An open economy New Keynesian DSGE model of the South African economy

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Abstract

In this paper an open economy New Keynesian model of the South African economy is presented. The model is constructed to provide for incomplete pass-through of exchange rate changes, external habit formation, partial indexation of domestic prices and wages to past inflation, and staggered price and wage setting. Furthermore, the model is estimated using Bayesian techniques on South African domestic and trade partner data for the period 1990Q1 to 2007Q4. The estimated model is analysed by means of impulse response functions and its forecasting performance is compared with that of the Reuters consensus forecasts of the relevant South African variables.

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1 Introduction

In recent years the use of small-scale dynamic stochastic general equilibrium (DSGE) models in the analysis of monetary policy has become quite popular. In this regard, New Keynesian models, such as those developed in Clarida, Gali and Gertler (1999) and Woodford (2003) have been a focus of attention. More recently, open economy versions of the New Keynesian model have been developed. Some of these models (e.g. Gali and Monacelli (2005), Clarida, Gali and Gertler (2001) and Benigno and Benigno (2002)) assume that the law of one price holds internationally but others allow for incomplete pass-through of exchange rate changes. Particular examples of the latter are the models developed in Monacelli (2003) and Justiniano and Preston (2004).

DSGE models of the South African economy have only very recently begun to appear in the literature. Liu and Gupta (2007) calibrate Hansen’s (1985) DSGE model to match South African data. The model is used to generate forecasts for a number of macroeconomic variables, which are then compared with the forecasts of a Bayesian and classical VAR. Ortiz and Sturzenegger (2007) use a version of the Gali and Monacelli (2005) model and Bayesian techniques to estimate the policy reaction function of the South African Reserve Bank (SARB). The authors find that the SARB puts a low weight on the exchange rate and more weight on output.

In this paper the open economy New Keynesian model is used as a basis to develop and estimate a DSGE model of the South African economy. The model exhibits the following features: imperfect pass-through of exchange rate changes, external habit formation, partial indexation of domestic prices and wages to past inflation, and staggered wage and price setting. The model is estimated with Bayesian techniques on South African domestic and trading partner data for the period 1990Q1 to 2007Q4. The estimated model is then analysed by means of impulse response functions.

The remainder of the paper proceeds as follows: Section 2 lays out the theoretical model. Section 3 provides details of the estimation method, the data used and the specification of the prior distributions of the estimated parameters. Section 4 covers the estimation results and the analysis of alternative shocks on the dependent variables in the model. The forecast performance of the model is considered in section 5. Section 6 concludes.

1 The South African Reserve Bank (SARB), which has a long tradition of constructing and using macro-econometric models for forecasting and policy evaluation, has joined an increasing number of central banks in developing DSGE models of their economies. This paper represents the second such output from the SARB’s research programme on DSGE modelling (the first being a closed economy DSGE model by Mathuloe and Steinbach (2008)).
2 The model

In this section a two-country New Keynesian DSGE model is developed, where the domestic economy is represented by South Africa and the foreign economy by the rest of the world. Firstly, the domestic economy, i.e. South Africa, is modelled as a small open economy and builds on the basic structure of Monacelli (2003) and Justiniano and Preston (2004), which provides for incomplete pass-through of exchange rate changes. Furthermore, the model is adapted to include real rigidity in the form of external habit formation in consumption, and additional nominal rigidities through partial indexation of domestic prices to its past inflation, staggered price and wage setting following Calvo (1983), and finally, partial indexation of wages to past consumer price inflation. Secondly, the rest of the world, i.e. the foreign economy in the context of this model, is assumed to be so large that it is not affected by developments in the South African economy and therefore approximates a closed economy. Hence, the structure of the rest of the world is modelled as a closed economy version of the domestic economy, with the only difference being that, for the sake of simplicity, wages in the foreign economy are flexible.

2.1 Domestic households and wage setting

A continuum of infinitely lived households (indexed by \( i \), where \( i \in [0, 1] \)) populate the domestic economy and consume aggregates of domestic \((C_{h,t})\) and imported \((C_{f,t})\) goods, according to the following composite consumption index:

\[
C_t \equiv \left( (1-\gamma) \left[ \frac{P_{h,t}}{P_t} \right]^{\frac{\eta-1}{\eta}} + \gamma \left[ \frac{P_{f,t}}{P_t} \right]^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}},
\]

where \( 0 \leq \gamma < 1 \) is the import share and \( \eta > 0 \) is the intratemporal elasticity of substitution between domestic and foreign goods. Households allocate expenditure according to the demand functions for domestic and foreign goods:

\[
C_{h,t} = (1-\gamma) \left[ \frac{P_{h,t}}{P_t} \right]^{-\eta} C_t \quad \text{and} \quad C_{f,t} = \gamma \left[ \frac{P_{f,t}}{P_t} \right]^{-\eta} C_t,
\]

where \( P_{h,t} \) and \( P_{f,t} \) are the respective price indices for domestic and imported goods, and

\[
P_t \equiv \left( (1-\gamma) P_{h,t}^{1-\eta} + \gamma P_{f,t}^{1-\eta} \right)^{\frac{1}{1-\eta}},
\]

is the consumer price index.

In addition, households are assumed to be monopolistically competitive and supply differentiated labour services \( N_t(i) \), to intermediate goods producing firms. The total demand for each household’s labour
by all firms is given by

\[ N_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\xi_w} N_t, \tag{4} \]

where \( N_t \) is per capita employment and the elasticity of labour demand, \( \xi_w \), is constant across workers. The aggregate wage index, \( W_t \), is specified as

\[ W_t = \left[ \int_0^1 W_t(i)^{1-\xi_w} di \right]^{1-\xi_w}. \tag{5} \]

In every period \( t \), each household maximises its expected lifetime utility, which is represented by the intertemporal utility function:

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t - H_t, N_t), \tag{6} \]

where \( \beta \) is the subjective discount factor and \( H_t \equiv \vartheta C_{t-1} \) represents external habit formation.\(^2\) While maximising their utility, households face the sequence of budget constraints:

\[ P_{h,t} C_{h,t} + P_{f,t} C_{f,t} + E_t\{Q_{t,t+1} D_{t+1}\} = W_t N_t + D_t, \tag{7} \]

where \( D_t \) denotes the pay-off of a portfolio of assets and \( Q_{t,t+1} \) is the corresponding one-period stochastic discount factor, so that \( E_t\{Q_{t,t+1} D_{t+1}\} \) represents the price of the portfolio purchased during period \( t \). A nominal wage \( W_t \) is earned for labour services supplied to firms.

It is assumed that the period utility function takes the form

\[ U(C_t - H_t, N_t) \equiv \frac{1}{1-\sigma} (C_t - H_t)^{1-\sigma} - \frac{1}{1+\varphi} N_t(i)^{1+\varphi}, \tag{8} \]

where \( \sigma \) is the inverse of the intertemporal elasticity of substitution for consumption and \( \varphi \) is the inverse of the elasticity of labour supply. The consumption Euler equation follows as

\[ U_{C,t} = \beta E_t \left\{ \frac{U_{C,t+1}}{Q_{t,t+1}} \left( \frac{P_t}{P_{t+1}} \right) \right\}, \tag{9} \]

where \( U_{C,t} = (C_t - \vartheta C_{t-1})^{-\sigma} \), \( Q_{t,t+1} = \mu_t^d R_t^{-1} \), in which \( R_t \) represents the gross nominal return on a one-period discount bond and the stochastic disturbance term \( \mu_t^d \), as in Smets and Wouters (2007), represents the differential between the policy rate controlled by the monetary authority and the return on assets held by households, i.e. a risk premium on asset holdings. A positive shock to this risk premium is equivalent to a negative demand shock, as it increases the required return on assets and

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2 External habit formation represents the notion that households are influenced by the lagged consumption of other households, i.e. a preference for "keeping up with the Joneses" (see Nimark (2007)). It serves to explain the inertia of aggregate output generally found in the data.
reduces current consumption.

As in Erceg et al. (2000), it is assumed that households set wages in Calvo-style staggered contracts (see Calvo (1983)). Hence, in every period $t$, each household may reset its existing wage contract with probability $1 - \theta_w$. In addition, following Rabanal and Rubio-Ramirez (2005), it is assumed that the wage rate of the proportion of households that do not reset in period $t$ is partially indexed to the previous period’s rate of gross consumer price inflation, $\Pi_{t-1} = P_{t-1}/P_{t-2}$. The first order condition for the household’s labour supply is

$$E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left\{ \left[ \frac{\hat{W}_t}{P_{t+k}} \Pi_{t+k-1}^{\alpha} (C_{t+k} - H_{t+k})^{-\sigma} - (1 + \mu_w)N_{t+k}^\phi \right] N_{t+k} \right\} = 0,$$

where $\hat{W}_t$ is the optimal reset wage, $\alpha$ captures the degree of partial indexation to the previous period's rate of gross consumer price inflation, and $(1 + \mu_w)$ is the gross optimal wage markup. Hence, combining the first order condition for labour supply, Eq. (10), with the aggregate wage index, Eq. (5), and applying the law of large numbers, overall wage level evolves according to

$$W_t = \left[ \theta_w (W_{t-1} \Pi_{t-1}^{\alpha})^{1-\xi_w} + (1 - \theta_w)\hat{W}_t^{1-\xi_w} \right]^{1/1-\xi_w}.$$

2.2 Domestic firms and pricing

Production is assumed to take place in two stages. Firstly, a continuum of identical intermediate goods producing firms (indexed by $j$, where $j \in [0, 1]$) each produce a differentiated good. These firms are monopolistically competitive and set prices in a staggered manner as advocated by Calvo (1983). Secondly, final goods producers operate in a perfectly competitive environment and combine the differentiated intermediate goods into a final good, which is then sold to households.

2.2.1 Intermediate goods producers

In a continuum of intermediate goods producing firms, each firm produces a differentiated good $Y_t(j)$ using a linear technology production function as follows:

$$Y_t(j) = A_t N_t(j),$$

3 In the special case where wages are completely flexible, i.e. $\theta_w = 0$, Eq. (10) reduces to the standard first-order condition for labour supply:

$$\frac{W_t}{P_t} = (1 + \mu_w)MRS_t,$$

where $MRS_t = -\frac{\partial C_t}{\partial C_t} = (C_t - \vartheta C_{t-1})^\vartheta N_t^\phi$. 

4
where $A_t$ represents the firm-specific productivity and $a_t \equiv \log(A_t)$, which follows an AR(1) process $a_t = \rho a_{t-1} + \varepsilon_t^a$, and $\varepsilon_t^a \sim i.i.d. N(0, \sigma^2_a)$. The labour input of each firm, $N_t(j)$, is described by the constant elasticity of substitution (CES) composite of individual household labour supply, $N_t(i)$, as follows:

$$N_t(j) = \left[ \int_0^1 N_t(i) \frac{\xi_w}{\xi_w} \right]^\frac{1}{\xi_w - 1}$$

The total cost function faced by the intermediate goods producer is thus defined as its nominal wage bill:

$$TC_t^m = W_t N_t(j),$$

where $TC_t^m$ denotes total nominal costs. When Eq. (14) is combined with the linear production function of Eq. (12), real marginal cost is derived as the differential between the real wage (in terms of domestic prices) and productivity, as follows:

$$MC_t = \frac{W_t}{\widehat{P}_{H,t} A_t^{-1}}.$$

In order to introduce further nominal rigidity into the model framework, it is assumed that intermediate goods firms set prices in a staggered manner as proposed by Calvo (1983). Hence, according to Calvo’s method, in every period $t$, each firm is allowed to reset its price with probability $(1 - \theta_h)$. Thus, in a given period $t$, not all firms are able to react to supply shocks immediately, which implies that the higher $\theta_h$, the more sticky prices are. Furthermore, following Smets and Wouters (2002), additional price inertia is introduced by assuming that domestic prices are partially indexed to the last period’s gross rate of domestic price inflation, $\Pi_{h,t-1}$. Firms that have the ability to change their price in period $t$, will choose the optimal reset price $\hat{P}_{h,t}$ such that they maximise the following objective:

$$E_t \sum_{k=0}^{\infty} \theta_h^k Q_{t,t+k} Y_{t+k}(j) \left[ \hat{P}_{h,t} \Pi_{h,t+k-1}^\delta - MC_{t+k} P_{h,t+k} \right],$$

subject to the demand for intermediate goods by final goods producers (discussed below):

$$Y_t(j) = \left[ \frac{P_{h,t}(j) \Pi_{h,t-1}^\delta}{\hat{P}_{h,t}} \right]^{-\xi_h} Y_t,$$

where $\delta$ is the degree of partial indexation to the last period’s gross rate of domestic price inflation. The optimal solution to this objective is therefore:

$$E_t \sum_{k=0}^{\infty} \theta_h^k Q_{t,t+k} Y_{t+k}(j) \left[ \hat{P}_{h,t} \Pi_{h,t+k-1}^\delta - (1 + \mu^p) MC_{t+k} P_{h,t+k} \right] = 0,$$
which intuitively states that the firm maximises its expected future profits by setting the optimal reset price equal to a markup \((\mu p)\) over a discounted stream of expected future nominal marginal cost.\(^4\)

### 2.2.2 Final goods producers

In order to produce the final good \(Y_t\), final goods producing firms use intermediate goods as their only input and employ the following CES technology production function:

\[
Y_t = \left[ \int_0^1 Y_t(j) \frac{\xi_h}{\xi_h - 1} dj \right]^{\frac{\xi_h}{\xi_h - 1}}. 
\]  

(19)

Both \(Y_t\) and \(Y_t(j)\) are normalised by population size. Through profit maximisation, i.e. the optimal demand for the input \(Y_t(j)\) where the price of the final good is given, the demand function for intermediate goods, i.e. Eq. (17), is obtained as well as the price index:

\[
P_{h,t} = \left[ \int_0^1 P_{h,t}(j)^{1-\xi_h} dj \right]^{\frac{1}{1-\xi_h}}, 
\]  

(20)

By incorporating the price-setting behaviour of intermediate goods producers into Eq. (20), the law of large numbers implies that the economy’s domestic price index will evolve as follows:

\[
P_{h,t} = \left[ \theta_h \left( P_{h,t-1} \Pi_{h,t} \right)^{1-\xi_h} + (1 - \theta_h) (\hat{P}_{h,t})^{1-\xi_h} \right]^{\frac{1}{1-\xi_h}}. 
\]  

(21)

### 2.3 Domestic importing retailers and incomplete pass-through

Following Monacelli (2003), it is assumed that there are retailers in the domestic economy who import differentiated goods. Furthermore, when the imported goods are received by the local importing retailers, they pay the world-market price in terms of the domestic currency, i.e. the law of one price still holds. However, these importing retailers face a downward-sloping demand curve in the domestic economy and therefore need to solve an optimal mark-up problem when setting the domestic currency retail price of the imported goods. This feature allows for deviations from the law of one price in the short run, while complete pass-through of exchange rate movements are achieved in the long run.\(^5\)

Deviations from the law of one price, i.e. the law of one price (l.o.p.) gap, is defined as follows:

\[
\Psi_{f,t} \equiv \frac{\xi_t P^*_t}{P_{f,t}}, 
\]  

(22)

\(^4\) In the special case where prices are flexible, i.e. \(\theta_h = 0\), all firms would be able to change their price in every period \(t\). Eq. (18) then reduces to \(P_t = (1 + \mu p)MC_t^n\) which implies that firms would choose a reset price that includes a constant markup over marginal cost.

\(^5\) See Campa and Goldberg (2002).
where $\Psi_{f,t}$ denotes the l.o.p. gap, $E_{t}P^*_t$ the world price in terms of the domestic currency and $P_{f,t}$ the domestic currency price of imports.

Similar to the domestic firms, the local importing retailers exhibit Calvo-style price setting behaviour and hence are able to reset the domestic retail price of the imported good in period $t$ with probability $(1-\theta_f)$. Thus, the importing retailer needs to set price $\hat{P}_{f,t}(j)$ for imported good $j$ such as to maximise:

$$E_t \left\{ \sum_{k=0}^{\infty} (\theta_f \beta)^k Q_{t,t+k} C_{f,t+k}(j) \left[ \hat{P}_{f,t}(j) - E_{t+k}P^*_t \right] \right\},$$

subject to

$$C_{f,t}(j) = \left[ \frac{\hat{P}_{f,t}(j)}{P_{f,t}} \right]^{-\xi_f} C_{f,t},$$

where $P^*_t(j)$ is the price of good $j$ in the world market and hence $E_{t+k}P^*_t$ is the domestic currency price paid by the local importer, i.e. the price for which the law of one price still holds. The optimal solution to the importing retailer’s mark-up problem is therefore:

$$E_t \sum_{k=0}^{\infty} (\theta_f \beta)^k Q_{t,t+k} C_{f,t+k}(j) \left[ \hat{P}_{f,t}(j) - (1 + \mu_f)E_{t+k}P^*_t \right] = 0,$$

where $\mu_f = \frac{1}{\xi_f - 1}$. Eq. (23) shows that the local importing retailer sets the optimal domestic currency retail price of the imported good equal to a mark-up $(\mu_f)$ over a discounted stream of expected future world-market prices in terms of the domestic currency, i.e. the price for which the law of one price holds.

By incorporating the price-setting behaviour of importing retailers into the price index for foreign (imported) goods:

$$P_{f,t} = \left( \int_0^1 P_{f,t}(j)^{1-\xi_f} dj \right)^{\frac{1}{1-\xi_f}},$$

the law of large numbers then implies that the economy’s overall imported price index will evolve as follows:

$$P_{f,t} = \left[ \theta_f P_{f,t-1}^{1-\xi_f} + (1 - \theta_f)(\hat{P}_{f,t}^{1-\xi_f}) \right]^{\frac{1}{1-\xi_f}}.$$

**2.4 Terms of trade and the real exchange rate**

Let the terms of trade, i.e. the relative price of imports to domestically produced goods, be defined as

$$S_t \equiv \frac{P_{f,t}}{P_{h,t}}.$$
and the real exchange rate be defined as the foreign CPI, in terms of the domestic currency, relative to
domestic CPI:
\[ Q_t \equiv \frac{E_t P_t^*}{P_t}, \]  
(27)
where \( E_t \) the nominal exchange rate, i.e. the price of one unit of foreign currency in terms of the
domestic currency.

2.5 International risk sharing and UIP

Assuming complete international asset markets implies that consumption risk is perfectly shared be-
tween the households of the domestic economy and those of the foreign economy. Furthermore, it can
be shown that a consumption Euler equation similar to Eq. (9) will hold for the representative household
in the foreign country. Hence, perfect risk sharing requires that, in equilibrium, the stochastic discount
factors in the domestic and foreign economy are equal, which yields the following condition:
\[ \beta E_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} \left( \frac{P_t}{P_{t+1}^*} \right) \right\} = Q_{t,t+1} = \beta E_t \left\{ \frac{U_{C,t+1}}{U_{C,t}^*} \left( \frac{P_t^*}{P_{t+1}^*} \right) \left( \frac{E_t}{E_{t+1}} \right) \right\}, \]  
(28)
where the superscript \( \ast \) refers to the foreign country. Under the assumption of symmetric initial condi-
tions of the relative net asset position, the consumption risk sharing condition becomes: \(^6\)
\[ U_{C,t} = \zeta U_{C,t}^* Q_t^{-1}. \]  
(29)

The consumption risk sharing condition implies that differences in the relative marginal utilities of con-
sumption, in equilibrium, are captured by movements in the real exchange rate. Subsequently, the
combination of the domestic and foreign consumption Euler equations with Eq. (29) yields the following
uncovered interest parity (UIP) condition for the real exchange rate:
\[ E_t \{ Q_{t+1} \} = Q_t \left( \frac{R_t}{R_{t+1}} \right) \left\{ \frac{\Pi_{t+1}}{\Pi_{t+1}^*} \right\} \Phi_t, \]  
(30)
where \( \Phi_t = \frac{\nu_t^d}{\nu_t^{d*}} \) is the stochastic risk premium.

2.6 Goods market equilibrium

Goods market clearing in the domestic economy requires that:
\[ Y_t = C_{h,t} + C_{h,t}^*, \]  
(31)

\(^6\) It can be shown that \( \zeta = \left( \frac{C}{C^*} \right) Q_1^{-1} \), and similar to Gali and Monacelli (2005), if it is assumed that the initial relative net
asset position is symmetric, in the symmetric perfect foresight steady state \( C_t = C^*_t = C \) and \( Q = 1 \), hence \( \zeta = 1 \).
where $Y_t$ is aggregate domestic output, from Eq. (2a) $C_{h,t}$ is the demand for goods produced in the domestic economy and $C^*_h$ is the foreign demand for domestically produced goods, i.e. exports.

### 2.7 Linearisation

Next, the model system is represented as a log-linear approximation around the steady state, where lower case variables indicate log deviations from the deterministic steady state. From Eq. (9), the consumption Euler equation can be expressed as a log-linear approximation around the steady state as follows:

$$c_t = \frac{1}{1 + \vartheta} E_t \{c_{t+1}\} + \frac{\vartheta}{1 + \vartheta} c_{t-1} - \frac{1 - \vartheta}{\sigma(1 + \vartheta)} [r_t - E_t \{\pi_{t+1}\}] + \hat{\mu}_t^d,$$

where $r_t$ is the nominal interest rate, $\pi_{t+1}$ is the consumer price inflation rate from period $t$ to $t+1$, and $\hat{\mu}_t^d$ is the risk premium on asset holdings which follows an AR(1) process, as follows $\hat{\mu}_t^d = \rho d \hat{\mu}_{t-1}^d + \varepsilon_d^t$, $\varepsilon_d^t \sim i.i.d. N(0, \sigma_d^2)$.

Combining the log-linearised optimal wage-setting rule with the overall wage index, i.e. Eqs. (10) and (11), yields the Phillips-curve type relationship for nominal wage inflation, partially indexed to consumer price inflation, as follows:

$$\pi_{w,t} - \alpha \pi_{t-1} = \beta \pi_{w,t+1} - \alpha \beta \pi_t + \frac{(1 - \theta_w)(1 - \theta_w \beta)}{\theta_w(1 + \varphi \xi_w)} \hat{\mu}_t^w$$

where the wage mark-up $\hat{\mu}_t^w$ serves as a wedge between the real wage and the marginal rate of substitution between labour and consumption, that arises due to wage stickiness:

$$\hat{\mu}_t^w = \frac{\sigma}{(1 - \vartheta)} (c_t - \vartheta c_{t-1}) + \varphi (y_t - a_t) - \omega_t + \eta_t^w,$$

where $\omega_t = p_t - w_t$ is the level of the real wage and it is assumed that the wage markup shock, $\eta_t^w$ follows an AR(1) process as follows: $\eta_t^w = \rho_w \eta_{t-1}^w + \varepsilon_t^w$, $\varepsilon_t^w \sim i.i.d. N(0, \sigma_w^2)$.

From Eq. (26), the terms of trade is

$$s_t = p_{f,t} - p_{h,t},$$

and the overall price level, from Eq. (3), is

$$p_t = p_{h,t} + \gamma s_t.$$  

Real marginal cost, from Eq. (15), is therefore expressed in terms of the real wage, productivity, and the terms of trade:

$$mc_t = \omega_t - a_t + \gamma s_t + \eta_t^p,$$
where $\eta_p$ serves as a stochastic cost-push shock and follows an AR(1) process: $\eta_p = \rho_p \eta_{p,-1} + \varepsilon_p$, $\varepsilon_p \sim i.i.d. N(0, \sigma_p^2)$.

Further, the New-Keynesian Phillips curve for domestic inflation (w.r.t. marginal cost), is derived through the combination of the log-linearised optimal price-setting rule and price level, i.e. Eqs. (18) and (21):

$$\pi_{h,t} = \frac{\delta}{1 + \delta \beta} \pi_{h,t-1} + \frac{\beta}{1 + \delta \beta} \pi_{h,t+1} + \frac{(1 - \theta_h)(1 - \theta_h \beta)}{\theta_h (1 + \delta \beta)} m_{ct}. \quad (38)$$

From Eq. (3) the log-linearised overall CPI inflation equation is

$$\pi_t = (1 - \gamma) \pi_{h,t} + \gamma \pi_{f,t}, \quad (39)$$

where imported inflation, $\pi_{f,t}$, can be expressed as a Phillips-curve type relationship by combining the log-linearised optimal price-setting rule for importing retailers with the import price index, i.e. Eqs. (23) and (25):

$$\pi_{f,t} = \beta \pi_{f,t+1} + \frac{(1 - \theta_f)(1 - \theta_f \beta)}{\theta_f} \psi_{f,t}, \quad (40)$$

and where, from Eq. (22), the l.o.p. gap is given as

$$\psi_{f,t} \equiv (e_t + p^*_t) - p_{f,t}. \quad (41)$$

On the open economy side, the international risk sharing condition, from Eq. (29), is:

$$\frac{\sigma}{1 - \vartheta} (c_t - \vartheta c_{t-1}) = \frac{\sigma}{1 - \vartheta} (c^*_t - \vartheta c^*_{t-1}) + q_t, \quad (42)$$

where $q_t$ is the real exchange rate, and hence, from Eq. (30), the uncovered interest parity condition for the real exchange rate is

$$E_t \Delta q_{t+1} = (r_t - \pi_{t+1}) - (r^*_t - \pi^*_{t+1}) + \phi_t, \quad (43)$$

where $\phi_t = \hat{\mu}_t^d - \hat{\mu}_t^{d*}$ is the exchange rate risk premium, and $\hat{\mu}_t^d$ is assumed to follow an AR(1) process: $\hat{\mu}_t^d = \rho_d \hat{\mu}_t^{d,-1} + \varepsilon_d$, $\varepsilon_d \sim i.i.d. N(0, \sigma_d^2)$. $\hat{\mu}_t^{d*}$ is assumed to follow an analogous AR(1) process. From Eq. (31), aggregate demand in the domestic economy follows as

$$y_t = (1 - \gamma) c_t + \eta \gamma (2 - \gamma) s_t + \gamma y_t^* + \eta \gamma \psi_{f,t}. \quad (44)$$

The rest of the world, i.e. the foreign economy in the context of this model, is modelled as a closed economy version of the domestic economy, hence $\gamma = 0$. The only structural difference lies in the wage-setting process, since wages in the foreign economy are assumed to be flexible for the sake of simplicity. Being an ‘approximated’ closed economy, all goods are sold domestically, and hence aggregate demand equals consumption, i.e. $y_t^* = c_t^*$. It also follows that since there is no imported
inflation component, domestic inflation equals overall CPI inflation, i.e. $\pi_{f,t}^* = \pi_t^*$. Furthermore, it can be shown that the closed foreign economy model collapses into an IS curve

$$ y_t^* = \frac{1}{1 + \vartheta} E_t \{ y_{t+1}^* \} + \frac{\vartheta}{1 + \vartheta} y_{t-1}^* - \frac{1 - \vartheta}{\sigma(1 + \vartheta)} \left[ r_t^* - E_t \{ \pi_{t+1}^* \} + \hat{\mu}_t^* \right], $$

and a New Keynesian Phillips curve (i.e. marginal cost):

$$ \pi_t^* = \frac{\delta^*}{1 + \delta^* \beta} \pi_{t-1}^* + \frac{\beta}{1 + \delta^* \beta} E_t \{ \pi_{t+1}^* \} + \frac{(1 - \theta^*)(1 - \theta^* \beta)}{\theta^*(1 + \delta^* \beta)} mc_t^*, $$

where

$$ mc_t^* = \left( \frac{\sigma^*}{1 - \vartheta} + \varphi^* \right) y_t^* - \left( \frac{\sigma^* \vartheta}{1 - \vartheta} \right) y_{t-1}^* - (1 + \varphi^*) a_t^* + \hat{\mu}_t^{ws}, $$

and productivity in the foreign economy follows an AR(1) process:

$$ a_t^* = \rho_{a}^* a_{t-1}^* + \varepsilon_t^{a*}, \quad \varepsilon_t^{a*} \sim i.i.d. N(0, \sigma_{a}^*). $$

To close the domestic and foreign economies, the behaviour of the respective monetary authorities is specified. The aim of the central bank, in the context of this model, is to stabilise both output and inflation in order to reproduce the specified equilibrium. Rather than setting out an explicit optimising framework for the monetary authority, as in Woodford (2003) and Clarida et al. (2001), it is assumed that the central bank follows a simple Taylor rule, similar to the policy rule used in Rabanal and Tuesta (2006):

$$ r_t = \rho_r r_{t-1} + (1 - \rho_r) \left[ \phi_\pi \pi_t + \phi_y (\Delta y_t) \right] + \varepsilon_t^r $$

where $\rho_r$ is the degree of policy smoothing, $\phi_\pi$ and $\phi_y$ are the relative weights on inflation and output growth, respectively, and $\varepsilon_t^r \sim i.i.d. N(0, \sigma_r^2)$ captures shocks to the policy rate. An analogous Taylor rule holds for the foreign economy.

Finally the monetary policy transmission in the model may be (briefly) described as follows: The aggregate demand channel is represented by the impact of interest rate changes on consumption expenditure ($c_t$) and output ($y_t$). These impact on domestic prices ($\pi_{h,t}$) and consumer prices ($\pi_t$) via marginal costs. In the open economy context the aggregate demand effect is strengthened through changes in the terms of trade (in response to the changes in domestic prices) which affect output ($y_t$) and also via marginal costs. The aggregate supply channel works through the impact of interest rate changes on the nominal and real exchange rates (i.e. $e_t$ and $q_t$) and then import prices ($\pi_{f,t}$). The changes in import prices affect inflation directly via the CPI and indirectly via the terms of trade as above.
3 Estimation methodology

3.1 Estimation

Parameter estimates for DSGE models can be derived via calibration or a variety of econometric estimation techniques (see De Jong and Dave (2007)). In recent years Bayesian estimation techniques appear to have become quite popular. This followed from the development of the appropriate numerical techniques and particular relevant advantages of the Bayesian methods. These advantages include the formal incorporation of prior information into the estimation process and dealing with potential model misspecification and possible lack of identification (see An and Schorfheide (2007) for a detailed analysis).

The Bayesian method may be formally described as follows (see Harjes and Ricci (2008)). Given a set of observables $Y^T$ over a sample period $T$ and a set of priors $p(\Theta)$, the posterior density of the model parameters $(\Theta)$ is given by:

$$p(\Theta|Y^t) = \frac{L(\Theta|Y^t)p(\Theta)}{\int L(\Theta|Y^t)p(\Theta)d(\Theta)},$$

where $Y^T$ is the set of variables for which data are provided and $\Theta$ is the vector of parameters in the model. The assumptions on the prior distributions of the parameters are presented in the next section below.

Given the highly non-linear mapping from the vector of structural parameters to the reduced form state-space representation of DSGE models, these models are prone to identification problems, and hence, it is therefore not possible to recover certain parameters values during estimation (see Lubik and Schorfheide (2005)). Until recently, the issue concerning parameter identification has not been considered to a great extent in the empirical DSGE literature. However, since then a number of studies have focussed on the importance thereof. See, for example, Iskrev (2007), Canova and Sala (2006) and Lubik and Schorfheide (2005). Fukac, Pagan and Pavlov (2006), suggest parameter identification is of greater importance if the impact of changes to structural parameters were to be analysed, as opposed to more standard policy uses of these models, such as impulse response analyses or forecasting. Nevertheless, in order to account for the lack of parameter identification in this model, certain parameters were calibrated. The selection of these parameters was guided by the singular value decomposition of the model’s information matrix.\(^7\)

The calibrated parameters are summarised in Table 1. The calibrated values chosen for the discount factor as well as the domestic and foreign labour substitution elasticity – a parameter that is notoriously

\(^7\) We would like to thank Michal Andrle for alerting us to the identification issue and for providing valuable assistance in this regard.
Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\beta$</th>
<th>$\vartheta$</th>
<th>$\gamma$</th>
<th>$\varphi$</th>
<th>$\xi_w$</th>
<th>$\delta$</th>
<th>$\theta_w$</th>
<th>$\rho_r$</th>
<th>$\rho_w$</th>
<th>$\sigma^*$</th>
<th>$\varphi^*$</th>
<th>$\delta^*$</th>
<th>$\rho^*_w$</th>
<th>$\rho^*_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.99</td>
<td>0.7</td>
<td>0.2</td>
<td>3.0</td>
<td>1.0</td>
<td>0.25</td>
<td>0.5</td>
<td>0.73</td>
<td>0.8</td>
<td>1.0</td>
<td>3.0</td>
<td>0.5</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

unidentified in DSGE models – are standard in the literature. The degree of habit formation for both
the domestic and foreign economy is set to 0,7 following estimates of Smets and Wouters (2007). The
import share in the domestic economy is set to 0,2 and is guided by the actual import penetration ratio
in total South African GDP of approximately 30% during the last 5 years of the sample period and an
import penetration ratio in consumption of approximately 7,5% during the same period. The labour
demand elasticity is set to 1,0 and the Calvo parameter for wage setting is set to 0,5, which implies that
on average, wage contracts are reset every two quarters. Justiniano and Preston (2004) find that under
a uniform distribution within the (0,1) interval, the degree of price indexation in small open economies
Australia, Canada and New Zealand is less than 0,2. Adolfson et. al. (2005) find price indexation to
the degree of around 0.22 in a small open economy model of Sweden. In the light of these findings, the
degree of price indexation is set to 0,25 for the domestic economy. Following the findings of Smets and
Wouters (2002) for the Euro Area, the degree of price indexation in the foreign economy is set to 0,5.
The intratemporal elasticity of substitution for the foreign economy is based on standard calibrations in
the literature. The AR(1) persistence parameters for the wage shock in both the domestic and foreign
economy, as well as the foreign economy’s monetary policy smoothing parameter were calibrated at
0,8. The monetary policy smoothing parameter for the domestic economy was set at 0,73, based on
the findings of Ortiz and Sturzenegger (2007) for South Africa. The remaining parameters of the model
are estimated using Bayesian techniques, as discussed above. The prior distributions for the param-
eters are presented in Table 2. The prior specifications are broadly based on those used in a number
of similar studies for open economies. The priors on the standard deviations of shocks in the domestic
economy are twice that of the foreign economy, allowing for the fact that frequent and extensive shocks
generally are more endemic in small open economies. These prior distributions are assumed to be
independent across the different parameters.

Estimation of the model’s parameters is done in two steps: firstly, the foreign economy’s parameters
are estimated independently from those of the domestic economy; hereafter, the domestic (open) econ-
yomy’s parameters are estimated, while the foreign economy’s parameters are calibrated to their re-
spective posterior mean values recovered in step one. This approach has no significant impact on the
estimation outcome, as the foreign economy is closed and therefore theoretically not influenced by the
domestic economy.\footnote{Justiniano and Preston (2004) note that from an econometric point of view, the foreign economy is not completely ex-
genous. The implied cross-equation restrictions from the uncovered interest parity condition allows the data generating
process of the foreign economy to be influenced by the domestic economy’s parameters. However, their estimates of the
foreign blocks for Australia, Canada and New Zealand yield no significant differences.}

8 Justiniano and Preston (2004) note that from an econometric point of view, the foreign economy is not completely ex-
genous. The implied cross-equation restrictions from the uncovered interest parity condition allows the data generating
process of the foreign economy to be influenced by the domestic economy’s parameters. However, their estimates of the
foreign blocks for Australia, Canada and New Zealand yield no significant differences.
3.2 Data

In order to estimate the model, domestic and international macro-economic time series for inflation, output and interest rates for the period 1990Q1 to 2007Q4 are used.\(^9\) For South Africa, the quarterly Consumer Price Index excluding interest rates on mortgage bonds (2000=100), hereafter CPIX, and quarterly real Gross Domestic Product (2000=100) data, which are compiled by Statistics South Africa, were used.\(^10\) For the domestic nominal interest rate, the South African Repo rate was used. On the foreign economy’s side, the quarterly price level is represented by a trade-weighted wholesale price index, which is weighted on the basis of trade in manufactured goods between South Africa and its most important trading partners, excluding Zimbabwe (2000=100).\(^11\) The foreign economy’s real Gross Domestic Product is proxied by the total GDP for member countries of the Organisation for Economic Co-operation and Development (OECD). The foreign interest rate is proxied by a trade weighted combination of the United States’ Federal Funds rate and the European Central Bank’s (ECB) key interest rate.\(^12,13\) All series are presented in Figure A.1 in the Appendix.\(^14\)

4 Estimation results

4.1 Posterior parameter estimates

The parameter estimates for the domestic economy are presented in Table 2.\(^15\) These estimates are represented by the mean values of the estimated posterior distribution of each parameter. Generally

\(^9\) The decision on the sample size of 72 observations was largely guided by the literature on Bayesian estimation of DSGE models, as well as specifics relating to the South African data. Justiniano and Preston (2007) use 70 observations to estimate a DSGE model for New Zealand with Bayesian techniques, while An and Schorfheide (2007) regard 80 observations as a realistic sample size. Furthermore, it is not uncommon in the US literature to use data sets that begin in the mid 1980s, i.e. the period after the high inflation of the 1970s and the subsequent disinflation. For almost similar reasons, the sample period 1990Q1 to 2007Q4 was selected in order to exclude the excessively volatile GDP, high interest rates and high inflation characterising the South African economy during the 1980s.

\(^10\) CPIX inflation, i.e. the consumer price index excluding interest rates on mortgage bonds, is the definitional equivalent of CPI inflation \((\pi_t)\) portrayed in the DSGE model.

\(^11\) The weight structure used in the calculation of the foreign price index corresponds to the weights used in the official calculation of the real effective exchange rate for South Africa.

\(^12\) Trade weights used in the calculation of the foreign interest rate series correspond to the specific weights that were used for the United States and Euro Area in the calculation of the foreign price index.

\(^13\) For the period 1990Q1 to 1998Q4, i.e. prior to the inception of the Euro Area, the Deutsche Bundesbank’s discount rate is used as a proxy for the ECB key interest rate.

\(^14\) For the purpose of consistency with respect to the linearised model, all series are expressed as log deviations from a deterministic trend, e.g. \(\left[\ln(GDP_t) - \ln(GDP_t^{\text{trend}})\right] \times 100\). Furthermore, the interest rate series are converted to quarterly rates and inflation is measured as the first difference of its respective price index, i.e. quarterly non-annualised inflation.

\(^15\) The posterior modes are determined via Sims’s algorithm, checking for local optima at these modes. Using Dynare, the posterior parameter distributions are then estimated using the Metropolis-Hastings algorithm with 5 blocks of 50000 replications each. The acceptance rate for each drawing was around 25 per cent. The Dynare program is an ongoing project under the guidance of Michel Juillard at the Center for Economic Research and its Applications (CEPREMAP) and can be downloaded at www.cepremap.cnrs.fr/dynare
speaking the estimates appear quite reasonable.

Table 2: Prior distributions and posterior estimates

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Prior</th>
<th>Prior</th>
<th>Prior</th>
<th>Posterior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>density</td>
<td>mean</td>
<td>std dev</td>
<td>mean</td>
<td>90% interval</td>
</tr>
<tr>
<td>Domestic economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structural parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$ Inverse of substitution elasticity</td>
<td>N</td>
<td>1</td>
<td>0.2</td>
<td>1.026</td>
<td>0.770 ; 1.263</td>
</tr>
<tr>
<td>$\eta$ Home/Foreign substitution elasticity</td>
<td>G</td>
<td>1</td>
<td>0.2</td>
<td>0.591</td>
<td>0.406 ; 0.766</td>
</tr>
<tr>
<td>$\alpha$ Wage-price indexation</td>
<td>B</td>
<td>0.75</td>
<td>0.1</td>
<td>0.696</td>
<td>0.515 ; 0.885</td>
</tr>
<tr>
<td>$\theta_d$ Calvo: domestic prices</td>
<td>B</td>
<td>0.75</td>
<td>0.1</td>
<td>0.539</td>
<td>0.409 ; 0.679</td>
</tr>
<tr>
<td>$\theta_f$ Calvo: imported prices</td>
<td>B</td>
<td>0.75</td>
<td>0.1</td>
<td>0.672</td>
<td>0.497 ; 0.855</td>
</tr>
<tr>
<td>Taylor rule weights</td>
<td>G</td>
<td>1.5</td>
<td>0.125</td>
<td>1.389</td>
<td>1.240 ; 1.525</td>
</tr>
<tr>
<td>$\phi_{\pi}$ Inflation</td>
<td>G</td>
<td>0.5</td>
<td>0.125</td>
<td>0.625</td>
<td>0.402 ; 0.864</td>
</tr>
<tr>
<td>Persistence parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a$ AR(1): productivity</td>
<td>B</td>
<td>0.8</td>
<td>0.1</td>
<td>0.727</td>
<td>0.531 ; 0.919</td>
</tr>
<tr>
<td>$\rho_d$ AR(1): demand</td>
<td>B</td>
<td>0.8</td>
<td>0.1</td>
<td>0.639</td>
<td>0.547 ; 0.724</td>
</tr>
<tr>
<td>$\rho_p$ AR(1): domestic prices</td>
<td>B</td>
<td>0.8</td>
<td>0.1</td>
<td>0.743</td>
<td>0.550 ; 0.933</td>
</tr>
<tr>
<td>Standard deviations of domestic shocks</td>
<td>IG</td>
<td>2</td>
<td>$\infty$</td>
<td>1.151</td>
<td>0.490 ; 1.854</td>
</tr>
<tr>
<td>$\sigma_a$ iid shock: productivity</td>
<td>IG</td>
<td>2</td>
<td>$\infty$</td>
<td>0.483</td>
<td>0.361 ; 0.599</td>
</tr>
<tr>
<td>$\sigma_d$ iid shock: demand</td>
<td>IG</td>
<td>2</td>
<td>$\infty$</td>
<td>1.403</td>
<td>0.499 ; 2.377</td>
</tr>
<tr>
<td>$\sigma_w$ iid shock: wages</td>
<td>IG</td>
<td>2</td>
<td>$\infty$</td>
<td>1.952</td>
<td>0.565 ; 3.353</td>
</tr>
<tr>
<td>$\sigma_p$ iid shock: costs</td>
<td>IG</td>
<td>2</td>
<td>$\infty$</td>
<td>0.366</td>
<td>0.312 ; 0.420</td>
</tr>
<tr>
<td>Foreign economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structural parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta^*$ Calvo: prices</td>
<td>B</td>
<td>0.75</td>
<td>0.2</td>
<td>0.557</td>
<td>0.472 ; 0.637</td>
</tr>
<tr>
<td>Taylor rule weights</td>
<td>G</td>
<td>1.5</td>
<td>0.125</td>
<td>1.277</td>
<td>1.159 ; 1.394</td>
</tr>
<tr>
<td>$\phi_{\pi}^*$ Inflation</td>
<td>G</td>
<td>0.5</td>
<td>0.125</td>
<td>0.651</td>
<td>0.399 ; 0.902</td>
</tr>
<tr>
<td>Persistence parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a^*$ AR(1): productivity</td>
<td>B</td>
<td>0.75</td>
<td>0.1</td>
<td>0.695</td>
<td>0.506 ; 0.900</td>
</tr>
<tr>
<td>$\rho_d^*$ AR(1): demand</td>
<td>B</td>
<td>0.75</td>
<td>0.1</td>
<td>0.657</td>
<td>0.575 ; 0.740</td>
</tr>
<tr>
<td>Standard deviations of foreign shocks</td>
<td>IG</td>
<td>1</td>
<td>$\infty$</td>
<td>0.610</td>
<td>0.383 ; 0.849</td>
</tr>
<tr>
<td>$\sigma_a^*$ iid shock: productivity</td>
<td>IG</td>
<td>1</td>
<td>$\infty$</td>
<td>0.280</td>
<td>0.206 ; 0.353</td>
</tr>
<tr>
<td>$\sigma_d^*$ iid shock: demand</td>
<td>IG</td>
<td>1</td>
<td>$\infty$</td>
<td>0.769</td>
<td>0.247 ; 1.356</td>
</tr>
<tr>
<td>$\sigma_w^*$ iid shock: costs</td>
<td>IG</td>
<td>1</td>
<td>$\infty$</td>
<td>0.211</td>
<td>0.179 ; 0.243</td>
</tr>
</tbody>
</table>

†Where $B = \text{beta}$, $N = \text{normal}$, $G = \text{gamma}$, $IG = \text{inverse gamma}$

For the domestic economy, the estimate for the inverse elasticity of temporal substitution is broadly in line with results for similar models. The elasticity of substitution between foreign and home goods is estimated at 0.591, which is fairly low (Chari, McGratten and Kehoe (2002), for example, calibrated this parameter at 1.5), perhaps reflecting South Africa’s status as a commodity exporter. The degree of wage indexation to previous CPI inflation (0.696) is relatively high when compared to similar studies such as Justiniano and Preston (2006), but reflective of the wage formation process in South Africa. The Calvo parameters for domestic firms (0.539) and importing firms (0.672) suggest that domestic and imported prices are re-optimised every 2 – 3 quarters, with domestic prices being less sticky than
imported prices in this respect. The estimates for the Taylor rule parameters are in accordance with the results of similar models for other countries. The parameters for inflation and output are estimated at 1,389 and 0,625, respectively. The estimate for output suggests that the South African monetary authorities are relatively sensitive to the output effects of interest rate changes, a result which concurs with a recent paper by Ortiz and Sturzenegger (2007). The persistence parameters for the AR(1) shocks to technology, demand and domestic prices are all fairly high (at 0,639 or higher). Finally, the standard deviations of domestic shocks indicate that price and wage mark-up shocks dominate in terms of magnitude.

The parameter estimates for the foreign economy are in general similar to those of the domestic economy. Foreign firms appear to re-optimize prices every 2 quarters on average. When compared to the domestic economy, the AR(1) persistence parameters of the shocks are also high, with demand shocks being slightly more persistent and productivity shocks slightly less persistent. Finally, shocks within the cost channel also dominate in terms of magnitude, albeit to a lesser extent when compared to the domestic economy’s estimates.

4.2 Variance decomposition

The relative importance of the different shocks in explaining the major macroeconomic variables in the model is obtained by the variance decomposition of the shocks. The domestic cost ($\varepsilon^w_t$) and wage mark-up shocks ($\varepsilon^p_t$) are grouped as supply shocks, and all four shocks of the foreign economy are grouped into one category as external shocks. These results are summarized in Table 3 below.

<table>
<thead>
<tr>
<th></th>
<th>Productivity</th>
<th>Demand</th>
<th>Supply</th>
<th>Monetary policy</th>
<th>External</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real wage</td>
<td>$\varpi_t$</td>
<td>0,09</td>
<td>0,02</td>
<td>0,86</td>
<td>0,02</td>
</tr>
<tr>
<td>Domestic inflation</td>
<td>$\pi_{h,t}$</td>
<td>0,39</td>
<td>0,10</td>
<td>0,37</td>
<td>0,14</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>$\Delta e_t$</td>
<td>0,02</td>
<td>0,35</td>
<td>0,02</td>
<td>0,29</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>$q_t$</td>
<td>0,29</td>
<td>0,16</td>
<td>0,20</td>
<td>0,14</td>
</tr>
<tr>
<td>Imported inflation</td>
<td>$\pi_{f,t}$</td>
<td>0,07</td>
<td>0,32</td>
<td>0,05</td>
<td>0,43</td>
</tr>
<tr>
<td>Consumer price inflation</td>
<td>$\pi_t$</td>
<td>0,35</td>
<td>0,13</td>
<td>0,34</td>
<td>0,18</td>
</tr>
<tr>
<td>Output</td>
<td>$y_t$</td>
<td>0,51</td>
<td>0,09</td>
<td>0,32</td>
<td>0,08</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>$r_t$</td>
<td>0,24</td>
<td>0,35</td>
<td>0,19</td>
<td>0,21</td>
</tr>
</tbody>
</table>

Supply shocks contribute more than 80 per cent to the variation in real wages, and productivity around 10 per cent. Domestic inflation and total CPI inflation are similarly dominated by productivity and supply shocks, with demand shocks contributing 10 per cent to domestic inflation and 13 per cent to total CPI inflation. By contrast, the variation in imported inflation is dominated by monetary policy and demand shocks (43 and 32 per cent respectively), while external shocks account for 13 per cent. The contributions to variation in the nominal and real exchange rates are largely in line with priori expectations. Shocks to demand, monetary policy and the foreign economy contribute 35, 29 and 32 per cent to variation in the nominal exchange rate respectively. The real exchange rate is dominated by productivity
shocks, accounting for 29 per cent of the variation, while supply and external shocks each account for 20 per cent. The variation in output is dominated by productivity (51 per cent) and supply shocks (32 per cent). Demand and monetary policy shocks each contribute less than 10 per cent to output variability. The nominal interest rate is dominated by demand shocks and appears to be insulated from external shocks.

4.3 Impulse response function analysis

In order to analyse the dynamic behaviour of the model, the impulse responses of a standard deviation temporary shock to the various structural shocks are presented in figures A.2 to A.5 in Appendix A. Solid lines represent the mean values of the posterior distribution and, due to parameter uncertainty, dashed lines represent the 90 per cent confidence interval.

The responses to a shock to the nominal interest rate (see Figure A.2) are in accordance with the results found in the literature (see e.g. Smets and Wouters (2002) and Adolfson et al. (2005)) and Harjes and Ricci (2008) for South Africa. Both consumer price inflation and output reflect the typical humped-shape response although the peak in the negative response of inflation occurs at two quarters, which is earlier than in the other studies referred to here. The impact on inflation (of about 0.6 per cent decline in the CPI in the first year) is, however, more than double the impact found by Harjes and Ricci (2008). The nominal exchange rate initially appreciates, but depreciates slightly in the second quarter and then stabilises after five quarters.

The impulse responses to a cost-push shock are also similar to other international studies (see e.g. Smets and Wouters (2002)) and to that obtained by Harjes and Ricci (2008) for South Africa (see Figure A.3 in Appendix A). Real wages decline as nominal wage increases lag the immediate reaction of consumer price inflation. Monetary policy reacts to rising inflation and peaks in the second quarter. Due to the rising real interest rate output reacts negatively, with a typical humped-shape response, peaking in the third quarter. Both interest rates and output peak earlier than found by Harjes and Ricci. The real exchange rate appreciates due to the positive real interest rate differential with the foreign economy, and peaks in the second quarter.

The impact of a positive wage shock (see Figure A.4 in Appendix A) leads to an immediate increase in consumer price inflation, with a magnitude of around 80 per cent of the initial increase in the real wage. The monetary policy contraction peaks in the third quarter and the consequent decline in output peaks around the fourth quarter. The real exchange rate appreciates, peaking in the third quarter.

Finally, the impulse responses to a negative demand shock (see Figure A.5 in Appendix A) are all of the expected signs. Output, inflation and real wages decrease and the monetary authority responds by lowering the nominal interest rate. In addition, the nominal and real exchange rates appreciate imme-
diately, which add further impetus to the declines in inflation and output. The nominal interest rate is lowered to a greater extent than the decline in inflation, which lowers the real interest rate and hence allows output (and inflation) to recover after six quarters.

In summary, the results of the impulse response analyses are in accordance with international and available South African literature but the peaks in the hump-shape responses are generally experienced noticeably earlier than otherwise found.

5 Forecast performance

In the literature, it is common practice to determine the forecasting capability of a DSGE model by means of comparison with forecasts generated by reduced-form models such as VARs or BVARs (see e.g. Smets and Wouters (2007) and Liu and Gupta (2007)). The forecasts generated by the DSGE model developed in this paper are compared to the consensus forecasts of economists polled by Reuters. Consequently, the quality of the forecasts generated by the DSGE model is measured against a benchmark equivalent to the ‘average economist’. The DSGE model’s forecasts for consumer price inflation (CPIX) and output growth (quarter-on-quarter GDP growth, annualised) are compared with the Reuters consensus forecasts for these variables. One- to seven-quarter-ahead forecasts are generated from the DSGE model by means of recursive estimations, with the first estimation being over the sample period 1990Q1 to 2003Q2, and the corresponding ex ante forecast over the period 2003Q3 to 2005Q1, while the last recursive estimation is over the sample period 1990Q1 to 2007Q3, and its corresponding ex ante forecast for the period 2007Q4. Hence, using the actual data for the entire sample period, i.e. 1990Q1 to 2007Q4, this recursive procedure allows for the calculation of 18 one-quarter-ahead forecast errors and 12 seven-quarter-ahead forecast errors for the DSGE model. Table 4 contains the root mean squared error statistics (RMSEs) of the DSGE and Reuters consensus forecasts for both CPIX inflation and GDP growth (qqa).

Table 4: Forecast error comparison: DSGE vs. Reuters consensus (RMSE)

<table>
<thead>
<tr>
<th>Quarters ahead</th>
<th>CPIX (yy)</th>
<th>GDP (qqa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DSGE</td>
<td>REUTERS</td>
</tr>
<tr>
<td>1</td>
<td>0.626</td>
<td>0.399*</td>
</tr>
<tr>
<td>2</td>
<td>0.984</td>
<td>0.779*</td>
</tr>
<tr>
<td>3</td>
<td>1.141</td>
<td>1.056*</td>
</tr>
<tr>
<td>4</td>
<td>1.050*</td>
<td>1.219</td>
</tr>
<tr>
<td>5</td>
<td>0.912*</td>
<td>1.357</td>
</tr>
<tr>
<td>6</td>
<td>1.069*</td>
<td>1.475</td>
</tr>
<tr>
<td>7</td>
<td>1.208*</td>
<td>1.532</td>
</tr>
</tbody>
</table>

* lowest RMSE

From Table 4 it is evident that the DSGE model outperforms the Reuters consensus with respect to forecasting quarter-on-quarter GDP growth (annualised) over the entire forecasting horizon. When considering the DSGE model’s ability to forecast inflation, it outperforms the Reuters consensus over
the medium- to longer-term forecasting horizon, i.e. 4 to 7 quarters ahead. The superior forecasting performance of the DSGE with respect to GDP growth, and especially inflation (over longer-term forecasting horizons), illustrates the valuable contribution which is to be made by DSGE models to macroeconomic forecasting as a whole.

6 Concluding remarks

In this paper an open economy New Keynesian model of the South African economy is presented. The model is broadly based on recent New Keynesian models designed for the purpose of monetary policy analysis, but more specifically on the models presented in Monacelli (2003) and Justiniano and Preston (2004). The structure of the model is specified to provide for imperfect pass-through (over the short term) of exchange rate changes to the prices of imported goods and thus to the CPI. In addition the model provides for external habit formation, partial indexation of domestic prices and wages to past inflation and Calvo (1983) wage and price setting.

The model is estimated with Bayesian techniques using data for the South African economy and its trading partners for the period 1990Q1 to 2007q4. The parameter estimates generally appear reasonable and in accordance with the literature. Impulse response analyses of the structural shocks yield results that are in accordance with prior expectations as far as the direction of the impacts is concerned, albeit that the responses generally peak and wear off earlier than the South African literature suggests. When comparing the DSGE model's forecasting capability with that of the Reuters consensus forecasts, the DSGE model's forecasts of GDP growth are consistently superior to the Reuters consensus forecasts over the entire forecast horizon (7 quarters), whereas inflation forecasts generated by the DSGE model are more accurate than the Reuters consensus forecasts over the medium- to longer-term forecasting horizons (i.e. 4 to 7 quarters).
References


**Appendix**

**Figure A.1: Loglinearised data**

![Loglinearised data](image-url)
Figure A.2: Impulse responses to a monetary policy shock

Figure A.3: Impulse responses to a cost-push shock
Figure A.4: Impulse responses to a wage mark-up shock

Figure A.5: Impulse responses to a demand (UIP risk premium) shock