

# South African Reserve Bank Working Paper

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## An Alternative Business Cycle Dating Procedure for South Africa

*Adel Bosch and Franz Ruch*

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South African Reserve Bank

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### An Alternative Business Cycle Dating Procedure for South Africa

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Authorised for external distribution by Johan van den Heever

September 2012

#### Abstract

This paper applies a Markov switching model to the South African economy to provide an alternative classification of the business cycle. Principal components analysis (PCA) is applied to 114 of the 186 variables used in the dating of the business cycle by the South African Reserve Bank. PCA establishes the co-movement in the dataset to calculate the reference turning points over the period 1982 to 2009. The large dataset broadens the information set available to date the turning points. The number of factors are chosen using a modified Bai and Ng (2002) method. The Markov switching model is also applied to Gross Domestic Product (GDP) as this is a commonly used variable to date the business cycle in the literature and provides a benchmark to the factor model. Our results indicate that the factor model accurately dates the South African business cycle and compares favourably to the SARB dating.

JEL classification: E32, C10

Keywords: Markov switching model, business cycles.

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The authors would like to thank Iaan Venter, Siobhan Redford, Noma Maphalala and participants at the 2011 ERSA/UP Workshop on Monetary Economics and Macroeconomic Modelling for helpful comments. A previous version of this paper appeared as an Economic Research Southern Africa (ERSA) working paper (Working Paper no 267).

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## 1. INTRODUCTION

The South African Reserve Bank (SARB) has been dating business cycle turning points since 1946 (Venter, 2009). SARB uses a combination of methods, closely following the Burns and Mitchell (1946) definition and Moore's (1980) approach.<sup>2</sup> It is, however, argued that the Burns–Mitchell and Moore approaches suffer from “measurement without theory” and lack “well-defined statistical properties” (Koopmans, 1947; Blanchard and Fischer, 1989). Banerji (2010) defends Moore's work and qualifies his defence by saying that Moore's process of identifying business cycle indicators was “rooted in business cycle theory: not falsifiable statistical models . . . but in a theoretical, conceptual understanding of the drivers of the business cycle, nevertheless”.

The aim of this paper is to determine an alternative methodology to dating business cycle turning points in South Africa, based on both “well-defined statistical properties” and a “firm understanding of the underlying drivers of business cycles”. Accurate and timeous business cycle turning-point dates for an economy are crucial for policy-making and private sector decision-making. Accurate turning points allow policy-makers to implement countercyclical policy measures and provide the basis for comparing current data with historic phases. For the private sector, accurate business cycle turning points assist in arriving at informed sales and investment strategies.

This paper makes use of both a Markov switching model, similar to that used by Kontolemis (2001), and the Bry–Boschan (BB) algorithm (Bry and Boschan, 1971). This paper follows the growth rate cycle approach. Two other approaches that could be followed are (i) the growth cycle approach, that is, where data are de-trended, and deviations from trend are used to date upswings and downswings; and (ii) the classical cycles approach, where recessions and expansions are dated.

Our paper is the first attempt at using a model-based approach to date the South African business cycle functionally. It differs from the available literature in three respects. First, unlike most of the literature that focuses on model estimation of the business cycle in quarterly terms (e.g., Moolman, 2004; du Plessis, 2006; Altug and Bildirici, 2010; Yadavalli, 2010), we use monthly data. Although monthly data pose certain challenges, this complementary method should provide policy-makers with more timely information regarding the state of the economy. Second, it is argued that gross domestic product (GDP) is not a sufficient measure of the business cycle and an attempt is made to provide further information regarding the state of the economy. To this end, principal components analysis (PCA) is employed on 114 stationary variables of the 186 used in the official dating of the SARB business cycle, which allows for the uncovering of the correlation structure determining the aggregate business cycle. GDP, although commonly used in the literature, does not conform to the generally accepted Burns and Mitchell (1946:3) definition of the business cycle which focuses on many economic activities. It was found that when using this method, business cycle turning points could be predicted more accurately than when using only GDP. Third, some model-based studies in the literature assume *a priori* that the SARB business cycle dates are correct (Moolman, 2003) and attempt to apply a model that predicts these dates using an indicator such as yield spreads and GDP. The Markov switching model uses a latent variable to model the regime shift and date the business cycle. Our paper reveals that within the Markov switching framework, the mean and variance of each variable are sufficient estimators to determine accurate turning points in the South African economy, and no durational dependence or other dependent variables are necessarily required in the dating process. For the purposes of comparison, turning points are also identified using the BB algorithm.

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<sup>2</sup> For more detail on SARB's dating procedure, refer to Venter (2005).

The paper is set out as follows: in section 2 literature on the dating of business cycles, particularly Markov switching models, and specifically their application in South Africa are considered. Section 3 is an outline of the methodological approach followed, including the Markov switching framework as described in Hamilton (1994), PCA and the BB method. Section 4 contains an elaboration of the data used in determining alternative business cycle turning points for South Africa. Section 5 presents the results, in which the Markov switching output is compared to SARB's reference turning points, the BB method, and to other studies. Section 6 contains the conclusion and suggestions for future work.

## 2. LITERATURE REVIEW

With the general move towards estimating turning points in business cycles, much attention has been given to which models best estimate these points. Such models include linear, non-linear (including Markov switching) parametric as well as non-parametric models. Some schools of thought however suggest that turning point determination should rather be based on the fundamental (theoretical) definition of the business cycle, as defined by Burns and Mitchell (1946:3), as opposed to model-based approaches. Burns and Mitchell (1946:3) defined business cycles as

a type of fluctuation found in the aggregated economic activity of nations that organise their work mainly in business enterprises: a cycle consist of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of change is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.

Bry and Boschan (BB) (1973) replicated the Burns and Mitchell approach to determining turning points and also later introduced a method for working with quarterly data. They coded the BB procedure into an algorithm that could easily be applied. A variant of this method for dealing with quarterly data was developed by Harding and Pagan (2002) and was called the BBQ.

Similarly, Moore (1980:4) noted that expansions and contractions should reflect an absolute rise and an absolute fall in trend-adjusted aggregate economic activity. It is important to note that expansions and contractions occur "at about the same time in many economic activities" and that no single index of economic activity is superior to another (Moore, 1980:5). Moore and Zarnovitz (1986) made use of a weighted average of several series rather than a single series. Burns and Mitchell (1946) also did not have a GDP series available to them at the time and instead extracted a reference cycle from many series to determine turning points. They dated turning points based the pattern of clustering around during peaks and troughs.

These methods however, did not require any understanding of the underlying data generating process. During the late 1980s model-based approaches became more popular, such as the work done by Stock and Watson (1989, 1991). It was also during this time that the application of Markov switching models to time series analysis began with the seminal work of Hamilton (1988; 1989). In the latter paper he formulated a nonlinear iterative filter that allowed for the maximum likelihood estimation of population parameters through a "discrete-valued unobserved state vector" (1989:358). He defined the algorithm as "formalising the statistical identification of 'turning points' of a time series" (1989:358). He applied the method to his analysis of the business cycle in post-war United States (US) gross national product (GNP) and found that a shift from positive (negative) growth to negative (positive) growth was a recurrent feature in the data. He also found that the results were similar to the National Bureau of Economic Research (NBER) dating procedure and that this approach could be used as an alternative objective algorithm for dating. Much work has

followed from this study, including that done by Phillips (1991); Goodwin (1993); Kim and Yoo (1995); Artis et al. (1997); Kim and Nelson (1999); Kontolemis (2001); Artis et al. (2004); and Altug and Bildirici (2010).<sup>3</sup>

More recently, a parametric approach to extracting reference cycles or a coincident index from multivariate series has become popular. The process can largely be described as one where a common component and idiosyncratic component is extracted from a large number of data series. Forni et al (2001) made use of a dynamic factor approach to extract the common component from a set of data series. Kontolemis (2001) used both univariate and multivariate Markov switching models, similar to Engel and Hamilton (1990), on the component variables of the US composite coincident indicator in order to determine the turning points of the business cycle. The variables used included the index of industrial production, non-agricultural employment, personal income (excluding transfer payments), and manufacturing and trade sales. The author found that the resultant dating from a multivariate model was similar to the official NBER reference cycle and improved on univariate models of the component variables.

One aspect of these models however, as mentioned by Harding and Pagan (2006), is that they rely on the underlying assumptions on the data generating process, and can therefore not be seen as a neutral measurement of the index.

For South Africa, du Plessis (1950) first published business cycle turning points for the period 1910 to 1949 in 1950. The SARB closely followed the Burns and Mitchell (1946) approach to dating business cycle turning points. SARB also back-dated the coincident index back to 1946 (see Smit and van der Walt, 1970). Since then, SARB has moved away from the clustering approach towards a diffusion index approach. The move towards model-based approaches to dating the business cycle largely took off during the 2000s with the works of Frank (2001).

Moolman (2003) investigated the feasibility of looking at one indicator to predict turning points in the South African economy. The author used a probit model to investigate the relationship between the turning points of the business cycle and several individual leading indicators. The results showed that based on goodness of fit, the short-term interest rate, with a lead of seven months, was most statistically significant; followed by SARB's composite leading indicator, which led by three months; and then the yield spread which had a lead of seven months. It was found that SARB's composite leading indicator gave two false signals over the period, while neither of the single variables gave false signals.

Later, Moolman (2004) introduced a non-linear markov switching and logit model into forecasting turning points for the South African economy and compares the results to that of a linear model. The yield spread is used as an explanatory variable in both models, similar to the research conducted by Nel (1996). The Markov switching model used by Moolman (2004) incorporates time-varying transition probabilities, which provide information on future movements of the business cycle. She follows Hamilton (1989) and makes use of an AR(4) two-regime Markov switching model. The data used are quarterly real GDP and the yield spread is from 1978 to 2001. The Markov switching model outperformed both the linear AR(4) model and the logit model. The turning points and estimated probabilities of the Markov switching model closely match the SARB business cycle reference turning points. However, the Markov switching model signals an expansion in 1985 and a recession in 1994. These signals only last for one quarter and are therefore not dated based on the common cycle dating rule.<sup>4</sup>

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<sup>3</sup> These works by no means constitute a comprehensive list.

<sup>4</sup> The common cycle dating rule defines 'a recession' as two consecutive quarters of negative real GDP growth. Layton and Banerji (2003) question the origin of this common cycle dating rule, as some attribute the origin to Arthur Okun, although this reference is debatable. It is more likely that this rule originated from an article

Altug and Bildirici (2010) used a univariate Markov regime switching model for GDP growth to characterise the business cycle of 22 developed and developing countries,<sup>5</sup> including South Africa. This cross-section allowed for the comparison of cyclical variation between developing and industrialised countries and the dating of individual business cycles. Their results were compared to other methods in order to determine the efficacy of the Markov switching model. They found evidence of a world factor that drove the cyclical fluctuations in both developed and developing countries, but there was also an important degree of heterogeneity among the countries studied. In the South African case evidence of non-linearity in GDP was found and a two-regime Markov switching model best fitted the data spanning 1972 Q1 to 2009 Q1. The authors show that South Africa experienced the smallest decline in output during contractions compared to other countries, but also low growth during expansions. The model also tracked the recessions over the sample period fairly well.

Botha (2004) aimed to gain a better understanding of the underlying data generating process of the business cycle. He found that changes in the business cycle were asymmetrical and should be modelled non-linearly. The non-linear models used in the analysis were, among others, various regime-switching models. Botha found that the most popular measures to model the business cycle were the composite business cycle indicators and real GDP. The regime switching models also outperformed the other models.

Other methods and properties of the South African business cycle have been thoroughly explored in papers published by Du Plessis (2004); Boshoff (2005); Venter (2005); Du Plessis, Smit and Sturzenegger (2007); Venter (2009) and Yadavalli (2010). The most influential is the method used by SARB to date the reference turning points in the South African economy as described in Venter (2005) and again during the dating of the November 2007 upper turning point in Venter (2009).

Current research is largely focused on the works of Du Plessis (2006). He uses a non-parametric dating algorithm (henceforth BBQ index) described by Harding and Pagan (2003), first suggested by Bry and Boschan (1971), to date turning points in the South African business cycle. However, this algorithm did not fit the South African GDP data well during the 1960s and the 1990s, as the economy experienced a prolonged expansion during both periods. Du Plessis (2006) makes use of a transformed series by subtracting a deterministic linear trend, as opposed to the rate of change in real GDP. The author implemented a concordance index, measuring the proportion of time that both the SARB coincident index and the BBQ index are either in an expansionary or contraction phase. The results show that the two indices are highly synchronised. The main differences, however, include the fact that the average duration of contractions is shorter with the BBQ index than in the SARB index, whereas expansions have a similar average duration.

The study of these models raised interesting questions regarding the duration dependence of the South African business cycle. Frank (2001) makes use of a parametric Weibull Hazard function to test whether the South African business cycle is time-dependent, that is, whether the length of an expansion or recession has an impact on the probability of the economy switching states (i.e. entering a recession after a significant period of expansion or vice-versa). The results showed that there is no evidence of time dependence in the South

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published in the *New York Times* in 1974 by Julius Shiskin (1974). He is often misquoted on what he refers to as a “quantitative definition of a recession” (Shiskin, 1974:222). In the article he defines a recession in terms of three dimensions (which should all be considered together): (i) duration, (ii) depth and (iii) diffusion. He is often only (incorrectly) quoted on duration, and hence the common cycle dating rule refers only to the duration of negative growth and considers real GDP as the only variable.

<sup>5</sup> The other countries are Australia, Brazil, Canada, Chile, France, Hong Kong, Germany, Malaysia, Italy, Mexico, Japan, South Korea, the Netherlands, Singapore, Spain, UK, US, Taiwan, Turkey, Argentina and Uruguay.

African business cycle, as the probability of a downward phase (upward phase) shifting to a new phase in South Africa does not rise the longer the duration of the upward phase (downward phase). Similarly, Du Plessis (2004) makes use of non-parametric methods to investigate the duration dependence of the South African business cycle using the exponential distribution as the null hypothesis for three different tests. The business cycle is divided into two periods: (i) pre-1972 and (ii) post-1972. The results show that there is some evidence of duration-dependence in the pre-1972 downward cycles, which is not so clear in the post-1972 downward cycles. There is also weak evidence of duration dependence during the downward cycle and the total cycle in the post-1972 period. However, similar to what has been the case internationally, the South African business cycle does not display strong duration dependence to support even weak forms of periodicity.

### 3. METHODOLOGY

Let  $\mathbf{y}_t = [y_{1,t}, \dots, y_{N,t}]$  be a vector of variables of length  $N$ . This vector can be modelled using a mean-variance Markov switching model such that:

$$\mathbf{y}_t = \boldsymbol{\mu}_{s_t} + \boldsymbol{\varepsilon}_t \quad (1)$$

where  $s_t \in \{1,2\}$  is a discrete time, discrete state Markov chain with the Markov property; i.e. a stochastic variable where the current regime only depends on the previous regime,  $\boldsymbol{\mu}_{s_t}$  is the intercept term and  $\boldsymbol{\varepsilon}_t \sim i.i.d.(0, \sigma_{s_t}^2)$  is an independently and identically distributed error. Two states are modelled in this paper to conform to the growth cycle contraction and expansion phase of the business cycle.<sup>6</sup> Both the mean and variance are regime dependent. The transition probabilities can be summarised in a transition matrix,  $P$ , for a two-state Markov chain as follows:

$$P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix} \quad (2)$$

where  $p_{ij} = P(s_t = j | s_{t-1} = i)$  or the probability that the current regime is  $j$  given the previous regime was  $i$ . For a more general representation of a Markov chain and the statistical properties, see Hamilton (1994).

In order to estimate the coefficients of equation (1),  $\theta = (\boldsymbol{\mu}_{s_t}, \sigma_{s_t}, p_{ij})$ , we need to maximise the log-likelihood of the unconditional density function of  $\mathbf{y}_t$ :

$$L(\theta) = \sum_t^{T-1} \log f(\mathbf{y}_t; \theta) \quad (3)$$

where

$$f(\mathbf{y}_t; \theta) = \sum_{j=1}^2 p(\mathbf{y}_t, s_t; \theta) \quad (4)$$

The unconditional density function is the product of the conditional density function and the unconditional probability of  $s_t$ . This is written as:

$$p(\mathbf{y}_t, s_t; \theta) = f(\mathbf{y}_t | s_t; \theta) p_j \quad (5)$$

$$= \frac{p_{s_t}}{\sqrt{2\pi\sigma_{s_t}}} \exp\left\{-\frac{(\mathbf{y}_t - \boldsymbol{\mu}_{s_t})^2}{2\sigma_{s_t}^2}\right\} \quad (6)$$

<sup>6</sup> Evidence of the appropriateness of two regimes is given in Altug and Bildirici (2010).

Due to the highly non-standard estimation requirements; the need to estimate the Markov chain  $s_t$ , the likelihood function is maximised using the expectations-maximisation (EM) algorithm proposed by Hamilton (1989).<sup>7</sup>

Estimates in our paper are derived from a model with no autoregressive component, with only the mean and variance regime dependent similar to Engel and Hamilton (1990) and Kontolemis (2001). This is done for a number of reasons. First, results from Frank (2001) and Du Plessis (2004) suggest that durational dependence is not present (or strong enough) to add autoregressive terms to the model structure in the South African context. Second, abstracting from the autoregressive parameters yields more appropriate results in determining the turning points. Third, since the variables used are coincident to the business cycle, we are only interested in the contemporaneous impact. Fourth, owing to the use of monthly data, the ability to maximise the EM algorithm given the necessary amount of autoregressive terms becomes computationally strenuous.

PCA is utilised in our paper in order to reduce the dimensionality of the diffusion data (123 variables), but still ensuring that the majority of the variation in the dataset is modelled. According to Jolliffe (2005), this is achieved by creating a set of new variables, the principal components or factors, ordered such that the first few variables contain most of the variation and are uncorrelated across variables. The  $k^{th}$  principal component (PC) of a  $k \times k$  vector of variables is  $\alpha'_k x$  and  $var(\alpha'_k x) = \lambda_k$  where  $\lambda_k$  is the  $k^{th}$  largest eigenvalue of the variance-covariance matrix ( $\Sigma$ ) and  $\alpha_k$  is the corresponding eigenvector.<sup>8</sup> Since the population variance-covariance matrix is unknown, it is replaced with a sample matrix (S). Generally, the PCs are derived subject to a normalisation restriction,  $\alpha'_k \alpha_k = 1$ .

In order to provide further comparison, the BB method for determining turning points in monthly series is applied<sup>9</sup>. The BB method makes use of an algorithm to determine turning points as established by the NBER. Table 1 describes the original BB monthly procedure (Bry and Boschan, 1971).

[Table 1 here]

#### 4. DATA

The data used in our analysis were selected in such a way as to test whether one series sufficiently captures business cycle turning points. The first variable modelled was the 12-month change in the log of real GDP at market prices. The quarterly series was interpolated using linear trending and adjusting for seasonality in order to convert it into a monthly series. Two factor models were then developed for the 186 series used in the SARB turning-point exercise. After visual inspection, some of the series were dropped due to starting date differences, structural breaks and stationarity concerns leaving 114 variables<sup>10</sup>. This data, however, still covers all the sectors of the SA economy. All the data were studied in log differences, to ensure compatibility with the growth rate approach and to induce stationarity<sup>11</sup>, over the period of 1982 to 2009. All data that were not available on a monthly basis were converted to a monthly frequency.

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<sup>7</sup> Hamilton (1994) states that the mixture density in equation 4 does not have a global maximum. However, Kiefer (1978) proved that the log-likelihood function has a bounded local maximum with consistent, asymptotically Gaussian estimates of the parameter coefficients.

<sup>8</sup> For a complete derivation of principal components analysis, see Jolliffe (2005).

<sup>9</sup> The BB algorithm is also adjusted, with a censoring rule, by Harding and Pagan (2003) to deal with quarterly data. This is referred to as the “BBQ algorithm”.

<sup>10</sup> Variable list available on request.

<sup>11</sup> Variables are stationary at a 5 per cent level of significance based on the Augmented Dickey-Fuller and/or Phillips-Perron tests.

## 5. RESULTS

The Markov switching models are estimated in Gauss, using Bellone's (2005) Markov Switching Vector Autoregressive library (MSVARlib),<sup>12</sup> and the BB method in Scilab, using the Grocer package.<sup>13</sup> The GDP model serves as a benchmark model for this analysis and as a way to validate GDP as an appropriate aggregate measure in dating the business cycle. GDP is often used because it is seen as an estimate of aggregate economic activity. However, a diffusion index aims to capture the movement as the change in aggregate economic activity moves and spreads from one economic process to the next. By only looking at one indicator, GDP, these movements are typically lost. The diffusion indicators aim to provide a deeper understanding of the motions that are put in place when the economy changes from a recession to an expansion (recession) or vice versa. This process is described in Banerji (2010) as a fall in income, leading to a fall in sales, followed by a fall in production and then in employment.

### 5.1 Gross Domestic Product Model

The GDP model infers that the 12-month growth rate in real GDP is subject to two regimes. Low regime periods are associated with lower or negative GDP growth, while high regime periods are associated with positive or high GDP growth. GDP is generally accepted as a good approximation of movements in the aggregate economy, although this may not necessarily mean that it accurately reflects the turning points of the business cycle. Other issues also exist. GDP is only available on a quarterly basis, while the diffusion index indicators are mostly available on a monthly basis. The GDP model uses linearly interpolated monthly data to test whether this would allow for adequate dating. GDP is also frequently revised, resulting in a change in the main indicator, while revisions in the diffusion indicators do not make such a big difference in the total diffusion index. The BBQ method adopted by Du Plessis (2006) is updated to test whether revisions to GDP do, in fact, make dating problematic. This model also provides information regarding the stylised facts of the South African business cycle which is lost when using PCA.

A mean–variance model (MSMH(2)-AR(0))<sup>14</sup> in which the mean and variance are regime-dependent was fitted for the period 1982–2009. The results are presented in Table 2.  $\mu_i$  and  $\sigma_i$  are the mean and variance respectively for regime  $i=1,2$ . Here, 1 is the downward phase and 2 is the upward phase.  $P_{11}$  is the probability that the current period is a downward phase, given that the previous period was a downward phase. The log-likelihood values, Bayesian Information Criterion (BIC) and the Jarque–Bera test statistic are also presented.

[Table 2 here]

Over the sample period, the average year-on-year growth rate during downward phases (regime 1) was a rate of contraction of 0,7 per cent, while the average growth rate during upward phases (regime 2) was 3,8 per cent. These estimates generally match Moolman (2004) and du Plessis (2006), who estimated a negative average growth rate during downward phases of 1,1 and 0,6 per cent and a 3,7 and 4,6 per cent positive average growth during expansion periods. This, however, is in contrast to Altug and Bildirici (2010) who find the mean growth rate during the contraction phase to be 0,02 per cent and 2,06 per cent during expansions.<sup>15</sup> The average growth rate based on the SARB business cycle turning points were 0,3 per cent during downward phases and 3,6 per cent during upward phases. A

<sup>12</sup> For more information, see Bellone (2005).

<sup>13</sup> For more information, see Dubois and Michaux (2009).

<sup>14</sup> Our paper follows the naming convention of Krolzig (1997).

<sup>15</sup> It is important to note the sample period differences between this paper and that of Altug and Bildirici (2010), and Moolman (2004). Altug and Bildirici (2010) studied the period 1972–2009, while Moolman (2004) studied the period 1978–2001.

possible reason why average growth is marginally positive during downward phases based on SARB's turning points, while average growth is negative based on the GDP model, can be attributed to the fact that some sectors in SARB's diffusion index only turn after GDP growth has already gained momentum. Finally, the BB method finds that growth averages 1,75 per cent during downward phases and 2,6 per cent during upward phases, differing substantially from all other results.

Similar to Altug and Bildirici (2010), the variance in growth in the GDP model is larger during contraction phases (0,023 per cent) compared to that during expansion phases (0,014 per cent). This result is expected, because, generally, downward phases during this period were exacerbated by large exogenous shocks, most significantly the financial crisis of 2007/2008, but also the debt standstill agreement and isolation policies of the late 1980s.

Figure 1 plots the density of GDP growth in each regime over the sample period. The figure shows that GDP growth during a downward phase is more dispersed (i.e., has a larger base) compared to growth during an upward phase. During downward phases, growth can be anywhere between -4,6 and 2,1 per cent, while during upward phases, the spread is between 1,6 and 7,4 per cent.

*[Figure 1 here]*

The transition probabilities show that over the sample period the average upward phase lasted just over 45 months, while the average downward phase lasted almost 29 months. Based on the SARB dating of the business cycle in South Africa since 1945, the average upward phase lasted close to 31 months (48 months for the sample period) and 20 months (35 months) in downward phases, excluding the current recession. The BB method applied to monthly GDP estimated an average upward phase of 18 months and an average downward phase of 20 months. The transition matrix also shows that the probability of the economy remaining in an upward phase given that the previous month was in an upward phase, is 97,7 per cent, while the probability of staying in a downward phase given that the previous month was also in a downward phase, is 96,5 per cent.

Figure 2 plots the smoothed probabilities, those obtained from estimates of the probability that regime  $j$  occurs at time  $t$  given all available observations, for the GDP Markov switching model against the SARB turning points and the BB method. The area shaded in grey, where the business cycle takes on the value 1, represents the upward phases of the business cycle. The discrepancy between the model estimates and SARB's business cycle is 17,3 per cent. This discrepancy is relevant due to the desire to establish robust turning-point dates using a number of possible complementary methods. Overall, the model performs relatively well in dating the business cycle, especially the start of the upward phases. However, it does not accurately date the start of any of the four recessions during the period under review. One area of concern is the dating of the final downward phase of the South African economy during the late 2000s. According to the GDP model, the current recession only begins in November 2008, 12 months after the dating of SARB's reference turning points and 15 months after the diffusion MSMH(2)-VAR(0) model. See Appendix B for the actual dating of each model.

*[Figure 2 here]*

For purposes of comparison, the BBQ methodology adopted in Du Plessis (2006) is updated with the results provided in Appendix C. The updated dating procedure, which provides some positive evidence for the plausibility of GDP as a good approximation of the business cycle, does not differ significantly from the initial estimation undertaken in Du Plessis (2006) even though the data has since been revised. The updated BBQ method differs from the initial estimation in two dates: (i) the start of the 1987 upward phase shifts to the fourth quarter from the first previously, and (ii) the 2004 upward phase begins in the fourth quarter of 2003

instead of the first quarter of 2004. This difference could also be attributed to the detrending technique. Canova (1999) and others have found that dating is sensitive to the type of detrending method applied. As more data are made available, the trendline no longer corresponds with the initial trendline calculated by Du Plessis (2006) and therefore deviation from trend will differ. A more robust method, as applied in this paper, is to determine turning points in growth rate cycles. This method is, however, applied to quarterly data, whereas the SARB dating procedure is based on monthly data. To find an adequate comparison, the BB method is adopted on the monthly GDP data series and found to be substantially different to the other approaches in this paper. It is not unusual for this method to date significantly more turning points.

## 5.2 Diffusion Model

The diffusion model applies PCA to 114 variables used in the determination of SARB's reference turning points of the business cycle. Not all the diffusion index data are used, due to inconsistency in starting dates, breaks and stationarity issues in some of the variables. Figure 3 plots the first PC from this analysis, clearly indicating the cyclical nature in economic activity. However, it only explains about 15 per cent of the co-movement in the 114 variables.

*[Figure 3 here]*

Two diffusion models are fitted to the PCs, one including only the first PC, namely an MSMH(2)-AR(0) model; and the other including the first eight PCs, namely MSMH(2)-VAR(0). The presumption is that these models would more accurately represent aggregate business cyclical nature, as compared to GDP, as more data are included from each sector. The eight PCs explain close to 64 per cent of the overall variation in our 114 variables. The strong correlation structure present in the data allows for a close to fifteen-fold decline in the number of variables needed in the estimation step.

We use a modified Bai and Ng (2002) information criteria as implemented by Alessi, Barigozzi and Capasso (2010) in order to determine the "true" number of factors to use in our model. This method chooses the number of factors by minimising the variance of the idiosyncratic component of the principal components. This is subject to a penalisation in order to avoid over-parameterisation. Figure 4 shows the estimated number of factors for our model. The results suggest that the number of factors should be eight.

*[Figure 4 here]*

Tables 3 and 4 present the results of the two diffusion models.

*[Table 3 and 4 here]*

Owing to the relatively small percentage of variation explained by the first PC, the MSMH(2)-AR(0) model poorly estimates the turning points of the business cycle and does not compare as favourably as other models with the business cycle dates published by SARB, with a 25,6 per cent discrepancy between the two dating methods. That said, the model still finds a significant difference in the means of each regime and accurately indicates the volatility differences between the two regimes. The transition probabilities show that over the sample period the average upward phase lasted just under 20 months (15 months based on the BB method), while the average downward phase lasted 47 months (21 months). However, due to the low explanatory power of the first PC (only 15 per cent), the model was not as effective in dating turning points as the eight PC diffusion model.

The MSMH(2)-VAR(0) model performance is highly correlated with the movements of the SARB business cycle, with a discrepancy of only 15,8 per cent. However, in this case much of the discrepancy arises from the May 2002 to February 2003 period, where the model

correctly predicts a slowdown in economic activity, although not officially dated by SARB. The discrepancy with SARB's reference turning points in dating the start of downward phases is also improved in this model as compared to the GDP model, with the only significant difference occurring in the recession in the late 1980s. A clear pattern in the mean and variance of this model is present. Generally, regime 1 coefficients are negative and regime 2 coefficients are positive. Furthermore, similar to the MSMH(2)-AR(0) model, the variance during the downward phase (regime 1) is, on average, higher. The average duration of downward phases is estimated in this model at 27 months while the average upward phase is 32 months. Figure 6 plots the smoothed probabilities of both the diffusion models against SARB's business cycle.

*[Figure 5 here]*

One possible area of concern is the non-normality of the residuals in the two diffusion models; in both cases the null hypothesis of normal residuals is rejected. However, Lanne and Lutkepohl (2009) point out a number of possible reasons for this. First, non-normality may be due to differing business cycle fluctuations in expansion or contraction periods generating differing statistical properties. If such differences arise then a Markov switching model will accurately identify these differences. In the present context, regime 1 is not normally distributed due to negative skewness. Second, the authors indicate that the normality assumption is made for convenience and is not necessarily required for asymptotic inference. Finally, the model assumes conditional normality which provides greater flexibility as compared to unconditional normality.

## 6. CONCLUSION

In this paper we applied a Markov switching model and BB method to date the South African business cycle turning points and found that the model estimates generally coincide with the business cycle turning points as dated by the SARB. Given the consensus that the business cycle refers to a cycle in aggregate economic activity, this paper moves away from only using GDP, to using PCA on the diffusion data, in order to model the aggregate co-movement in economic variables. This method was found to be more accurate at predicting business cycle turning points than GDP. However, given the simplicity of the GDP approach, this cannot be effectively disputed. This paper also reveals that within the Markov switching framework, the mean and variance are sufficient estimators to determine accurate turning points in the South African economy, and no durational dependence or other dependent variables are necessarily required in the dating process. However, this could be investigated further.

This paper suffers from some caveats. First, the data are detrended using only one procedure, log differencing, therefore focusing on growth rate cycles rather than classical business cycles. This method was deliberately chosen to enable a comparison between the Markov switching output and SARB's business cycle reference turning points. Second, it is difficult to determine whether the advantages of statistical methods to detect the turning points of the business cycle outweigh the advantages of other approaches such as the current method adopted by SARB. Third, the method applied above was unable to detect the current upswing in the business cycle, even though Krolzig (1997) states that one of the advantages of Markov switching models is their ability to detect recent regime shifts.<sup>16</sup> However, the SARB approach also requires a sufficient amount of lag before dating is possible.

Future work includes investigation into the impact of different detrending methods on the dating of business cycle turning points and testing for the inclusion of other dependent

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<sup>16</sup> This is due to the cut-off date of the data at the end of 2009. By extending the data to the most recent observation, we were able to date the turning point in mid-2009.

variables in the Markov switching model framework. Other types of non-linear models could also be estimated to provide further robust estimates of the turning points.

Appendix B1: Business cycle dating using the Markov switching procedure

| SARB's dating                  |          | Gross Domestic Product<br>MSMH(2)-AR(0)     |          | Diffusion MSMH(2)-AR(0)       |          | 8 PC diffusion MSMH(2)-VAR(0)         |          |
|--------------------------------|----------|---|----------|-------------------------------|----------|---------------------------------------|----------|
| Upward phase                   | Duration | Upward phase                                | Duration | Upward phase                  | Duration | Upward phase                          | Duration |
| April 1983 – June 1984         | 15       | March 1982<br>November 1983 – November 1984 | –<br>13  | October 1983 – June 1984      | 9        | April 1982<br>August 1983 – June 1984 | –<br>11  |
| April 1986 – February 1989     | 35       | March 1987 – August 1989                    | 30       | July 1987 – January 1989      | 19       | October 1986 – October 1989           | 37       |
| June 1993 – November 1996      | 42       | August 1993 – September 1997                | 50       | February 1994 – November 1995 | 22       | July 1992 – December 1996             | 54       |
| September 1999 – November 2007 | 99       | June 1999 – October 2008                    | 113      | December 1999 – November 2000 | 12       | November 1999 – July 2001             | 21       |
|                                |          |   |          | December 2002 – June 2006     | 43       | October 2002 – August 2007            | 59       |
| <b>Downward phase</b>          |          |   |          |                               |          |                                       |          |
| September 1981 – March 1983    | 19       | April 1982 – October 1983                   | 19       | – September 1983              | –        | May 1982 – July 1983                  | 15       |
| July 1984 – March 1986         | 21       | December 1984 – February 1987               | 27       | July 1984 – June 1987         | 36       | July 1984 – September 1986            | 27       |
| March 1989 – May 1993          | 51       | September 1989 – July 1993                  | 47       | February 1989 – January 1994  | 60       | October 1989 – June 1992              | 33       |
| December 1996 – August 1999    | 33       | October 1997 – May 1999                     | 20       | December 1995 – November 1999 | 48       | January 1997 – October 1999           | 34       |
|                                |          |   |          | December 2000 – November 2002 | 24       | August 2001 – September 2002          | 14       |
| December 2007 –                | –        | November 2008 –                             | –        | July 2006 –                   | –        | September 2007 –                      | –        |

Appendix B2: Business cycle dating using the Bry–Boschan method

| SARB's Dating                  |          | Bry-Boschan Method<br>Gross Domestic Product |          | Du Plessis (2006) BBQ<br>method<br>Updated |                 | Bry-Boschan Method<br>Diffusion  |          |
|--------------------------------|----------|--|----------|--|-----------------|----------------------------------|----------|
| Upward phase                   | Duration | Upward phase                                 | Duration | Upward phase                               | Duration<br>(q) | Upward phase                     | Duration |
| April 1983 – June 1984         | 15       | March 1983 – May 1984                        | 15       | 1983 Q3 – 1984 Q2                          | 4               | January 1983–October 1983        | 10       |
| April 1986 – February 1989     | 35       | June 1985 – August 1988                      | 39       | 1987 Q4 – 1989 Q1                          | 6               | June 1985 – February 1988        | 33       |
|                                |          | December 1992 – February 1995                | 27       | 1993 Q1 – 1994 Q4                          | 8               | January 1991 – October 1992      | 22       |
| June 1993 – November 1996      | 42       | December 1995 – November 1996                | 12       | 1996 Q1 – 1996 Q4                          | 4               | January 1994 – December 1994     | 12       |
| September 1999 – November 2007 | 99       | December 1998 – May 2000                     | 18       | 1999 Q1 – 2001 Q1                          | 9               | December 1997 – June 1998        | 7        |
|                                |          | December 2001 – August 2002                  | 9        | 2001 Q4 – 2003 Q1                          | 6               | July 1999 – March 2000           | 9        |
|                                |          | December 2003 – November 2004                | 12       | 2003 Q4 – 2008 Q2                          | 19              | March 2002 – February 2003       | 12       |
|                                |          | December 2005 – February 2007                | 15       |  |                 | February 2004 – April 2005       | 15       |
| Downward Phase                 |          |  |          |  |                 |                                  |          |
| September 1981 – March 1983    | 19       |  |          |  |                 | January 1982 – December 1982     | 12       |
| July 1984 – March 1986         | 21       | June 1984 – May 1985                         | 12       | 1984 Q3 – 1987 Q3                          | 13              | November 1983 – May 1985         | 19       |
| March 1989 – May 1993          | 51       | September 1988 – November 1992               | 51       | 1989 Q2 – 1992 Q4                          | 15              | Mar ch1988 – December 1990       | 34       |
|                                |          |  |          |  |                 | November 1992 – December<br>1993 | 14       |
|                                |          | March 1995 – November 1995                   | 9        | 1995 Q1 – 1995 Q4                          | 4               | January 1995 – November 1997     | 35       |
| December 1996 – August 1999    | 33       | December 1996 – November 1998                | 24       | 1997 Q1 – 1998 Q4                          | 8               | July 1998 – June 1999            | 12       |
|                                |          | June 2000 – November 2001                    | 18       | 2001 Q2 – 2001 Q3                          | 2               | April 2000 – February 2002       | 23       |
|                                |          | September 2002 – November 2003               | 15       | 2003 Q2 – 2003 Q3                          | 2               | March 2003 – January 2004        | 11       |
|                                |          | December 2004 – November 2005                | 12       | 2008 Q3 –                                  | –               | May 2005 – December 2009         | 56       |
| December 2007 –                | –        | March 2007 –                                 | –        |  |                 |                                  |          |

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## Tables and figures

*Table 1: Bry and Boschan determination of turning points*

|  |
|--|
| 1. Determination of extremes and substitution of values.   |
| 2. Determination of cycles in a 12-month moving average (extremes replaced). <ul style="list-style-type: none"> <li>a. Identification of points higher (or lower) than 5 months on either side.</li> <li>b. Enforcement of alternation of turns by selecting highest of multiple peaks (or lowest of multiple troughs)</li> </ul>  |
| 3. Determination of corresponding turns in Spencer curve (extremes replaced). <ul style="list-style-type: none"> <li>a. Identification of highest (or lowest) value within +/- 5 months of selected turn in 12-month moving average.</li> <li>b. Enforcement of minimum cycle duration of 15 months by eliminating lower peaks and higher troughs of shorter cycles.</li> </ul>  |
| 4. Determination of corresponding turns in short-term moving average of 3 to 6 months, depending on months of cycle dominance. <ul style="list-style-type: none"> <li>a. Identification of highest (or lowest) value within +/- 5 months of selected turn in Spencer curve.</li> </ul>   |
| 5. Determination of turning points in unsmoothed series. <ul style="list-style-type: none"> <li>a. Identification of highest (or lowest) value within +/- 4 months, or months of cycle dominance term, whichever is larger, of selected term in short-term moving average.</li> <li>b. Elimination of turns within 6 months of beginning and end of series.</li> <li>c. Elimination of peaks (or troughs) at both ends of series that are lower (or higher) than values closer to the end.</li> <li>d. Elimination of cycles whose duration is less than 15 months.</li> <li>e. Elimination of phases whose duration is less than 5 months.</li> </ul> |
| 6. Statement of final turning point.   |

*Table 2: MSMH(2)-AR(0) model of real gross domestic product (1982-2009)\**

| Coefficients       | GDP MSMH(2)-AR(0)   |
|--------------------|---|
| $\mu_1$            | -0.007043 (0.001804)  |
| $\mu_2$            | 0.038352 (0.001014)   |
| $\sigma_1$         | 0.000226 (0.000035)   |
| $\sigma_2$         | 0.000140 (0.000016)   |
| $P_{11}$           | 0.965041 (0.016081)   |
| $P_{22}$           | 0.977817 (0.010004)   |
| <b>Diagnostics</b> | Log-likelihood = 953.34766404<br>BIC = -8.767<br>JB-stat = 2.53 |

\* Standard errors are included in brackets.

*Table 3: Diffusion MSMH(2)-AR(0) model (1982-2009)\*\**

| Coefficient        | MSMH(2)-AR(0)   |
|--------------------|---|
| $\mu_1$            | -0.339863 (0.033814)  |
| $\mu_2$            | 0.738557 (0.040107)   |
| $\sigma_1$         | 0.214647 (0.020476)   |
| $\sigma_2$         | 0.097865 (0.016287)   |
| $P_{11}$           | 0.978672 (0.009579)   |
| $P_{22}$           | 0.949340 (0.021234)   |
| <b>Diagnostics</b> | Log-likelihood = -208.35228988<br>BIC = -1.805<br>JB-stat = 128.157 |

\* Standard errors are included in brackets.

Table 4: Eight-principal component diffusion MSMH(2)-VAR(0) model (1982-2009)\*\*

| <b>Coefficient</b> | <b>Regime 1</b>  | <b>Regime 2</b>      |
|--------------------|--|----------------------|
| $\mu^1$            | -0.549605 (0.039080)   | 0.456295 (0.035021)  |
| $\mu^2$            | 0.018669 (0.086353)  | -0.015515 (0.043652) |
| $\mu^3$            | -0.077474 (0.041102)   | 0.064323 (0.046166)  |
| $\mu^4$            | -0.056221 (0.045189)   | 0.046689 (0.034427)  |
| $\mu^5$            | 0.063738 (0.042821)  | -0.052916 (0.028472) |
| $\mu^6$            | -0.030285 (0.040009)   | 0.025144 (0.033398)  |
| $\mu^7$            | 0.003917 (0.025697)  | -0.003254 (0.032165) |
| $\mu^8$            | 0.011686 (0.035260)  | -0.009701 (0.032819) |
| $\sigma^1$         | 0.182625 (0.021540)  | 0.251221 (0.029668)  |
| $\sigma^2$         | 0.174286 (0.018817)  | 0.130058 (0.013945)  |
| $\sigma^3$         | 0.410449 (0.049387)  | 0.199938 (0.023379)  |
| $\sigma^4$         | 0.260369 (0.029706)  | 0.130982 (0.013927)  |
| $\sigma^5$         | 0.210897 (0.027114)  | 0.099279 (0.011914)  |
| $\sigma^6$         | 0.318470 (0.035180)  | 0.152818 (0.016306)  |
| $\sigma^7$         | 0.272194 (0.032317)  | 0.132440 (0.016238)  |
| $\sigma^8$         | 0.193568 (0.021058)  | 0.009481 (0.009276)  |
| P                  | 0.963612 (0.015129)  | 0.969227 (0.012687)  |
| <b>Diagnostics</b> | Log-Likelihood = -1555.88151071<br>BIC = -12.560<br>JB-stat = 49.229 |                      |

\* Standard errors are included in brackets.

Figure 1: Density plot of real gross domestic product growth

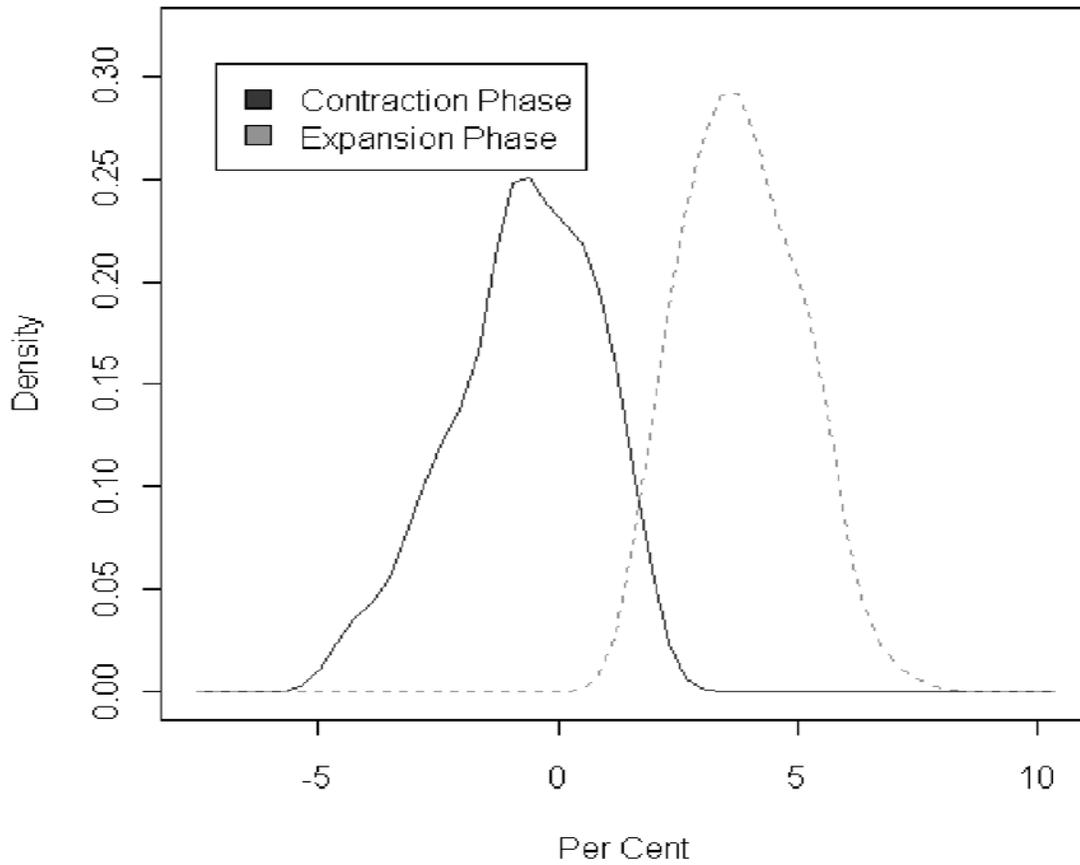


Figure 2: Gross domestic product MSMH(2)-AR(0)

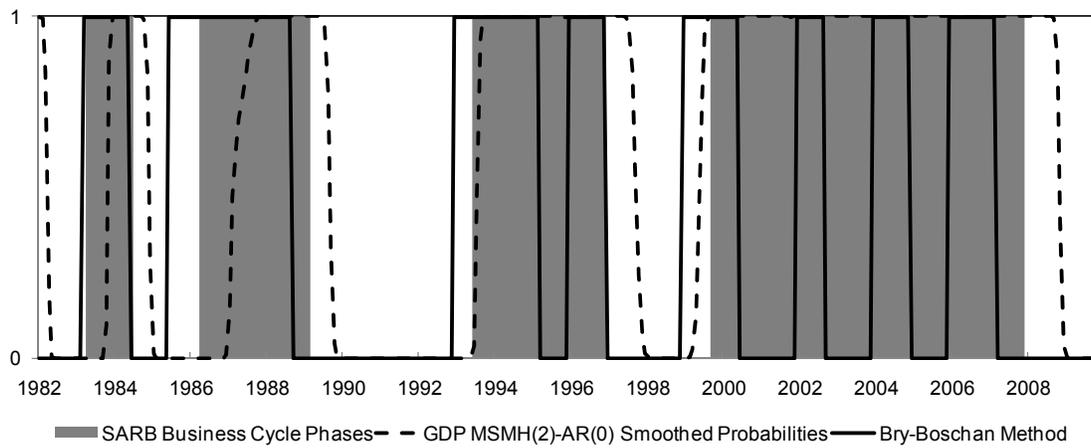


Figure 3: Diffusion first principal component

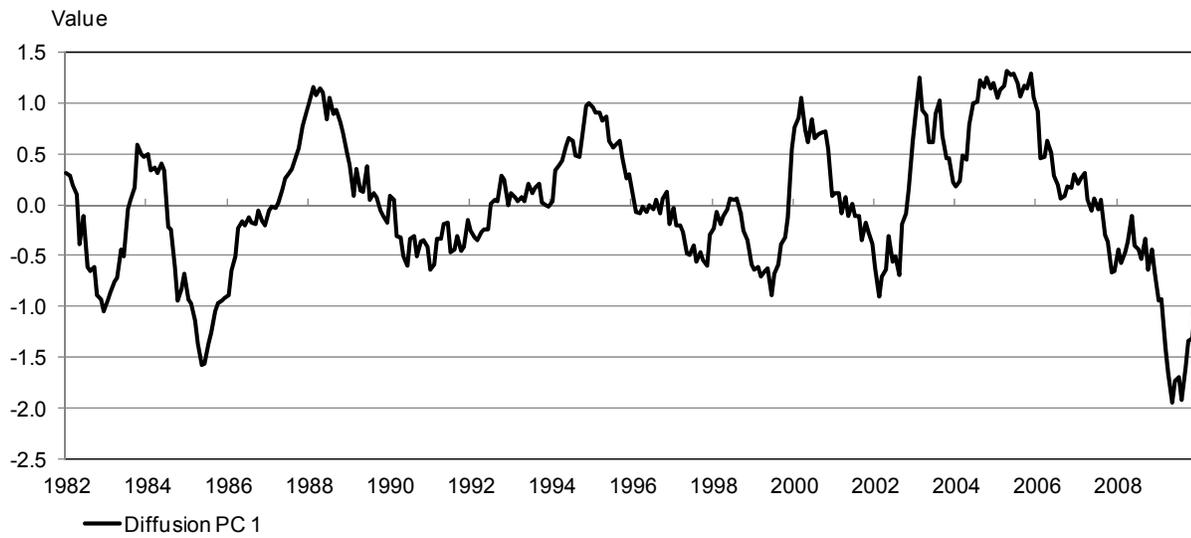


Figure 4: Estimating the number of factors

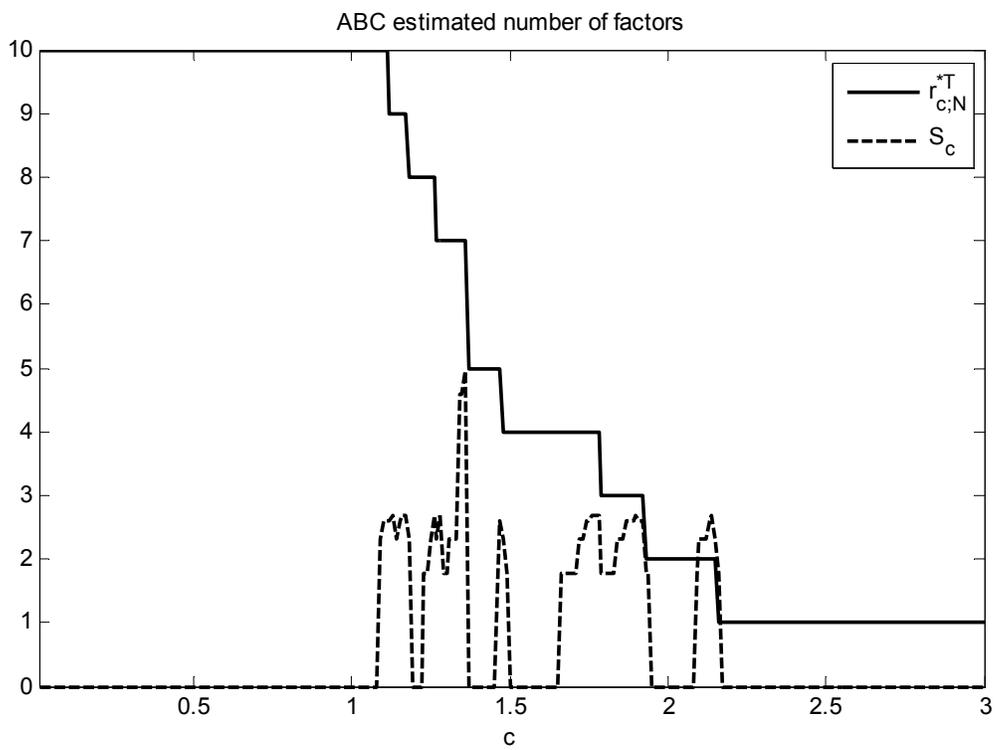


Figure 5: Diffusion MSMH(2)-AR(0) and diffusion MSMH(2)-VAR(0)

