has been created with specific examples, scenarios and supporting calculations. At the time of writing this document there is not enough market depth to bootstrap realistic forward and discount curves. For this reason, synthetic curves generated using the Vasicek model have been used for illustrative purposes. Obviously, it will be a simple matter to replace this data with the values from market data when they become available.

An accompanying pricing template ("*ZARONIA-based non-linear derivatives model.xlsm*") has been developed by the DWS. This workbook contains a working model that demonstrates all of the key calculations for caps/floors and swaptions. Take note that the workbook highlights features and nuances of the specific conventions that are recommended rather than an exhaustive presentation of all possible conventions.

Glossary

Abbreviations

ACFR annualised cumulative floating rate. 8, 9, 10, 19, 21, 22, 23, 26, 28, 30

ARR alternative reference rate. 4

ATM At-the-money. 8, 9, 10, 11

bd business day(s). 6, 7, 8, 9, 10, 11, 13, 19, 22, 23, 28

BIS Bank for International Settlements. 4

CCBS cross-currency basis swap. 5

CSA Credit Support Annexure. 6, 7, 8, 9, 10, 11, 26, 27

DWS Derivatives Workstream. 4, 5, 30

EOM End-of-Month (business day convention). 6, 8, 9, 10, 16, 17

FSCA Financial Sector Conduct Authority. 4

GBLO London banking calendar - London Financial Center. 6, 7, 11

GBP British pound. 3, 6, 7

IBOR interbank offered rates. 4

IRS interest rate swap. 16, 18, 19

ISDA International Swaps and Derivatives Association. 11

Jibar Johannesburg Interbank Average Rate. 4, 7, 28

MPG Market Practitioners Group. 4, 5, 8, 9, 10, 19

ONRR overnight reference rate. 4, 8, 9, 10, 14, 18, 19, 22

OIS overnight indexed swap. 5, 7, 9, 10, 11, 12, 13, 15, 18, 23, 24, 25

OTC over-the-counter. 4

SARB South African Reserve Bank. 4, 8, 9, 10, 19

SAST South African standard time. 19

SOFR secured overnight financing rate. 6, 7, 8, 9, 10, 11, 26

SONIA Sterling Overnight Index Average. 6, 7

TBRR term-based reference rate. 14, 18, 19

UK United Kingdom. 6, 7

US United States of America. 4, 6, 7

USD United States dollar. 3, 6, 7, 8, 9, 10, 11, 26, 27

USGS United States Government Securities business days, also referred to as the Securities Industry and Financial Markets Association (SIFMA) Calendar. 6

USNY United States banking calendar - New York Financial Center. 6, 7

ZAJO South African calendar - Johannesburg Financial Center. 8, 9, 10, 11, 17

ZAc South African cent. 8, 9, 10, 25

ZAR South African rand. 25, 27

ZARONIA South African Overnight Index Average. 5, 7, 8, 9, 10, 11, 13, 19, 24, 26, 27, 28, 30

References

BIS (2023). "Statistical release: OTC derivatives statistics at end-June 2023". In: URL: https://www.bis.org/publ/otc_hy2311.pdf (cited on page 4).

Hou, D. and D.R. Skeie (2014). "LIBOR: Origins, economics, crisis, scandal, and reform". In: *FRB of New York Staff Report* 667 (cited on page 4).

ISDA (2023). "2021 ISDA Interest Rate Derivatives Definitions". In: (cited on page 11).

— (2024). "2021 ISDA Interest Rate Derivatives Definitions Settlement Matrix for Settlement, Early Termination and Swaptions". In: (cited on page 11).

Lyashenko, A. and F. Mercurio (2019a). "Looking Forward to Backward-looking Rates: A Modeling Framework for Term Rates replacing LIBOR". In: *Available at SSRN* 3330240 (cited on page 29).

— (2019b). "Looking Forward to Backward-looking Rates: Completing the Generalized Forward Market Model". In: *Available at SSRN* (cited on page 29).

SARB (2018). "Consultation paper on selected interest rate benchmarks in South Africa". In: URL: https://www.resbank.co.za/content/dam/sarb/publications/markets-consultation-paper/2018/8722/Consultation-Paper-28-08-2018.pdf (cited on page 4).

— (2020). "Draft Statement of methodology and the policies governing the South African Reserve Bank - administered interest rate benchmarks". In: URL: https://www.resbank.co.za/content/dam/sarb/publications/markets-consultation-paper/2020/10021/Draft-Statement-of-methodology-and-policies-for-benchmarks.pdf (cited on page 19).

SARB-DWS (2023). "Market conventions for ZARONIA-based derivatives". In: URL: https://www.resbank.co.za/content/dam/sarb/publications/media-releases/2023/zaronia-based-derivatives/Market%20conventions%20for%20ZARONIA-based%20derivatives.pdf (cited on pages 5, 7–12).